# Curriculum as a vehicle for reform in U.S. mathematics education

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> *Mathematics curriculum - what it should be, what it is, how it is organized and sequenced, how it is taught, and what students learn — is the core around which mathematics education revolves.*
>
> Center for the Study of Mathematics Curriculum (2004-present)

Throughout the history of American education, debate over defining and implementing the school mathematics curriculum has been a central focus of educational reform initiatives. While most experts agree that teaching is more directly related to student learning, curriculum is tangible and changeable — more easily legislated and governed than teaching. Thus, reformers and policy makers have often used curriculum as the lever of choice for promoting change and improvement in school mathematics programs (NRC, 2002).

The term *curriculum* is used in the United States to refer to the broad construct of what society values and expects elementary, middle and secondary students to learn as well as the materials (textbooks) organized by authors and publishers and used by teachers and students regularly in mathematics classrooms. Both standards and textbooks are specific curriculum products — used by teachers and students on a daily basis to organize and guide instruction — that influence student opportunities to learn and the quality of learning experiences.

Figure 1 illustrates how these and other forms of curriculum connect to and influence other components of the educational system as well as the forces that influence type of curriculum. Since the U. S. may be unique in how curricula are constructed and disseminated, this model may be limited in its applicability elsewhere. However, the model provides a reminder of the significant role that mathematics curriculum development and implementation play in influencing U.S. classrooms and school mathematics programs.

The model displayed in Figure 1 distinguishes three major types of curriculum — intended, textbook and assessed. The *intended curriculum* refers to the official specification of what is to be taught and learned. The *textbook curriculum* translates the intended curriculum in the form of sequenced lessons that students and teachers use on a daily basis. Since up until recently in the U.S. there has been no national consensus on the intended curriculum, textbook materials have included many topics, often repeated from earlier grades. Consequently, teachers needed to make decisions, often on a daily basis, about what to use from the textbook, what to skip, and what to supplement from other resources. In this sense, teachers have been active developers of the implemented curriculum, influenced by experiences that occur within the mathematics classroom as well as by the instructional materials available to them. The *assessed curriculum* noted in Figure 1 is the mathematics that is the focus of high-stakes assessments used to monitor student progress. In the U.S. these assessments are a prominent accountability tool so schools and teachers are motivated to focus on this content although it generally represents only a part of the intended curriculum. The interaction of these three forms of curriculum influence the *implemented curriculum* — the actual learning opportunities that play out in U.S. classrooms.

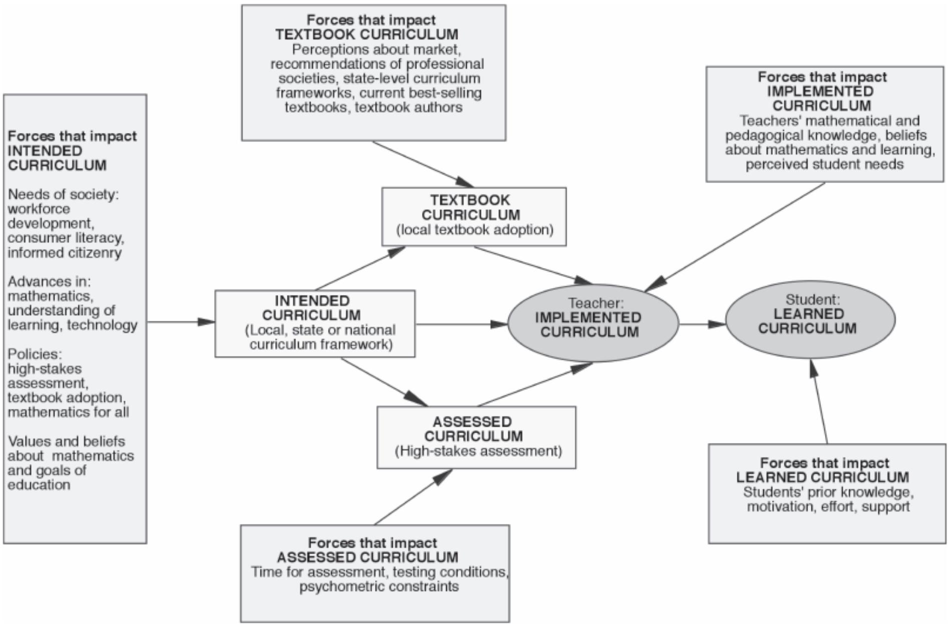

**Figure 1** — Model depicting the relationship of various types of curriculum and the forces that influence the content of those curricula.

Figure 1 also includes some of the major forces that influence each form of curriculum and, in turn, student opportunities to learn. For example, current federal educational policies call for increased articulation of learning goals accompanied by annual assessments to measure student progress in relation to these goals. (U.S. Department of Education, *No Child Left Behind Act*, 2001). These policies influence the textbook curriculum as well as the implemented curriculum emphasized by the teacher (Jennings

& Renter, 2006). In addition, the teachers' own beliefs and experiences as well as student's prior knowledge and motivation influence the ways in which students interact with mathematics. Instructional strategies, whether explicitly suggested by the textbook lesson or applied by the teacher independent of available curricular materials is another key influence on student learning.

The current decade is likely to witness an unprecedented focus on curriculum in the U.S. The adoption of the *Common Core State Standards for Mathematics* (CCSSM) by all but a few states (e.g., Alaska, Minnesota, Texas, Virginia) represents a revolution. For the first time ever in the U.S., a large majority of schools, teachers and students will focus on common, specific and grade-level focused learning goals for mathematics. Coupled with mandated common annual grade-level assessments aligned to CCSSM, this initiative is likely to impact other important systems and factors critical to student learning including instruction, curriculum materials, teacher training, and course-taking/graduation policies.

