Mathematical competence for all: options, implications and obstacles

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Introduction

Twenty years ago, as a response to the back-to-the-basics movement, the community of mathematics education started to emphasise the need to enlarge the components of what was generally considered as basic skills in mathematics. The inclusion of problem solving, reasoning, applications and the use of technology became major proposals in several programmatic documents. For example, the Agenda for Action (NCTM, 1980) stated that "basic skills in mathematics [should] be defined to encompass more than computational facility" (p. 1), and criticised "the 'back-to-the-basics' movement for its tendency to place a low ceiling on mathematical competence" (p. 6).

In the last two decades, the continuous evolution of the society, together with the developments in science, technology and education, led us to consider an increasing number of aspects and to deal with more and more complex problems when discussing what school mathematics for all should be. Quoting again the NCTM, the notion of 'mathematical power' was referred to in the early nineties in the following terms:

> Mathematical power includes the ability to explore, conjecture and reason logically; to solve nonroutine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other intellectual activity. Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate and use quantitative and spatial information in solving problems and in making decisions. Students' flexibility, perseverance, interest, curiosity and inventiveness also affect the realization of mathematical power.
>
> (NCTM, 1991, p. 1)

About ten years later, a similar view seems to have guided the new 'standards document' of the NCTM. Together with a number of improvements and clarifications, a stronger emphasis is put on 'equity', which becomes the first principle for school mathematics: "the vision of equity in mathematics education challenges a pervasive societal belief [...] that only some students are capable of learning mathematics" (NCTM, 2000, p. 12).

We are indeed dealing with well-known and persistent problems. More than a decade ago, Resnick (1987) pointed out, as an outcome of cognitive research, that complex thinking processes are involved in even the most apparently elementary mental activities and therefore 'basic' and 'higher order' skills cannot be clearly separated. However, she added, a new and very demanding challenge to educational reform results from the fact that "no educational system has ever been built on the assumption that everyone, not just an elite, can become a competent thinker" (p. 45).

We may use a broader notion of 'basic skills', or some definition of mathematical 'power' or 'proficiency', or any other terms, in association to the mathematical education that should be given to all students. However, the way we define and interpret it and, additionally, how we can make some progress in ensuring that this is related to actual educational change, remains a crucial issue.

Recently, in the context of an innovative movement in Portuguese basic[1] schools, the need to re-conceive the view about the curriculum led us to consider the concept of 'competence' and the process of innovation as major aspects. With respect to mathematics, the focus is on the way mathematical competence for all may be interpreted. The options that were available, some implications and the emerging obstacles raise a number of points for reflection that might be of general interest.

Recent developments in portuguese basic education

In 1996, the Portuguese Ministry of Education proposed a national debate about the major problems of 'basic education' — which means, from 1986 on, general and compulsory education for 6–15 year olds. Although a comprehensive and 'inclusive' basic school of nine years for all, similar to the Scandinavian traditional organisation, had been created, it became plagued with high dropout and failure rates. Since the restoration of democracy in 1974, there was a rapid development of the educational system with very 'advanced' laws in areas such as the extension of compulsory education or teacher education, and even a clear tendency to give a larger administrative autonomy to the schools. However, this development strongly contrasted with the tradition of centralised and rigid views about the curriculum, as a set of disciplines with detailed syllabi indicating for each subject what (and how) must be 'covered' in each school grade (for more details see Abrantes, 2001).

Curriculum development as a project

For this reason, the emphasis of the debate was put on the curriculum. Central issues included the concept of curriculum, the way it should be formulated at a national level and the role of schools and teachers in the process of developing and managing it. In 1997, after one year of discussion and preparation, schools were invited to join a national project, labeled as 'flexible management of the curriculum', which aimed at giving the schools a larger autonomy in the decisions about all aspects of the teaching and learning process. In particular, this autonomy was related to activities to be developed and the time and space dedicated both to the various disciplines and to three new cross-curricular areas: a 'project area', an 'oriented study area' and a 'civics area'.

