High school mathematics teachers' inquiry-oriented approaches to teaching algebra

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Introduction

Algebra, as a central aspect of school mathematics, continues to attract attention to make it accessible to all students. For example, it is a central theme of the standards of the National Council of Teachers of Mathematics [NCTM] that promote it as fundamental to the basic education of all students from prekindergarten through grade 12 (NCTM, 2000). While the NCTM standard for algebra provides guidelines of how to interpret it in the mathematics classroom, it is still dependent on the teacher for it to be realized as intended. In particular, the instructional approach used by the teacher will determine the meaning and level of understanding students develop of algebra. Traditional instructional approach of high school algebra generally involves the teacher introducing students to a new concept by demonstrating two or three "worked examples", with emphasis on procedure and symbol manipulation, followed by intensive practice exercises to reinforce the procedure demonstrated. This paper focuses on high school mathematics teachers who deviate from this approach. It reports on a study of the teachers' inquiry-oriented approaches based on their practice. Specifically, it addresses two questions: (1) What are the central features that characterize the teachers' inquiry-oriented approaches based on teaching systems of equations? (2) What are central features in their thinking that allow them to make sense of their approaches?

Literature review

Algebra education has been an active field of research covering a range of issues in terms of the nature, teaching, and learning of algebra (*e.g.*, Bednarz, Kieran, & Lee, 1996; Herscovics & Linchevski, 1994; Kaput, Carraher, & Blanton, 2008; Sleeman, 1986; Stacey & MacGregor, 1999; Stacey, Chick, & Kendal, 2004). Concerns about students' inadequate understandings and preparation in algebra, algebra being difficult to learn, algebra curricula and algebra instruction have been a focus of this body of research. A key question raised by Kieran (1992) is whether the comprehension of school algebra is a difficult task for the majority because of the nature of the content of algebra or the way it is taught. While no definitive answer is offered to support either or both as the cause of students' dilemmas in learning algebra, there are many suggestions about meaningful instruction offered in the literature.

In general, reform perspectives of mathematics teaching (*e.g.*, NCTM, 1991; 2000) are considered meaningful to make a difference to the learning of algebra. So, for example, French (2002) provides a comprehensive guide to the teaching and learning of algebra through a constructivist approach. Smith, Silver, and Stein (2005) provide cases of teaching patterns and functions in middle school algebra for use in teacher development, cases that reflect a constructivist perspective of instruction. Swan (2000) suggests different lesson designs in teaching algebra to encourage students to construct and reflect on the meanings for expressions and equations particularly through discussions, *e.g.*, students work in groups on card sorting activities matching two different representations of a concept. Others have focused on a variety of manipulative tools including computer simulations to bring a more concrete understanding of algebra to students (*e.g.*, McArthur, 1989).

While this body of research has provided us with insights about teaching and learning school algebra, there is an underrepresentation of studies based on high school teachers' non-traditional instructional practices in particular. In fact, studies on teachers' practice in general tend to focus on elementary school teachers and prospective teachers (Ponte & Chapman, 2006) and when they involve secondary school teachers they tend to be at the lower secondary level. In particular, very few studies involving secondary school teachers' practice (i.e., instructional approaches) were found based on a review of current literature. Escudero and Sánchez (2007) is one of these studies. They studied the practice of a secondary school mathematics teacher, exploring how his pedagogical approaches on mathematics, mathematics learning, and mathematics teaching were related to the relational architecture he established in the classroom during an instructional unit of similarity and if that relationship could be explained in terms of his underlying perspective. The results showed the characteristics of the relationship and the important role that the teacher's knowledge of the students' difficulties played both in making decisions and in developing his actions. In another study, Escudero and Sánchez (2002) addressed similar issues, now concerning the teaching of Thales theorem. Discussing the cases of two secondary school teachers, they concluded that pedagogical content knowledge and subject matter knowledge were integrated in the teachers' decisions, but they used different structures and their initial decisions regarding the structures were linked to different characteristics of the domains of knowledge. In the case of Mendick (2002), the focus was on the practices through which teachers, explicitly and implicitly, answered the students' question, "Why are we doing this?" The author presented a case study of a secondary school class in which preparing for examination, competition among students, and procedural work were prominent features. She argued that the practices in which students and teachers engaged, the meanings they gave to them, and the possibilities these made available for the development of their identity were critical to understand students' success and failure in mathematics.