This paper provides a summary of current curriculum reform initiatives in the U.S. The discussion is based on a set of core assumptions about the role and impact of curriculum, including:

- *A well-articulated, coherent, and comprehensive set of K-12 mathematics learning goals/standards is necessary to large-scale improvement of school mathematics.*
- *Mathematics curriculum materials play a central role in any effort to improve school mathematics.*
- *High quality curriculum materials require a continual development cycle of research-based design, field-testing, evidence gathering, and revision.*
- *Teaching and curriculum materials are highly interdependent and increasing opportunities for student learning rests on better understanding the relationship between curriculum and instruction.*

These assumptions have guided the work of the Center for the Study of Mathematics Curriculum (CSMC), a partnership involving curriculum developers, users and researchers. Major areas of work include: developing leadership capacity related to K-12 mathematics curriculum design, analysis, implementation, and evaluation; and advancing a research agenda related to K-12 mathematics curriculum, including the impact of curriculum materials on student and teacher learning (see `http://www.mathcurriculumcenter.org/` for additional information about CSMC).

In this paper, we provide a brief overview of the current movement toward common (national) curriculum standards in the U.S. We examine some of the procedures used in developing textbooks and discuss some of the challenges textbook authors and publishers face. We reflect on the critical role that mathematics textbooks play in determining the mathematics content students have an opportunity to learn. Finally, we speculate about the future of curriculum materials and the impact of CCSS on school mathematics programs.

## Moving Toward Common Curriculum Standards for Mathematics in the U.S.

Unlike most countries, the U.S. places governance for educational decisions at the state or local level, rather than at the federal (national) level. However, in 2001 the U.S. government exercised unusual authority by requiring states seeking support for particular federal initiatives to adopt "challenging academic content standards" in mathematics and to annually assess the progress of schools in supporting student learning matched to these standards (U.S. Department of Education, *No Child Left Behind Act*, 2001). However, no federal model of standards was offered. Instead, each state was encouraged to develop and implement its own curriculum standards and to monitor student learning of the standards using an annual assessment (generally, developed by the state) it deemed appropriate.

During this period, state standards were heavily influenced by two publications of the National Council of Teachers of Mathematics (NCTM). The first was the *Curriculum and Evaluation Standards for School Mathematics* published in 1989 and then revised in *Principles and Standards for School Mathematics* (NCTM, 1989; 2000). While not obligatory, the NCTM *Standards* influenced the general content of state and district level curriculum standards as well as the discourse about good teaching and assessment practices. While the NCTM *Standards* served as a general template, state standards were more specific. The increased specificity of state standards were due, in part, to the need to specify (for assessment purposes) the mathematics all students within a state were expected to learn at particular grades/courses. With the increased specificity came increased variance across state standards.

A review of state-level mathematics curriculum standards by the Center for the Study of Mathematics Curriculum (Reys 2006) confirmed that state level mathematics learning expectations varied along several dimensions including grain size (e.g., level of specificity), terms used to convey learning goals (e.g., some state standards avoided terms such as "understand" or "explore" because they were difficult to assess) and the grade placement of specific learning expectations. In particular, when mathematics topics were introduced, their trajectory of development across grades and the grade at which students were expected to know and apply particular mathematical content differed dramatically across the states. For example, some states' standards introduced computation with fractions (with common fractions such as 1/2) as early as grade 1 while others began instruction on the topic in grade 3 or 4. Some states' standards included an expectation for students to attain computational fluency with fractions by the end of grade 5 and other states called for this expectation at grade 8. The variability in grade placement (introduction, development, specification of size of numbers and expectation for fluency) of this and many other topics has had significant implications for textbook publishers, as well as state and national assessments of student learning. It has also spurred debate about the need for national consensus on curriculum standards for mathematics.

Discussion about national standards for school mathematics (or any other subject) have included heated debates. Goertz (2010) summarizes the typical positions,

> The arguments in support of national standards today echo those of the past: they will promote democracy, equity, and economic competitiveness. The arguments against national standards are also familiar: they will lead to the establishment of a national curriculum; one size does not fit all; and local communities, not the federal government, know what is best for their students. (p. 52)

Until recently, the NCTM *Standards* represented the strongest movement to provide a national framework for standards. In addition, many states were examining standards from high-performing TIMSS countries to increase the rigor of their standards. However, few states looked to each other for guidance or collaboration.

At a meeting of state governors in the March 2009 an agreement was formulated to work across state lines to develop common standards in mathematics and language arts/English. The goal was to increase the quality and rigor of mathematics standards. The National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) assembled a small writing group to produce a set of common core standards for mathematics and language arts/English. Fifteen months after the governor's meeting, the *Common Core State Standards-Mathematics* (CCSSM) were released in June 2010 (see `http://www.corestandards.org/`).

The writers drew upon a variety of expertise and resources in developing the CCSSM including mathematical content experts, cognitive scientists and mathematics education researchers and practitioners. Standards from high performing TIMSS countries (particularly Singapore and Japan) were also reviewed. In addition, writers drew upon, "learning progressions detailing what is known today about how students' mathematical knowledge, skill, and understanding develop over time." (CCSSM, p. 4)

CCSSM includes two types of standards — standards for mathematical practice and standards for mathematical content. The standards for mathematical practice (see Figure 2 next page) describe different types of expertise that mathematics educators at all levels seek to develop in their students. They are based on the "process standards" outlined in the NCTM *Standards* (problem solving, reasoning and proof, communication, representation, and connections) and on the strands of mathematical proficiency specified in the National Research Council's report, *Adding It Up* (2001) (adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition).

The standards for mathematical practice are underlying themes of emphasis throughout K-12 mathematics programs, continually connecting to and with important mathematical content. However, while the CCSSM begins its presentation with the mathematical practices, the majority of the document is devoted to specifying mathematical content that students in grades K-12 should study and learn. Very little attention is given to the mathematical practices beyond the initial description at the beginning of the doc-

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

**Figure 2** — CCSSM Standards for Mathematical Practice.

ument. Therefore, it will be critical that curriculum and professional development attend to the mathematical practices in ways that support teachers (and their students) if the vision of these standards is to be realized.

The K-8 Standards for mathematical content are organized within domains (Number and Operations, Operations and Algebraic Thinking, Measurement and Data, Geometry, Expressions and Equations, Statistics and Probability, Ratios and Proportional Reasoning, the Number System, Functions). The high school standards are organized by conceptual categories: Number and Quantity, Algebra, Functions, Modeling, Geometry, Statistics and Probability. The standards are intended to communicate with parents, teachers, and the general public. They do not specify any particular pedagogy. They emphasize both understanding and procedural fluency. A sample of the standards from grade three (CCSSM, 2010, p. 24) is provided in Figure 3 (see next page).