The ultimate goal of the movement was to support the gradual creation of a new curricular organisation based on a more autonomous and responsible role of the teachers and their collective structures in school. In fact, this movement was justified by the need to promote a new conception of the curriculum, both the intended curriculum and the implemented one. On the one hand, it required educational authorities to express the curriculum in terms of 'essential competences' and types of 'educational experiences' that should be considered for all pupils (in each cycle), rather than in the form of 'programmes' indicating the content topics to be covered (syllabi) and the corresponding methods of teaching (for each grade). On the other hand, it challenged the teachers and the schools to make adequate decisions for the specific pupils they work with, taking into account their cultural and social backgrounds, their educational needs and the resources that exist or can be made available. Under the guidance of a national curriculum expressed in broad terms, the process of curriculum implementation is seen as a project to be conceived of and developed by the school, including more specific projects concerning each individual class.

Schools could join this movement by proposing their own curricular projects, under the condition of satisfying a minimal number of general rules. In 1997/98, only 10 schools participated, but this number increased significantly all over the country in the following years: 33 (in 1998/99), 92 (in 1999/00), and 184 (in 2000/01). These schools constitute a sort of network exchanging materials and points of view, and participating in local, regional or national meetings organised by educational authorities or by the schools themselves. Many teachers have been involved in in-service initiatives, namely workshops and small projects — for which teachers have been credited in their careers in the same way as for taking traditional courses. The co-operative work among teachers inside the school has become indeed the hallmark of the movement. Another interesting result was that leading teachers of the network schools started to receive invitations from colleagues in other schools and from meeting organisers to participate in conferences, debates and workshops, the kinds of activities traditionally reserved for researchers and teacher educators.

In the meanwhile, Ministry of Education started to produce discussion documents regarding the so-called 'essential competences'. Some documents focused on aspects transversal to all school subjects, while others were related specifically to particular disciplines. This activity has been developed by working groups with strong participation of members of associations of teachers, researchers and other professionals. Between 1999 and 2001, these documents have been discussed, criticised and modified in a process that took into account the feedback from the schools and the contributions of universities and professional associations.

In January 2001, a new law was adopted. From now on, there would be no compulsory uniform regulations about the exact amount of weekly time and the precise topics to be considered in each grade and in each discipline. Instead, under the guidance of the national document stating the essential competences and educational experiences for all

(DEB, 2001), schools would conceive their own curricular projects, both at a school level and at a class level. For each cycle, the document indicated the minimal time to be dedicated to each curricular component (group of disciplines or interdisciplinary areas) and the maximal number of hours per week to be devoted to compulsory activities.

This new law includes a number of recommendations that emerged from the experience of the schools involved in the movement — which in any case may be adapted or modified. One of them is to organise class activities in periods of 90 minutes, instead of the traditional lessons of 50 minutes. The justification is the creation of better conditions to promote practical and investigative work in the classroom, as well as the use of technology and other materials, and the goal of reducing the number of different subject matters in each day.

A new stage of the process of curriculum innovation will now begin. Refusing a top-down model for development, the guidelines are far from being completely 'ready'. The way in which programmes will evolve to constitute working materials for teachers, or the evolution of textbooks, are examples of problematic issues for debate.

The process of educational change

The most original aspect of the recent development in the Portuguese educational system is probably the fact that a curricular innovative movement at a national level is not following the RDD (standing for 'research-development-dissemination') model.

The criticism of this strongly dominant model of curriculum development and implementation is far from being recent. In the area of mathematics education, twenty years ago, Howson, Keitel and Kilpatrick (1981) have discussed its origins, assumptions and consequences, pointing out emergent alternative perspectives. More recently, in the context of the so called realistic mathematics education, Dutch researchers have carried this discussion forward in new and promising directions.

Gravemeijer (1994) explains that, in his approach, curriculum development is embedded in a holistic framework, taken from the concept of 'educational development' in Freudenthal's (1991) sense. The central idea is that the process of curriculum innovation has to consider all the actions needed from the initial purpose to the actual change, incorporating teacher education, counseling, assessment and opinion shaping. Furthermore, unlike the RDD model, initial theory is more like a philosophy or a vision and it will evolve in the interaction between theoretical and empirical justifications.