Two studies that address secondary school mathematics teachers' beliefs or views about mathematics teaching, but not their own practice, are Cai and Wang (2010) and Wilson, Cooney, and Stinson (2005). Cai and Wang investigated Chinese and U.S. teachers' cultural beliefs concerning effective mathematics teaching from the teachers' perspectives. Although sharing some common beliefs, the two groups of teachers thought differently about both mathematics understanding and the features of effective teaching with the U.S. teachers putting more emphasis on student understanding with concrete examples and the Chinese teachers putting more emphasis on abstract reasoning after using concrete examples. Both groups agreed that memorization and understanding cannot be separated. However, for the U.S. teachers, memorization comes after understanding, but for Chinese teachers, memorization can come before understanding. For Wilson, Cooney, and Stinson (2005), they examined experienced and professionally active teachers' views of good mathematics teaching and how it develops. In general, the nine teachers thought good teaching requires a sound knowledge of mathematics, promotes mathematical understanding, engages and motivates students, and requires effective management skills. They felt that good teaching is developed from experience, education, personal reading and reflection, and interaction with colleagues, with experience being the primary contributor.

One study that explicitly focused on teacher thinking and algebra at the lower secondary level is by Agudelo-Valderrama, Clarke, and Bishop (2007). They investigated the relationship between grade 8 Colombian mathematics teachers' conceptions of beginning algebra and their conceptions of their own teaching practices. Focusing on the perspectives of teachers afforded opportunities that exposed the powerful role that the teachers' conceptions of social/institutional factors of teaching played in their conceptions of their practices.

This sample of studies indicates what has been considered about secondary teachers' practice as well as the need for more attention to this area of mathematics education. This study, then, contributes to this sparse body of literature by focusing on high school teachers and their teaching of algebra based on their actual practice and thinking. It also addresses a perspective of practice based on inquiry that is currently advocated as a more effective way of teaching mathematics than the traditional teacher-directed approach.

Perspective of inquiry teaching

Inquiry is considered to be an effective method for teaching both content and process skills. In mathematics education, this is supported by professional standards and the presence of inquiry in reform mathematics curricula and learning materials. For example, a major influence on the acceptance of inquiry-based approaches to learning mathematics has been the NCTM Standards documents (1991; 2000). There is also a growing body of research to provide evidence of positive impacts of inquiry on students' learning in mathematics (*e.g.*, Heibert, 2003). In inquiry-based teaching the focus is on the learner and learning. As the NCTM (2000) Teaching Principle states: "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (p. 16). Also, Polya (1965) argued that the primary aim of mathematics teaching is to teach people to think while Davis and Hersh (1981) made a case for "reasoning together" (p. 282) as a central aspect of teaching mathematics. These views support the importance of an inquiry perspective of teaching mathematics to facilitate students' development not only of content knowledge, but also mathematical processes, inquiry, and thinking skills and collaborative skills.

Inquiry as a basis of learning is well established in the literature (*e.g.*, Dewey, 1938; National Research Council, 2000; Schwab, 1962; 1966; Wells, 1999; 2001). Based on this body of theories about inquiry, the key components that define it as a process to facilitate learning are: posing a question, investigating it, creating new knowledge, communicating the knowledge, reflecting on the knowledge in relation to the question posed, and considering new questions that could start a new cycle of inquiry. In this process, students' construction of their understanding is the central focus. Thus inquiry-based instruction actively engages students in investigating mathematics concepts and allows them to direct their own investigations and find their own answers. It allows students' questions and curiosities to drive curriculum, honours previous experience and knowledge, makes use of multiple ways of knowing, and allows for creation or adoption of new perspectives when exploring issues, content, and questions. In general, inquiry provides opportunities for students to be actively engaged in the construction of mathematical knowledge with deep understanding; to make connections between prior, existing and new knowledge and experience; to work and learn collaboratively as a team and co-construct knowledge; and to take responsibility for their own learning.