Following the release of the final draft of CCSSM in June 2010, each state was encouraged to review and consider adoption of CCSSM to replace their existing state standards. The federal government did not develop the CCSSM nor did it carry federal endorsement or mandate. So, in theory there was no political pressure on states to adopt these standards. Nevertheless, the CCSSM was initiated at the request of the National Governors Association and the CCSSO so there was strong support at the state level for the *Common Core State Standards*. In addition, the U.S Department of Education sponsored a grant competition (*Race to the Top*) challenging states to develop and submit plans for "implementing coherent, compelling, and comprehensive education reform" (see `http://www.ed.gov/blog/topic/race-to-the-top/`). In order to qualify for *Race to the Top* funds, the state needed to adopt CCSSM.

Eleven states and the District of Columbia submitted winning *Race to the Top* proposals and collectively received nearly $4 billion dollars to carry out their plans for educational improvement. Despite the financial incentive, some states (e.g., Alaska, Minnesota, Texas, and Virginia) opted out of adopting the CCSSM. Two primary reasons were offered by these states — the desire to retain local control for curriculum governance and the belief that their current state standards are at least as rigorous as CCSSM.

While not all states have adopted CCSSM, those that have represent nearly 90 percent of the U.S. student population. Thus, a new day is emerging in the landscape of

Domain: Number and Operations — Fractions
Cluster: Develop understanding of fractions as numbers.
Standards:

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by $a$ parts of size $1/b$.

2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

    a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into $b$ equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
    b. Represent a fraction $a/b$ on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size $a/b$ and that its endpoint locates the number a/$b$ on the number line.

3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

    a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
    b. Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
    c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate 4/4 and 1 at the same point of a number line diagram.
    d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

**Figure 3** — Example of domain, cluster and standards, CCSSM grade 3.

mathematics curriculum. Work is now underway to introduce and support teacher learning associated with CCSSM, to develop CCSSM-aligned curriculum materials and assessments, and to monitor the impact of common standards on the educational system in the U.S.

## The Role and Influence of Curriculum Materials

The influence of standards is strongest when translated into curriculum materials that guide the day-to-day decisions of teachers and help them focus on important mathematical learning goals in significant ways. Curriculum materials include textbooks typically designed for a semester or academic year of study, modules focusing on smaller amounts of mathematical content, workbooks, and/or computer software designed to support instruction and learning.

U.S. teachers of mathematics rely heavily on textbooks in their day-to-day teaching, and this is perhaps more characteristic of the teaching of mathematics than of any other subject in the curriculum. (Tarr, et al., 2006; Stein, Remillard, & Smith, 2007). Tyson-Bernstein and Woodward (1991) describe as ubiquitous the role of textbooks in American schools, and as a prominent, if not dominant, part of teaching and learning. U.S. teachers decide what to teach, how to teach it, and what sorts of exercises to assign to their students largely on the basis of what is contained in the textbook authorized for their course. (Robitaille & Travers, 1992

In spite of the acknowledged role and importance of mathematics textbooks, only recently (Schoenfeld 2002; Senk and Thompson 2003) has there been an expansion of research to study the effects of textbooks on students' learning of mathematics. Kilpatrick (2003) calls for additional scholarly work in this critical area, including establishing design principles for instructional materials that draw on research on how students learn (NRC, 1999). A National Research Council Committee chaired by Jere Confrey recommended a framework for the design of evaluation studies to assess the effectiveness of textbooks and other curriculum materials and offered recommendations to guide future efforts in this area (NRC, 2004).

Historically, U.S. mathematics curricula have been characterized as superficial, highly repetitive, and copious in the amount of content reviewed in any given year (Flanders 1987; Schmidt et al. 1997). Reviews of textbooks conducted by the American Association for the Advancement of Science and the U.S. Department of Education characterize many commercially developed middle school mathematics textbooks as "unacceptable" with regard to content emphasis (Kulm et al., 2000). Furthermore, these researchers found that, "many textbooks provide little development in sophistication of mathematical ideas from grades 6 to 8" and "most of the textbooks are inconsistent and often weak in their coverage of conceptual benchmarks in mathematics" (p. 1).

In an effort to influence and strengthen the quality of U.S. mathematics textbooks, in the early 1990s the National Science Foundation (NSF) invested over $90 million in K-12 mathematics curriculum development efforts (NRC, 2004). Teams consisting of mathematics educators, mathematicians, and classroom teachers worked together to produce mathematics textbooks that embodied "standards-based" characteristics (e.g., aligned to the NCTM Standards) including active engagement of students, a focus on problem solving, and attention to connections within mathematical strands as well as to real-life contexts (Reys, Robinson, Sconiers, & Mark, 1999; Trafton, Reys, & Wasman, 2001).

Pilot materials were field-tested in schools, some more extensively than others, and then revised before becoming commercially available. The resulting mathematics curricula represent notable exceptions to traditional textbooks that typically lack a research and development phase prior to release (Trafton et al. 2001). Figure 4 summarizes common design features of the NSF-funded *Standards*-based curricula (Hirsch 2007).

- Updated content including data analysis, probability, and, in the high school curricula, topics from discrete mathematics
- Focus on "big ideas" across grade levels and multiple representations
- Real-world situations that provide a connection between mathematics and the world in which students live and consider interesting
- Connections among ideas across mathematical strands and grade levels
- Incorporation of technological tools, especially calculators
- Attention to issues of equity and access
- Active engagement of students through investigations of important mathematical ideas and solving more-challenging problems
- Focus on depth over coverage to promote deeper understanding of important mathematical ideas
- Support for teachers to become stimulators and guides of inquiry
- Learning opportunities for teachers through extensive teacher guides and professional development opportunities
- Assessment embedded in the curriculum materials and used to guide instruction

**Figure 4** — Curriculum design features of the NSF-funded *Standards*-based curricula.

The mathematics textbooks that resulted from NSF-funded development grants provide the basis for enacting a different vision for teaching and learning — one that emphasizes student exploration, conceptual understanding, and use of contextualized mathematics problems. The high school materials integrate the study of algebra, geometry and statistics rather than treating these topics in separate courses.