The new ideas have influenced both research and the development of innovative small-scale projects. However, at a national level, it is the RDD perspective that, generally, shapes curricular reforms, as it was the case of the Portuguese reform of 1990: teams of invited experts prepared new curricula; these curricula were implemented in a small number of 'experimental schools' where motivated teachers prepared their own materials; finally, after slight corrections, the curricula were extended to all schools. The 'consumers' were introduced to the new finished 'product', usually in the form of new textbooks. It is not surprising that later studies have pointed out visible problems of low take-up, dilution and corruption of major ideas of the intended curriculum — to use the terms of Burkhardt (1989). Based on a simplistic analysis of the situation, some voices argued that the 'solution' was the promotion of intensive training courses to introduce the new topics and methods to the teachers.

The current movement is of a very different nature. Past reforms have always introduced interesting innovations, but all of them left unharmed the power of central authorities in defining the curriculum, the usual way of testing and implementing it, the traditional separation between curriculum guidelines and school organisation, and the nature of teachers' professional activity. For the first time in the history of our education, we are now dealing with changes in all these aspects.

The tendency to view the curriculum as a project, in the context of school as a *learning organisation* has been increasingly discussed in the literature (for example, Fullan and Hargreaves, 1992; Fullan, 1993; Goodson, 1997). From a theoretical perspective, the recent evolution in Portugal is far from being original. However, this approach to curriculum change is not so common as a political option at a national level and it seems to be relevant to identify and discuss emergent obstacles and problems. Before we get to this point, however, it is useful to elaborate on the adopted concept of 'competence' and on the way this term is interpreted in the specific context of the mathematics curriculum.

The concept of 'competence'

The shift from content topics and objectives to competences requires a clarification of the meaning of the term 'competence'. First of all, as Perrenoud (1997) observes, although there is a possible confusion with a behaviourist interpretation, the term 'competence' does not indicate some kind of specific behaviour that 'can be observed', neither does it refer to performance. In this author's view, competence is related to the process of activating resources (knowledge, skills, strategies) in a variety of contexts, namely problematic situations. Perrenoud quotes Chomsky (1977) to support the distinction between competence and performance, and the idea that competence is related to the ability to improvise, but emphasising the fact that, in his view, competence develops as a result of learning and not in a spontaneous fashion.

Short (1985) has shown that the concept of competence may be used (or misused) with several different meanings ranging from a connotation with behaviour and performance to identification with a quality of a person or a state of being. In this last conception, the holistic nature of competence is emphasised. Knowledge is obviously involved, as well as the skill necessary to use it, but this use is an emancipatory action, based on reflection and implicating some degree of autonomy.

A broad conception of competence allows us to distinguish it from a possible confusion with task-oriented skill. Maybe this connotation comes from the fact that vocational training may have been an entrance door for the use of the term in education. However, it seems to be relevant to observe how a recent document of the European Centre for the Development of Vocational Training tries to clarify the distinction of concepts such as skill, qualification and competence:

> The concept of competence is more comprehensive (. . .). In the literature,

> the increasing use of competence reflects the attempt to legitimise the active participation of individuals in coping with mastering or orientating usual or changing situations.
>
> CEDEFOP, 1998

It may also be interesting to note that there is a parallel evolution of the key concept used by the studies on literacy. Initially, 'alphabetisation' was identified with school attendance. In a second phase, the important matter was the acquisition of knowledge, whether or not the person had attended a given school level. Finally, the focus of literacy moved from the acquisition to the use of knowledge in concrete situations (Kirsch and Mosenthal, 1993).

The concept of competence that was adopted in the Portuguese innovative movement is in fact related to the reflexive and purposeful use of knowledge and to autonomy. In this sense, it intends to emphasise the integration of knowledge, skills and attitudes, where integration is the key idea. In the reform of 1990, the programmes for the various disciplines stated three lists of general goals, corresponding separately to knowledge, skills and attitudes, while the guidelines for each school grade indicated the content topics to be covered, together with 'specific objectives'. As a consequence, a common interpretation of the intended curriculum tended to view skills (for example, deductive reasoning or problem solving strategies) and attitudes (for example, persistence or solidarity) as elements to be 'added' to the content knowledge.