In this study, in considering teacher's inquiry-oriented approaches, the focus is on how the teachers make sense of it in their practice and thinking. The goal is not to investigate any particular view of inquiry, but to identify what the teachers are able to do. Describing it as inquiry-oriented is intended to highlight aspects of their practice that can be associated with the inquiry perspective. Thus inquiry-oriented could reflect some, but not all, of the key common notions associated with inquiry learning such as: learner-focused, question-driven, investigation, communication, reflection, collaboration, student autonomy, and emphasis on experience in learning.

Methodology

This study is part of a larger research project with a focus on elementary school teachers and secondary school mathematics teachers' thinking and use of contextual problems [CPs] in their teaching. It emerged from the data for three of the high school participants who described personal thinking and instructional actions that indicated inquiry-oriented approaches to their teaching. Thus it is based only on those aspects of the data that provided information about these approaches. The participants, who are the focus of this study, were three experienced practicing high school (grades 10–12) mathematics teachers (Teacher-A, Teacher-B, and Teacher-C). They had 16 to 20 years of teaching experience. They were from different local public schools. They were considered in their school systems to be excellent mathematics teachers and they consistently used CPs in their teaching, which were bases for their selection for the larger project. CPs refer to "word problems" in the broadest sense and include problems situated in a context, real or imagined, and involve closed or open solution processes. Examples from participants:

> Closed CP: *A car rental agency charges $200 per week plus $0.15 per mile to rent a car. How many miles can you travel in one week for $320?*
>
> Open CP: *When eggs in a basket are removed 2, 3, 4, 5, and 6 at a time, 1, 2, 3, 4, and 5 eggs remain, respectively. When they are taken out 7 at a time, none are left over. Find the smallest number of eggs that could be in the basket.*
>
> Not a CP: *For $2x + 5y = 0$, write a second linear equation that would create an inconsistent system.*

While the larger project focused on CPs, the teachers also used projects or inquiry-oriented tasks, with or without context, which were captured in the data and formed an integral part of this study that is not specifically about CPs.

The main sources of data for the larger project were as follows. Two open-ended interviews (two to three hours each depending on participant) prior to classroom observations explored the participants' thinking and experiences with CPs in three contexts: past experiences as students and teachers, current practice, and future practice (i.e., expectations). In addition to focusing on their explicit thinking of CPs and teaching, the interviews also addressed implicit conceptions by considering the relevant prior knowledge, abilities, and expectations they brought to their experiences with CPs in their teaching; task features, classroom processes and contextual conditions relating to CPs; and planning and intentions for CPs in their teaching. The interviews did not suggest particular attributes of CPs to talk about or any definition of CPs, which allowed them to talk about all types of tasks they used in their teaching. Interview questions were framed in both a cognitive context to allow the teachers to share *their* way of thinking by providing "theoretical responses" (*e.g.*, explicit conceptions) and a phenomenological context to allow them to describe their teaching behaviors as lived experiences (i.e., stories of actual events that embodied implicit conceptions). For example, some questions were of the form: How do you view CPs? What do CPs mean to you? What do you think is the role of CPs in mathematics, in the curriculum, in your teaching? How do you use CPs in your teaching? Other questions were in the form of open situations to address, *e.g.*, telling stories of memorable, liked and disliked mathematics classes involving CPs that they taught; role-play giving a presentation on CPs at a teacher conference; role-play having a conversation with a prospective teacher about CPs; and analyzing a list of CPs prepared by the researcher. The interviews were audio-recorded and transcribed.