The NSF-funded materials have been reviewed by committees of the U.S. Department of Education (1999) and AAAS (Kulm et al., 2000) and judged of exemplary quality compared to other commercially available textbooks. Studies have reported positive growth in the mathematics learning, particularly related to reasoning and problem solving, as a result of use of the new curriculum materials (Senk and Thompson 2003; Schoenfeld 2002; Tarr, et. al., 2008; Harwell, et. al., 2007; 2009).

Estimates of the market share of NSF-funded textbooks (percentage of textbooks currently in use) range from 10–20 percent of students and teachers at the secondary level and from 20–30 percent at the elementary level (Education Market Research 2006). Use of these textbooks by a significant segment of the school population is evidence that NSF's effort to stimulate new models of textbooks has been successful. In addition, commercially developed textbooks are beginning to incorporate some of the features of the

NSF materials (e.g., extended investigations, focus on multiple strategies, and encouragement of students to discuss, explain and defend their thinking). Nonetheless, the differences in organization and pedagogical focus of the new curriculum materials led to heated discussions in the U.S. based on differing philosophical and pedagogical perspectives. This serves as a reminder that "decisions about educational reform are driven far more by political considerations, such as the prevailing public mood, than they are by any systematic effort to improve instruction" or learning (Dow 1991).

## Improving the Quality of Mathematics Curriculum Materials

It is said that if one observes sausage being made, then forever after one's appetite or interest in eating sausage dwindles. In many ways, the same metaphor can be used for the making of textbooks in the U.S. The more one learns about how mathematics textbooks are prepared in publishing houses and adopted in schools, the more one is skeptical about their value in promoting learning (Finn & Ravitch, 2004).

Given the for-profit ambitions of the U.S. publishing industry, a primary goal is to develop a product (textbook series) that acquires a sizeable share of the school market and produces maximum profit for the publishing company. From a publishing company's point of view, whether or not textbooks influence student learning of mathematics is of secondary importance to their marketability. If a company develops and markets a successful textbook series, the financial rewards for the company and its authors are great. However, several factors have, in the past, made the development of mathematics textbooks challenging for publishers in the United States, including: 1) conflicting curriculum standards; 2) different textbook selection procedures; 3) varying timelines for textbook adoption; 4) short timelines for producing new textbooks; 5) insufficient evidence regarding the impact of textbooks on student learning; 6) risks associated with doing something different; and 7) consolidation of publishing houses.

*Conflicting curriculum standards.* Although the 1989 NCTM *Standards* offered a vision and some curriculum suggestions for school mathematics, they did not result in uniform state standards in mathematics. In fact, states continued to establish their own standards for mathematics. Disparities among state standards presented major dilemmas for textbook publishers who intended to sell their product across many states. The most popular solution was to address the same topic in multiple grades so that the textbook series aligned with multiple state standards. Unfortunately, this approach led to duplication of content across grades (Flanders, 1987) and resulted in voluminous textbooks of 600–1000 pages per grade (Reys & Reys, 2006). The CCSSM will likely reduce the variability of content within school mathematics textbooks. However, the development of CCSSM-aligned textbooks is underway and it is too early to assess the degree and nature of impact of common standards on the content and quality of curriculum materials.

*Different textbook selection procedures.* Throughout the U.S., individual school districts choose the textbooks that will be used in their schools. Historically, about one-half the states have state-mandated textbook adoption policies/requirements (Tyson-Berstein, 1988). In these states, state-level textbook adoption committees are appointed to review available textbooks and make recommendations for approval and use within the state. If a textbook is approved, then schools may use state funds to purchase the materials. If a textbook is not on the approved list, then state money cannot be used to purchase the books. Therefore, there is significant motivation for textbook companies to get their materials on the state approved list. This leads to political maneuvering to get textbooks approved (Finn & Ravitch, 2004; Seeley, 2003). Thus the pressure to get on textbook adoption list for states with large populations, such as California, Florida and Texas is great.

The remaining states are 'open' states. An open state is one in which each school district in the state has the responsibility of evaluating and choosing textbooks without input from the state. District adoption decisions in these states are made by a committee, an individual (e.g., district curriculum coordinator, department chair, principal) or by some combination of these constituents. Textbook adoption decisions in open states can also be influenced by politics (Becker & Jacob, 2000; Newman, 2004; Mark, et al., 2010). Companies wanting to sell textbooks must market their product individually to each school district. Thus, publishing companies that have a large sales force network are generally the most successful in getting schools to adopt their textbook product. The need for a large sales force makes it difficult for a new or small publisher to successfully market a product.

*Varying timelines for textbook adoption.* There are over 25,000 different school districts in the U.S. Regardless of whether the state is a 'state adoption' or an 'open' state, school districts typically choose new mathematics textbooks every 6–8 years. Unfortunately there is no coordination of adoption cycles, even among the largest adoption states, such as California and Texas. Therefore, thousands of school districts across the U.S. are looking for new mathematics textbooks every year. Since these districts strive to adopt contemporary textbooks, publishers feel pressure to have textbooks with current copyright continuously available. In some cases new editions are made exclusively for larger states, but often the changes tailored to a specific state are cosmetic (perhaps updating the copyright, mentioning cities in the state edition or providing photos of local landmarks), rather than reflecting deep or significant changes in the content or sequencing of the mathematics content (Willoughby, 2010).

*Short timelines for producing new textbooks.* Since some school districts are evaluating and adopting new textbooks each year, most textbook companies cater to the larger adoption states when making major changes or launching a new textbook series. This means that the standards in the large adoption states influence the mathematics content that will be included as well as the grade level in which the content is addressed.

Large adoption states typically release their new or revised standards 1–2 years prior to when textbooks will next be adopted. That means that textbook publishers have 1–2 years to develop a comprehensive program that is aligned with the new or revised state standards. It is difficult for authors/publishers to prepare new materials — as well as the wide range of ancillaries for a comprehensive textbook series — in that time frame. Consequently, most textbook companies hire consultants or small textbook writing companies to develop components of their textbooks. A by-product of this approach is that most textbook programs (at elementary, middle or high school level) are the result of dozens of different writers, and are assembled rapidly. Even with this large cast, the time crunch to make the finished product available to meet the state deadline is tight and also very expensive.