On the other hand, the choice of the expression 'essential competences' is a deliberate attempt to distinguish what is being proposed from the 'basic skills' or the 'minimal objectives', which were common expressions in the official discourse some years ago. This distinction is a particularly important pedagogical and political issue in a country where education for all is a relatively recent principle and it is necessary to resist the systematic proposals to achieve this goal by creating hierarchies and inequalities among students.

The case of school mathematics

In Portugal, the community of mathematics education (teachers, teacher educators and researchers) has become quite strong in the last fifteen years. For example, the Association of Teachers of Mathematics (APM), created in 1986, is the biggest association of this kind, having had, in 2001, about 6000 members. This number would be equivalent (proportionally to the population) to 24000 in Spain and more than 30000 in France, Italy or UK! A considerable number of teachers are following post-graduate studies on education, in particular on various aspects of the didactics of mathematics, and some of them are participating in projects, which involve both a strong component of curricular innovation and a research dimension. Since the creation of APM, and following the experience of the MAT789 project (Abrantes, 1993), the co-operation between teachers and researchers became an interesting feature of Portuguese mathematics education — see, for example, Oliveira et al. (1997) or Porfírio and Abrantes (1999).

However, this community is under a strong public pressure, as a consequence of the results of students' assessment in the exams at the end of secondary school, or the very low position of Portugal in the rankings of some international comparative studies. Like elsewhere, as Keitel and Kilpatrick (1998) have pointed out, these rankings are frequently used without any serious consideration about what they mean or do not mean. They are used, for example, as an argument to propose not only the return to a greater emphasis on training routine skills, but also 'solutions' like more exams and more comparative studies! Many teachers seem to have mixed feelings. They lean towards the new ideas about curriculum development, but they also have to live in a culture of school still dominated, inside and outside of the school, by traditional ideas and values.

Mathematical competence in a national curriculum for basic education

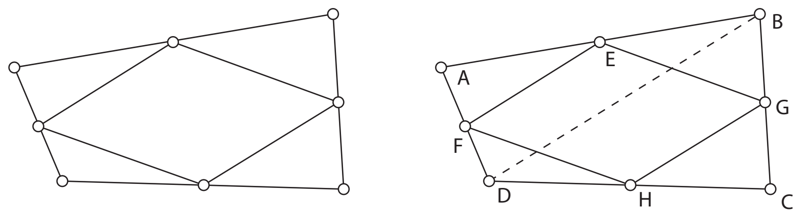
In the national document about the "essential competences" in basic education, the section dedicated to mathematics was based on a previous work of Abrantes et al. (1999), with a number of changes introduced as a result of the public debate. The ultimate goal of teaching mathematics in basic schools was stated in the following terms:

> Mathematics is part of the cultural patrimony of mankind and a way of thinking, which should be made accessible to all. Every child and adolescent should have the opportunity
>
> – to become acquainted, at an adequate level, with the fundamental ideas and methods of mathematics, and to appreciate its value and nature;
> – to develop a capacity of using mathematics to solve problems, reason and communicate, as well as the self-confidence to do it.
>
> Major aspects of 'mathematical competence' for all was expressed as follows:
>
> The mathematical competence that all pupils should develop through basic education integrates attitudes, skills and knowledge, and includes:
>
> – the disposition to think mathematically, this is, to explore problematic situations, search for patterns, formulate and test conjectures, make generalisations, think logically;
> – the pleasure and self-confidence in developing intellectual activities involving mathematical reasoning and the conception that the validity of a statement is related to the consistence of the logical argumentation rather than to some external authority;
> – the capacity to discuss with others and communicate mathematical thoughts through the use of both written and oral language adequate to the situation;