*Insufficient evidence regarding the impact of textbooks on student learning.* The situation described above means that the time between the release of the state standards and meeting the deadline for producing textbooks for review by adoption committees is consumed in product development. There is simply no time to field-test draft materials in classrooms to fine-tune or determine their impact on student learning. While some publishers elicit feedback from focus groups of teachers and parents, the attention is generally on formatting and appearance of the materials. Thus, the result is a newly developed mathematics textbook series that is marketed, sold and used in schools without product testing that is common in other commercial enterprises (Reys, 2001). The exception to this scenario are materials developed by small publishers and/or with funding from the National Science Foundation.

*Risks associated with doing something different.* One approach employed by textbook publishers to increase market share is to examine the textbooks of the market leaders (those with the largest sales over the past few years) and emulate prominent or well-received features of those textbooks. This approach results in textbooks that tend to look and be alike. It promotes homogeneity rather than stimulating change or encouraging creativity. Another factor restricting major changes in school textbooks in the U.S. is the expectations of parents and teachers. Parents, particularly in the elementary grades, want to help their children with mathematics. If the textbooks their children use do not look familiar, or if the tasks are not consistent with their view of what is important to study, then parents may complain. A few vocal parents can become a powerful voice within a school or district and promote the status quo with regard to textbook materials (Dow, 1991; Meyer, Delagardelle, & Middelton,1996).

Teachers too are often reluctant to change to textbooks that look different from what they have grown accustomed to using. Initially it may be difficult to get teachers to adopt new textbooks, and even if adopted it may be difficult to get the new mathematics curriculum materials implemented faithfully in accordance with the philosophy of the program authors (Newman, 2004; Tarr et. al., 2008). The value of providing professional development to help teachers successfully implement a new mathematics program has been well documented (Ball & Cohen, 1996; Reys & Reys, 1997). However, professional development adds additional cost to the adoption of new textbooks and is often not sustained beyond the initial period of adoption (first year).

*Consolidation of publishing houses.* It takes tens of millions of dollars to develop a comprehensive K-6 elementary mathematics textbook series, so only large companies choose to enter the school textbook publishing market. In fact, the last 30 years has seen a significant decline in the number of companies producing K-12 mathematics textbooks for the U.S. market. During this period some publishers have disappeared entirely (eg., American Book Company, Ginn and Company) while other publishing companies who used to be fierce competitors (e.g., Addison-Wesley and Scott-Foresman; and Holt, Rinehart & Winston and Harcourt, Brace & Jovanovich) have been consolidated into a single publishing company. In fact, today there are only three major publishing companies vying for the mathematics textbook market in the United States[1], and the majority of them are foreign-based companies. The motivation to buy the U.S. textbook companies was to produce a profit, and underscores that school publishing is a lucrative market. It also reinforces the pressure on these textbook publishing companies to earn a significant portion of the market to enhance their profit margin.

## Teachers Use of Curriculum Materials Varies and Impacts Students Opportunity to Learn

As noted earlier, it is well documented that textbooks are a strong influence on what mathematics students have an opportunity to learn (Tarr, et al., 2006; Stein, Remillard, & Smith, 2007). However, this same research documents that American teachers vary greatly in the manner and extent to which they use school textbooks. For example, a majority of teachers follow their textbooks closely, covering topics methodologically (including demonstrating examples and assigning all exercises) in the same order they appear in their textbook. Other teachers treat their textbook as a smorgasbord of mathematical topics, where they can pick and choose sections they feel are important for their students to learn. Still other teachers use their textbook as one of multiple resources to support their teaching. (Ball, 1988; Lloyd, 1999; Remillard, 2000).

The various ways mathematics textbooks are used by teachers makes it difficult to do research related to the impact of textbooks on mathematics learning. It is not sufficient to know what textbook is being used in a classroom, school, or district. In addition to knowing the textbooks, data must be collected to determine what parts of the mathematics books were used. Such information can be gathered by observations or teacher logs documenting portions of the textbooks used daily. (Tarr, et al 2006).

Another confounding variable is determining the extent to which the teacher supplemented their textbook. Research shows that a high percent of K-12 mathematics teachers use instructional activities/materials from sources other than their textbook (Seymour &

Davidson, 2003). Supplemental materials may be sought and used to provide opportunities to learn more about a particular mathematical topic or to provide a different model or approach to help students learn. With the increasing attention to assessments as well as the increasing availability of free, web-based supplementary materials, teachers may also be selecting supplemental materials specifically to prepare students for end-of-course tests. Regardless of why these additional instructional resources are used, their use introduces a confounding variable associated with studying the impact of the school district textbook on student learning.

In addition to knowing what portions of the textbook are used, it is important to know how the textbook is used. If the textbook is written to promote collaborative student learning, does the teacher employ this instructional strategy? If the textbook is intended to provide opportunities for discovery learning, are they capitalized upon? Questions such as these reflect the degree to which teachers use the textbook as intended by the authors, including consistency with the philosophy underlying the development of the textbook.

Studying the impact of textbooks on student learning of mathematics is a tedious and complex process. It requires knowledge of what textbook was used, how much of the textbook was used, what parts of the textbook were used, knowing the kind and extent of other materials used, and the manner in which the textbook was used. Together these factors determine the fidelity of implementation of the textbook. Although difficult, determining the fidelity to the district mathematics textbook is essential if accurate interpretations are to be made with regard to the impact of textbooks on student learning of mathematics.

## Mandated Annual Assessments Influence Teaching and Curriculum Materials

Since 2001, schools and districts success in promoting student learning has been closely monitored via annual criterion-referenced assessments produced at the state level. Results from these state assessments provide a measure of what students know and are able to do, an indication of the *achieved curriculum*. However, as with state standards, the annual assessments are unique to individual states. That is, each state retains governance not only of their curriculum standards but also of the assessments used to monitor student learning. As might be expected, the costs of such a localized system of assessment development and delivery are huge. In addition, the current system is inefficient and drains state departments of education from valuable staff recourses.

Following the release of the *Common Core State Standards*, the U.S. Department of Education issued a call for proposals to develop and use common assessments beginning in 2014–15. Two grants, each of which represents a partnership among multiple states, were approved in September 2010:

*Partnership for the Assessment of Readiness for College and Careers (PARCC)*
26 states — Alabama, Arizona, Arkansas, California, Colorado, Delaware, District of Columbia, Florida, Georgia, Illinois, Indiana, Kentucky, Louisiana, Maryland, Massachusetts, Mississippi, New Hampshire, New Jersey, New York, North Dakota, Ohio, Oklahoma, Pennsylvania, Rhode Island, South Carolina and Tennessee

*SMARTER Balanced Assessment Consortium (SBAC)*
31 states — Alabama, Colorado,Connecticut, Delaware,Georgia, Hawaii, Idaho, Iowa, Kansas, Kentucky, Maine, Michigan, Missouri, Montana, Nevada, New Hampshire, New Jersey, New Mexico, North Carolina North Dakota, Ohio, Oregon, Oklahoma, Pennsylvania, South Carolina, South Dakota, Utah, Vermont, Washington, Wisconsin, West Virginia

These state consortia plan to provide both formative and summative assessments, allowing the information to be used both for improving instruction during the year and for monitoring student progress at key times during and at the end of the year. In addition, both consortia plan to use computer-based systems for administering assessments and to collect, analyze and share responses. Collaboration across so many states allows for investment in innovative assessment tasks and tools that have the potential to influence classroom instruction.

## A Perfect Storm for Advancing New Models of Mathematics Textbooks

In the next decade, it is likely that major changes will be made in the format, features and delivery of school mathematics textbooks. Reductions in state funding to school systems have opened the door for creative use of open source curriculum materials, sometimes in place of traditional textbook adoption policies. A combination of economic factors, consensus on curriculum standards and emerging technology provide a “perfect storm” setting for the creation and use of new models of school textbooks. Advances in technology and the widespread use of educational media (from e-books to applets) make it possible to envision dramatic improvements in what we’ve traditionally called textbooks. Jeremy Roschelle (2010) describes these changes in the following way:

> It is conceivable that a totally interactive, continually updatable e-book (linked to numerous external sources of data, images, and research tools) will provide more inviting and effective learning environments than the conventional printed textbooks that students currently tote from class to class and home and back. It is also conceivable that a science, technology, or mathematics classroom that engages students in regular communication with teachers, other students, scientists, engineers and mathematicians, and makes accessible data from around the world could be more engaging

> and effective than an environment bound by the walls of conventional classrooms. Old boundaries may become less relevant, even as new knowledge generated by the learning sciences opens new paths for personalized learning. Effective use of such new instructional resources will require rethinking the ways that education is delivered and managed. Most important, those new ideas and their embodiments in experimental instructional resources must be developed and carefully tested before it makes sense to implement broad transformation of STEM learning both in and out of schools. (p. 37–38)

Roschelle warns that there is no guarantee that advances in technology will lead to curriculum resources that promote increased learning. He likens the situation to that of how the quality of music has (or has not) been affected by new music media,

> Consider the change to iTunes or Kindle for music and books. iTunes has not changed the structure of music; we still listen to 3 minute songs, a length that was dictated by recording time available on a vinyl disc spinning at 78 rotations per minute. We still read the same books, too. Quality has not been improved (e.g. music quality is of lower quality than on CDs or vinyl records), rather cost and convenience factors have dominated consumers' transition to digital media. Following the analogy, it is possible that schools will purchase digital curricula for cost and convenience factors as well, and that these materials could be of even lower quality than today's textbooks …. if digital learning materials have the same structure, content and quality of paper learning materials, the present opportunity will have been wasted. Our nation's students will not be better prepared in critical STEM disciplines merely because instructional content is now accessed in digital form. Our children need the transition to digital materials to be a transition to higher quality. (p. 41)

## Summary

One thing is certain — there are major curriculum changes in store for the U.S. over the next decade. These changes will result from the momentum of the Common Core State Standards as well as from taking advantage of the potential of new electronic vehicles for delivering mathematics content and instruction. Many questions remain regarding the pace, substance and quality of these changes.

*Will educators (teachers, teacher leaders, curriculum developers) use the momentum of the common standards movement in the U.S. to acknowledge and consider the need for other changes, including use of sound pedagogical practices?* For example, how and to what extent will the 'mathematical practices' (depicted in Figure 2) be visible in forthcoming mathematics textbooks and become the focus of instructional attention? It will be of little long-term benefit to rearrange the content focus of school mathematics without also paying close attention to the mathematical practices described in CCSS and to support widespread use of these practices by teachers.

*Will the U.S. transform the current accountability system that is based too narrowly on student performance on a single assessment?* To what extent will the new assessments developed by state consortia address different cognitive levels of knowing, and how will they balance the attention to procedural skills and conceptual understanding? How will these assessment data be used to improve student learning? A major challenge for the state assessment consortia is to develop methods that can be used at scale to monitor student learning of the full scope of the CCSS, including the mathematical practices.

*Will the potential of new delivery formats for curriculum resources (e.g., e-textbooks) spur the creation of more interactive and engaging student learning opportunities?* Early prototypes of e-textbooks vary from online pdf versions of traditional textbooks to systems that allow teachers to construct course materials from an online bank of resources. The adoption of CCSS by most states provides new opportunities for commercial textbook publishers to streamline and focus textbooks, particularly in grades K-8 and to take advantage of electronic delivery models. This new generation of CCSS-based mathematics textbooks are still under development and their ability to utilize effective electronic delivery models warrants close attention.

*Will the recent agreement among many states on common K-12mathematical learning goals lead to establishing a system for continual review and revision of CCSS?* CCSS represents a first attempt at consensus across the states in specific mathematics learning goals. However, adjustments will be needed as more is learned about learning trajectories of particular mathematical topics and as the mathematical expertise needed within the society shifts. A public and transparent system for continual review and improvement of CCSS is needed.