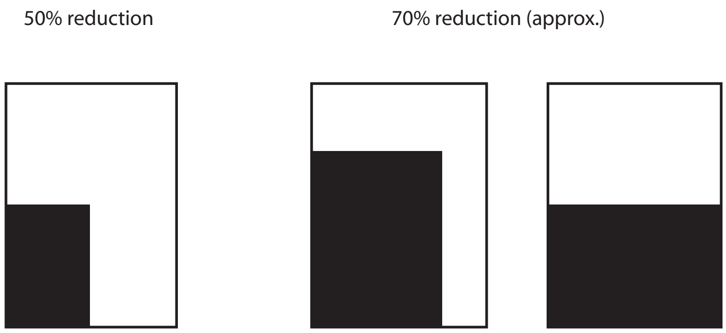
– the understanding of notions such as conjecture, theorem and proof, as well as the understanding of the consequences of the use of different definitions;
– the disposition to try to understand the structure of a problem and the capacity to develop problem solving processes, analyse errors and try alternative strategies;
– the capacity to decide about the plausibility of a result and to use, according to the situation, mental computational processes, written algorithms or technological devices;
– the tendency to see and appreciate the abstract structure underlying a situation, from daily life, nature or art, involving either numerical or geometrical elements or both:
– the tendency to use mathematics, in combination with knowledge from other areas, to understand real world situations, and a critical attitude towards the use of mathematical methods and results. (DEB, 2001)

In the second part, the document elaborated on what this means, in terms of each of the main areas of the mathematics curriculum — Numbers and Operations, Geometry and Measurement, Statistics and Probability, Algebra and Functions. Finally, pointing to problem solving as a general guideline, it stated that all students should be involved in mathematical investigations, projects, practical tasks, discussions, reading and writing about mathematics, exploration of connections inside mathematics and relating it to other areas. Moreover, they should have the opportunity of using technology, manipulatives and games in relation to their mathematical activities.

In Abrantes et al. (1999) we can find some discussion about the meaning of the aspects of mathematical competence mentioned above. For example, understanding of notions such as conjecture, theorem and proof is illustrated by a well-known example. Joining the midpoints of many different quadrilaterals, by using appropriate dynamic geometry software (Geometer's Sketchpad or Cabri Geometry, for instance), we can conjecture that we always get a parallelogram. But this does not prove such a statement and, maybe more importantly, it does not explain why this happens. In fact, proof may be relevant to understand why something happens rather than to be sure of something that we do not doubt. With some help from the teacher, if necessary, and using elementary properties of triangles, students may get a convincing proof of the statement.



Another example is used to support the discussion on the disposition to try to understand the structure of a problem. If we try to reduce a text or a drawing printed on an A4 format to an A5 format, by making a copy, we may introduce a 50% reduction, since an A5 sheet of paper is exactly the half of an A4 sheet. In many machines, our text or drawing would then occupy not a half but a quarter of the initial area! If we think about what happened we may note that, in fact, $0.5 \times 0.5 = 0.25$ and, then, even mentally, we may estimate 0.7 as a fair reduction. Obviously, the situation may be explored in many different directions.



Mathematical competence for all: Options and implications

The intention to integrate knowledge, skills and attitudes is quite apparent in the above list of major characteristics of mathematical competence for all. The role of attitudes is especially emphasised, it being assumed that school should cultivate a broad disposition to higher order thinking (Resnick, 1987). This author also focuses on the relations between cognitive abilities and motivation: "Motivation for learning will be empty if substantive cognitive abilities are not developed, and the cognitive abilities will remain unused if the disposition to thinking is not developed" (p. 50). In this aspect, the current formulation seems to be stronger than the definition of the NCTM (1991): attitudes not only 'affect' [the realisation of 'mathematical power'], they are an integral part of it.

A second characteristic of the above formulation is the clear concern with beliefs and conceptions about mathematics. Although there is evidence of the important role that beliefs and conceptions play in students' learning processes (Schoenfeld, 1992) — "the invisible hand operating in mathematics instruction" (to quote Borasi, 1990) — this aspect has been almost always absent in the curricular guidelines defined at an official and national level.