Curriculum reform provides a basis for major changes and improvements in the delivery, substance and quality of school mathematics programs in the U.S. It is our hope that looking to the future and considering the needs of society, best practices and research evidence will drive these changes.

## Note

[1] These companies are McGraw-Hill (including Macmillan, Glencoe, SRA, Everyday Learning, Laidlaw, Merrill, Open Court, Wright Group); Houghton Mifflin/Harcourt (including also McDougal-Littell, Heath, Holt, Saxon, Heinemann); and Pearson (including Scott Foresman, Prentice Hall, Addison-Wesley, Ginn, Silver Burdette, Dale Seymour, and Globe-Fearon).

## References

Ball, D. L. (1988). Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education. Unpublished doctoral dissertation, Michigan State University.

Ball, D. & Cohen, D. (1996). Reform by the Book: What Is--Or Might Be--the Role of Curriculum Materials in Teacher Learning and Instructional Reform? *Educational Researcher*, *25*(9), 6–14.

Becker, J. & Jacob, B. (2000). The politics of California school mathematics: The anti-reform of 1997–99. *Phi Delta Kappan*, *81*(7), 529–537.

Common Core State Standards-Mathematics. (2010) Retreived December 15, 2010 from http://www.corestandards.org/

Dow, P. B. (1991). *Schoolhouse Politics: Lessons from the Sputnik era*. Cambridge, MA: Harvard University Press.

Education Market Research (2006). Retreived December 1, 2010 from http://www.educationmarketresearch.com/

Finn, C. E. & Ravitch, D. (2004). *The Mad, Mad World of Textbook Adoption*. Washington, DC: Thomas Fordham Institute.

Flanders, J. R. (1987). How much of the content in mathematics textbooks is new? *Arithmetic Teacher*, *35*(1), 18–23.

Goertz, M. (2010). National standards: Lessons from the past, directions for the future. Reflections on Five Decades of Curriculum Controversies. B. Reys & R. Reys (Eds.) *Mathematics Curriculum: Issues, Trends, and Future Directions*, (pp. 51–64). Reston, VA: National Council of Teachers of Mathematics.

Harwell, M., Post, T., Maeda, Y., Davis, J., Cutler, A. & Andersen, E. (2007). Standards-based mathematics curricula and secondary students' performance on standardized achievement tests. *Journal for Research in Mathematics Education*, *38*(1), 71–101.

Harwell, M., Post, T., Cutler, A., Maeda, Y., Anderson, E. & Norman, K. (2009). The preparation of students from National Science Foundation-funded and commercially developed high school mathematics curricula for their first university mathematics course. *American Educational Research Journal*, *46*(1), 203–231.

Hirsch, C. (2007). *Perspectives on Design and Development of School Mathematics Curricula*. National Council of Teachers of Mathematics.

Jennings, J., & Renter, D. S. (2006). Ten big effects of the No Child Left Behind Act on public schools. *Phi Delta Kappan*. *88*(2), 110–113.

Kilpatrick, J. (2003). What Works? In S. Senk and D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* Mahwah, NJ: Lawrence Erlbaum Associates.

Kulm, G., Morris, K. & Grier, L. (2000). *Middle grades mathematics textbooks: A benchmarks-based evaluation.* Washington, DC: American Association for the Advancement of Science.

Lloyd, G. (1999). Two teachers' conceptions of a reform-oriented curriculum: Implications for mathematics teacher development. *Journal of Mathematics Teacher Education*. *2*(3), 227–252.

Mark, J., Spencer, D., Zeringue, J., & Schwinden, K. (2010). How do districts choose mathematics textbooks? B. Reys & R. Reys (Eds.) *Mathematics Curriculum: Issues, Trends, and Future Directions*, (pp. 199–213). Reston, VA: National Council of Teachers of Mathematics.

Meyer, M., Delagardelle, M. & Middelton, J. (1996). Addressing Parents' Concerns over Curriculum Reform. *Educational Leadership*. *53*(7), 54–57.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics*, Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*, Reston, VA: National Council of Teachers of Mathematics.

National Research Council (1999). *How people learn: Bridging research and practice*. M.S.Donovan, J.D.Bransford & J.W.Pellegrino (Eds.). Committee on Learning Research and Educational Practice, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

National Research Council. (2004). *On evaluating curricular effects: Judging the quality of K-12 mathematics evaluations*. J. Confrey & V. Stohl (Eds.). Committee for a Review of the Evaluation Data on the Effectiveness of NSF-Supported and Commercially Generated Mathematics Curricular Materials. Mathematical Sciences Education Board, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J.Kilpatrick, J. Swafford, and B.Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

National Research Council. (2002). *Investigating the influence of standards: A Framework for research in mathematics, science and technology education*. I.R.Weiss, M.S.Knapp, K.S.Hollweg, and G.Burrill (Eds.). Committee on Understanding the Influence of Standards in K-12 Mathematics, Science, and Technology Education, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Newman, W. (2004). Serving on a mathematics text selection committee: A tale of woe. *School Science and Mathematics*. *104*(8), 361–367.

Remillard, J. (2000). Can curriculum materials support teachers' learning? Two fourth-grade teachers' use of a new mathematics text. *The Elementary School Journal*. *100*(4), 331–350.

Reys, B. (Ed.) (2006). *The intended mathematics curriculum as represented in state-level curriculum standards: Consensus or Confusion?* Charlotte, NC: Information Age Publishing.

Reys, B. & Reys, R. (1997). Standards- Based Mathematics Curriculum Reform: Impediments and Supportive Structure. *NCSM Journal of Mathematics Education Leadership*, *1*(2), 3–8.

Reys, B. & Reys, R. (2006). The development and publication of elementary mathematics textbooks: Let the buyer beware! *Phi Delta Kappan*, *87*(5), 377–383.

Reys, B., Robinson, E., Sconiers, J. & Mark, J. (1999). Mathematics curricula based on rigourous national standards: What, why, and how? *Phi Delta Kappan*. *80*(6), 454–456.

Reys, R. (2001). Mathematics Curricula and Guinea Pigs. *Mathematics Teacher*, *94*(1), 6–7.