A third relevant characteristic is the explicit attention to the nature of mathematics. As Bishop (1991) points out, it is not enough to teach (some) mathematics, it is indeed necessary to educate *about*, *through* and *with* mathematics. To achieve this goal, mathematical competence cannot be seen as independent from the educational experiences that all children should live in school. Investigations and projects, involving both mathematical ideas and their relations with different sorts of problems, become central guidelines, not mere methodological suggestions. The ultimate goal is to develop understanding and appreciation of the nature of mathematics, rather than 'enriching' the knowledge of facts and the training of procedures with some sort of rhetoric about it.

Several approaches have been proposed to organise the curriculum around relevant mathematical activities, as opposed to a curriculum based on content topics. Bishop (1991), in the search for 'mathematical similarities', points out six activities that are "significant (…) for the development of mathematical ideas in any culture' (p. 23) — counting, locating, measuring, designing, playing and explaining. Goldenberg (1996) proposes 'habits of mind' as organisers of the curriculum — for example, the tendency to describe relations and processes or the tendency to look for invariants. The NCTM (2000) formulates 'process standards' in reference to "ways of acquiring and using content knowledge" (p. 29).

We may adopt different approaches, concepts and terms to organise the curriculum, but if we want to help all children to develop their mathematical competence, we have to question the basis of a 'technique-oriented' curriculum (to use the terms of Bishop, 1991). Such a curriculum is built on the perspective of training procedures, skills and rules (for all) with the expectation that this kind of training will constitute (for some) a prerequisite to future uses of mathematics. However, "a technique curriculum cannot educate (…), for the successful child it is at best a training, for the unsuccessful child it is a disaster" (Bishop, 1991, p. 9). The formulation of the curriculum in terms of competences and relevant mathematical experiences does not automatically guarantee success. It is necessary to connect it to the purpose of striving against school failure and taking into account all children, namely those with a cultural background not similar to that of the 'traditional school' (Perrenoud, 1997).

This perspective has several implications and consequences. One of them is the need to reconsider the extension and complexity of topics included in the curriculum. Bishop (1991) argues that curriculum should be relatively broad (in the variety of contexts offered) and elementary (in the mathematical content). Similarly, when discussing the problem of the construction of competences in school, Perrenoud (1997) points out that, if our option is education rather than instruction, then it is necessary to reverse the tendency to include more and more topics in the compulsory school curricula.

Obstacles

The innovative movement in Portuguese basic education — changing the traditional way of formulating and developing the curriculum — has been raising a number of problems and obstacles that should be identified and discussed. Four of these obstacles will be briefly discussed.

Political pressure and 'popular' views of education

Creating an alternative to the RDD model for curriculum development is a difficult task even in small-scale projects but it becomes much harder when dealing with a reform at a national level. This is particularly evident where there is a tradition of a centralised system. A dominant conception of development calls for well-defined and 'teacher-proof' curricula, carefully tested before generalisation, and high quality textbooks as key factors to improve teaching and learning. This seems to be a popular view shared by influential sectors of the scientific community and the society at large. Public opinion is a necessary, yet very complex, element to be considered. The strength of a movement based on the interaction between theoretical and practical developments, which is a gradual and long-term process in nature, seems to be at the same time its weakness. At a political level, it is not easy to respond to the accusation of delaying quick and clear answers. Guidelines, which appear to be ill defined, as well as the absence of new curricula with exact and precise indications, become a basis for criticism. It is interesting to note that, in this context, the single proposal to give schools the possibility of organising classes in periods of 90 minutes is pointed out almost like a 'revolution', provoking unusual public debates about education.

Tension between autonomy and security

Even inside schools, together with the public pressure mentioned above, it is obviously difficult to deal with uncertainty. If the current movement constituted an opportunity for innovative teachers and school leaders to organise teaching and learning contexts more adequate to their students, for others it is a source of problems. There is a tendency to look for models in the initiatives of 'more experienced' schools; however, this becomes difficult when there are several different models and there is not an official one. This tension between autonomy and security is amplified by the emergence of the rhetoric associated with the educational change. This is a common phenomenon in periods of reform, but it is particularly negative when change is a matter of process, not only of content. The tendency to emphasise the 'pedagogically correct' and criticising all 'deviations', characteristic of all sectors including some educational authorities and researchers, is in fact a force towards the adoption of uniform solutions, which is contradictory with the goal of a larger autonomy of schools and teachers.

A broad concept of competence is a difficult one to accept

The adopted concept of competence is not easy to be explained and misunderstanding tends to emerge. Doubts and criticism on the proposed framework showed that a broad concept of competence is difficult to be widely accepted.

In the case of the mathematical competence, terms like *disposition* (to think mathematically), *pleasure* (in developing intellectual activities) or *tendency* (to look for the abstract structure) have been especially criticised with the argument that it is very difficult to make such things 'operational'. This seems to reflect the difficulty in getting the understanding or the acceptance of the idea that integration of cognitive and non-cognitive components is essential to the concept of competence.

This is not a new problem, caused by the adoption of a terminology based on the concept of competence. Similar discussions tend to occur, regardless of the terms in which we couch our definitions. Proposals aligned with using and applying mathematics in schools (de Lange, 1996), valuing mathematical investigations (Ernest, 1991) or adhering to the 'rebirth' of project teaching (Bishop, 1995) are consistent with the development of students mathematical education in a broad sense. Although they are sometimes accepted as complementary methods or a sort of applications, they are not generally viewed as the essence of the curriculum. The problem seems to be the resistance to question and discard the technique-oriented curriculum and a central aspect of this problem has to do with assessment and control.

Assessment

Very often, conceptions and practices about assessment are not aligned with developments in other curricular components. Although this is not a recent problem, it seems that there is "an increasing mismatch and tension between the state of mathematics education and current assessment practices" (Niss, 1993, p. 4). Assessment of the development of mathematical competence requires observation in different situations and confidence in the teacher's professional judgments, while the central role of standardised tests and exams may become a strong obstacle to flexibility, adequacy and diversity.

A wider range of assessment modes and instruments — for example students' written productions — has begun to be increasingly accepted and used in the last years but the recent influence of the way in which international comparative studies tend to be interpreted and used has a powerful effect against educational change. These studies could be useful to provide information about aspects of mathematical competence. However, by presenting scores as indicators of curriculum achievement, tending to view curriculum as unproblematic, context-free and culture-free (Keitel and Kilpatrick, 1998) and emphasising rankings, they constitute a serious obstacle to new conceptions and practices of curriculum development.

Keitel and Kilpatrick (1998) show how, in the USA and in Germany, lower scores in the TIMSS test in comparison with Asian countries have been used as an argument to urge teachers to return to a curriculum based on 'core knowledge' or to claim for funding to develop more sophisticated instruments for measuring students' performance. In the UK, the Secretary of State for Education and Employment stated that "numeracy is an important life skill, but evidence shows that standards of school mathematics have not been high enough to enable us to compete internationally" (Department for Education and Employment, 1998).

In Portugal, the situation is similar, with the difference that scores were even lower than in other western countries. Porfírio and Abrantes (2000) presented a paradigmatic example of popular notions of culture, school and mathematics, taken from a TV programme organised in the wake of the publication of the international rankings. All evening, the moderator and two other opinion-makers criticised school for the low scores of students in mathematics tests. At the same time, they identified 'culture' with knowledge about facts related to poetry, history and geography. They had no idea about the nature of mathematical activity or the way in whichmathematical ideas are generated, how they develop and relate to other ideas, and they could hardly remember any 'pieces' of mathematical content knowledge. "For them, however, this fact was not relevant in cultural terms" (Porfírio and Abrantes, 2000, p. 278).

In Portugal, while the movement of curriculum innovation tries to emphasise flexibility and adequacy of teaching methods to students' characteristics and consideration of their social and cultural backgrounds, the 'societal' values of competitiveness and standardisation – of guidelines, methods and 'objective' results – tend to favour the reinforcement of a technique-oriented curriculum. The Ministry of Education is strongly criticised for not organising rankings of schools based on students' scores in national tests. A popular argument is that everybody has the right to know what are the 'best schools', the 'best teachers' and the 'best teaching methods'. The need to compete with other countries is generally added as well, together with the argument of 'globalisation'. As Keitel (2000) observes, this concept is ambiguous, having frequently a connotation opposite to the values of cross-country collaboration, interaction and co-operation at different levels.