Robitaille, D., & Travers, K. (1992). International studies of achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 687–709). New York, NY: MacMillan.

Roschelle, J. (2010). Blue Sky STEM Learning Designs for Emerging CyberLearning: The Need for a Timely, Targeted and Ambitious Investment. In B. Reys (Ed.) *The Future of STEM Curriculum and Instructional Design: A Research and Development Agenda for Learning Designers*, Center for the Study of Mathematics Curriculum.

Schmidt, W. H., McKnight, C., & Raizen, S. *A Splintered Vision: An Investigation of U.S. Science and Mathematics Education.* Dordrecht, Netherlands: Kluwer, 1997.

Schoenfeld, A. (2002). Making Mathematics Work for All Children: Issues of Standards, Testing, and Equity. *Educational Researcher*. *31*(1), 13–25.

Seeley, C. (2003). Mathematics Textbook Adoption in the United States. In G. Stanic & J. Kilpatrick (Eds.). *A History of School Mathematics*, Volume 2 (pp. 957–988). Reston, VA: National Council of Teachers of Mathematics.

Senk, S. & Thompson, D. (2003). *Standards-based school mathematics curricula: What are they? What do students learn?* Mahwah, NJ: Lawrence Erlbaum.

Sewall, G. T. (2005). Textbook publishing. *Phi Delta Kappan*, *86*(7), 498–502.

Seymour, D. & Davidson, P. (2003). A History of Nontextbook Materials, In G. Stanic & J. Kilpatrick (Eds.), *A History of School Mathematics*, Volume 2 (pp. 989–1036). Reston, VA: National Council of Teachers of Mathematics.

Stein , M. K., Remillard, J. & Smith, M. S. (2007). How curriculum influences student learning, F. Lester (Ed.) *Second Handbook of Research on Mathematics Teaching and Learning*, pp. 319–370. Charlotte, NC: Information Age Publishing.

Tarr, J., Chávez, O., Reys, R. & Reys, B. (2006). From the written to enacted curricula: Intermediary role of middle school mathematics in shaping students' opportunity to learn. *School Science and Mathematics*, *106*(4), 191–201.

Tarr, J. Reys, R., Reys, B., Chávez, O., Shih, J. & Osterlind, S. (2008). The Impact of Middle Grades Mathematics Curricula on Student Achievement and the Classroom Learning Environment. *Journal for Research in Mathematics Education*, *39*(3), 247–280.

Trafton, P., Reys, B. & Wasman, . (2001). Standards-based mathematics curriculum materials: A phrase in search of a definition. *Phi Delta Kappan*, *83*(3), 259–264.

Tyson-Berstein, H. (1988). *A conspiracy of good intentions: America's textbook fiasco*. Washington, D.C. Council for Basic Education.

Tyson-Bernstein, H. & Woodward, A. (1991). *Nineteenth century policies for twenty-first century practice: The textbook reform dilemma*. In (Eds.), *Textbooks in American Society*. Albany, NY: State University of New York Press.

United States Department of Education. (1999). *Exemplary and Promising Mathematics Programs*. Washington, D.C.: U. S. Department of Education. Retrieved on Feb. 1, 2011 from http://www2.ed.gov/PressReleases/10-1999/mathpanel.html.

United States Department of Education. (2001). *No Child Left Behind*, Washington, DC.

Willoughby, S. (2010). Reflections on Five Decades of Curriculum Controversies. B. Reys & R. Reys (Eds.) *Mathematics Curriculum: Issues, Trends, and Future Directions*, (pp. 77–86). Reston, VA: National Council of Teachers of Mathematics.

**Resumo.** A década em curso testemunhará, muito provavelmente, uma atenção sem precedentes sobre nos EUA. A adopção dos *Common Core State Standards for Mathematics* (CCSSM) por todos os estados, com poucas excepções, representa uma grande mudança na administração curricular. Pela primeira vez nos EUA, a grande maioria das escolas, professores e estudantes vão concentrar-se em objectivos de aprendizagem comuns específicos e estabelecidos por ano de escolaridade. Esta iniciativa, associada com as exigidas avaliações anuais comuns, também em cada ano de escolaridade, alinhadas com os CCSSM, poderá ter impacto em outros sistemas e factores importantes para a aprendizagem dos alunos, como o ensino e os materiais curriculares, a formação de professores e as políticas de escolha de cursos e graduação.

Este artigo apresenta um resumo das iniciativas correntes de reforma curricular nos EUA, proporcionando um breve panorama do movimento actual tendo em vista as normas curriculares comuns (a nível nacional) nos EUA. Analisamos alguns dos procedimentos utilizados na elaboração de manuais, discutimos alguns dos desafios que os autores e editores enfrentam e apresentamos uma reflexão sobre o importante papel que os manuais de Matemática desempenham na determinação do conteúdo matemático que os alunos têm oportunidade de aprender. Por fim, especulamos sobre o futuro dos materiais curriculares e sobre o impacto dos CCSS nos programas da Matemática escolar.

*Palavras-chave:* Matemática, Currículo, Manuais escolares, Normas curriculares

**Abstract.** The current decade is likely to witness an unprecedented focus on mathematics curriculum in the U.S. The adoption of the Common Core State Standards for Mathematics (CCSSM) by all but a few states represents a major change in curriculum governance. For the first time ever in the U.S., a large majority of schools, teachers and students will focus on common, specific and grade-level focused learning goals for mathematics. Coupled with mandated common annual grade-level assessments aligned to CCSSM, this initiative is likely to impact other important systems and factors critical to student learning including instruction, curriculum materials, teacher training, and course-taking/graduation policies.

This paper provides a summary of current curriculum reform initiatives in the U.S. We provide a brief overview of the current movement toward common (national) curriculum standards in the U.S. We examine some of the procedures used in developing textbooks and discuss some of the challenges textbook authors and publishers face. We reflect on the critical role that mathematics textbooks play in determining the mathematics content students have an opportunity to learn. Finally, we speculate about the future of curriculum materials and the impact of CCSS on school mathematics programs.

*Keywords:* Mathematics, Curriculum, Textbooks, Standards.

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