Conclusions

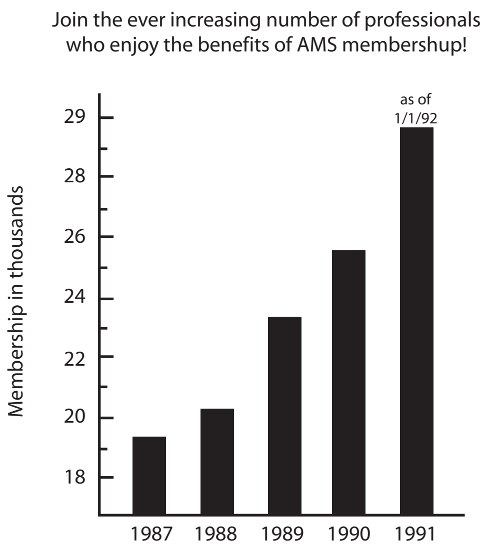
In conclusion, I propose three remarks that intend to suggest avenues for future work and research.

The first remark is that, in spite of the development of research on education, and specifically on mathematics education, the focus has been almost always on individual learning processes or small-scale projects. We need to know much more about the processes of curriculum evolution on a large scale. In fact, the observation of Burkhardt (1989) that the study of curriculum change on a large scale is neglected, "partly for practical reasons but mainly because of a lack of attention to system issues" (p. 9) seems to have lost nothing of its actuality.

The second remark is related to the first one. A major obstacle to develop promising approaches to curriculum innovation may be probably found in political and social issues, namely in 'popular' conceptions about education and educational change as well as about mathematics and mathematics learning in schools. We must be aware of, and study, the influence of these conceptions in favouring the perpetuation of a technique-oriented curriculum. This is not a story of teachers and students only. What is the role of the scientific community, and the society at large, and how does it work? Borasi (1990) admitted, "social stereotypes (…) may certainly play a role in shaping students' conceptions" (p. 177). Freudenthal (1991) included 'opinion shaping' in the set of actions needed in a process of educational development. However, we need to know much more about these issues.

The third remark is that, if we are dealing with actual change at a national level, research and debate on the evolution of mathematics education cannot be isolated from the processes of educational change at large. Moreover, we have to focus on the ways mathematics curriculum relates to school curriculum as a whole, which seems to be especially relevant as far as basic education is concerned. In particular, when characterising the mathematical competence for all, it is necessary to make more explicit the uses of mathematics in relation to other areas and the role of mathematics in education for democratic citizenship.

The following final example raises some interesting issues for reflection in this respect. The illustration shows a bar graph taken from a publicity flyer of an organisation. It tries to emphasise the increasing number of members. A mistake in the position of the point corresponding to 29 (thousand) and, especially, the fact that the membership scale begins at 17 (thousand), instead of zero, are responsible for an illusion. Concerning the last two years represented, taking the approximate values, we can estimate an increase not greater than 15%; however, taking the sizes of the bars the increase seems to be greater than 45% — and, in fact, this is the visual effect that we get. We all know that there are many examples of this kind of situations. The original aspect of this one is the fact that it comes from a powerful organisation of professional mathematicians!



Note

[1] The term 'basic schools' (básico, in Portuguese) was introduced in 1986; it refers to a general, compulsory education for all 6 to 15 year old children in Portugal.

[2] This article is adapted from a plenary talk at CIEAM 53, the fifty-third meeting of the International Commission for the Study and Improvement of Mathematics Education in Verbania, Italy, July 21–27, 2001.

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Abstract. In the recent evolution of Portuguese basic education, the concept of competence and the process of curriculum development became major aspects of an innovative movement at a national level. Concerning school mathematics, this evolution provided an opportunity to explore the way in which mathematical competence for all could be defined and interpreted with the purpose of abandoning a technique-oriented view of the curriculum and the process of developing it. After describing the context and exploring the concept of competence, this paper presents a formulation for the characteristics of mathematical competence for all and discusses some of its implications. Finally, the paper tries to identify the obstacles that tend to emerge when such an approach is proposed in the context of a national curriculum.

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