Classroom observations and field notes focused on the teachers' actual instructional behaviors during teaching units involving CPs. Ten lessons (60 to 85 minutes each in grades 10 and 11) were observed during a school term and audio-recorded for each teacher. Timing of observations was partly dependent on availability of the researcher who often showed up without notifying the teacher, which allowed for observation of their normal teaching approaches. Post-observation discussions, when necessary, focused on clarifying the teachers' thinking in relation to their actions. Most of the lessons observed involved algebra topics. The focus here is on units/lessons on "systems of equations" because there were data on this topic for the three teachers which provided a basis for comparison of their approaches. However, for Teacher-A and Teacher-B, the data consisted of observations of lessons of systems of two linear equations, while for Teacher-C the data consisted of observations of lessons on systems of one linear and one quadratic equation. This difference was due to scheduling conflicts in conducting the classroom observations for the larger project. But discussion with Teacher-C indicated that she used a similar approach in teaching systems with two linear equations.

Data analysis for the larger project involved the researcher and two research assistants working independently to thoroughly review the data and identify attributes of the teachers' thinking and actions that were characteristic of their conceptions of CPs and teaching with CPs. Transcripts were read, initially to gain a general impression of the participants' thinking and then significant statements and behaviors were identified. The open-ended coding focused on statements and actions that reflected judgments, intentions, expectations, and values of the teachers regarding CPs that occurred on several occasions in different contexts of the data. The coded information were categorized and validated through an iterative process of identification and constant comparison and grouped under broad themes of the teachers' thinking.

Inquiry-oriented teaching emerged as one of the themes in terms of the ways the teachers engaged students in teaching with CPs and is the basis of this paper with a focus on the three teachers who used this approach more consistently throughout their teaching. For this purpose, this theme was further investigated by examining the data in relation to the two research questions. First, key components of the inquiry process and structure were identified by analyzing the structure of the teachers' instructional approaches. This included: isolating the stages of the lessons on systems of equations; highlighting the different ways in which students were engaged in each stage; comparing the stages to identify patterns/cycles and prominent features in students' engagement; comparing teachers to identify similarities and differences in patterns/cycles and features; assigning themes based on patterns emerging to represent the central features of the approaches; and comparing themes to theories on inquiry to determine the inquiry-oriented nature of the teachers' approaches. This process resulted in three themes involving three perspectives of inquiry that described the teachers' inquiry-oriented approaches: problem solving, research, and dialogic discourse. Second, the coded data from the larger project were examined to identify central features of the teachers' thinking that related to their inquiry approaches. This was guided by theory of key characteristics of inquiry instruction, *e.g.*, learner-focused, question-driven, investigation, communication, reflection, and collaboration. Four themes emerged that represented central features in the teachers thinking that supported their sense-making of their approaches, i.e., their thinking of the algebra concept, task, inquiry, and peer interactions. The findings are presented in terms of the three themes of the central features of their inquiry-oriented approaches and the four themes of their thinking that allowed them to make sense of their approaches.

Findings

On a surface level, there were similarities in how the teachers engaged students in the lessons based on the structure of the lessons. All three teachers used an inquiry-oriented structure with components of posing a task, students working on the task with little or no input by teacher to allow students to create knowledge, students sharing and discussing knowledge, and students reflecting on knowledge and process. However, there were significant differences in the nature of the components and students' engagement that resulted in three perspectives of their inquiry approaches with central features specific to each, i.e., inquiry as problem solving, inquiry as research, and inquiry as dialogic discourse. Similarly, the four central features identified in the teachers' thinking that supported their sense-making of their inquiry-oriented approaches emerged as the same themes, i.e., algebra concept (systems of equations), task, inquiry, and peer (student-student) interactions, given the data available, but there were significant differences from each teacher's perspective. The findings are presented to highlight these central features in terms of their differences for the approach and thinking of each teacher.

Teacher-A's Case

Inquiry as problem solving

The theme "inquiry as problem solving" represents the central features of Teacher-A's inquiry-oriented approach. The approach consists of a four-stage inquiry cycle with features that parallel the problem-solving process (*e.g.*, Polya, 1945). In general, it involves Teacher-A presenting students with a problem involving the concept being taught, students working on the problem in groups to try and solve it on their own, then sharing and reflecting on the solution during whole-class discussion. The problems, although algorithmic in nature, were considered by Teacher-A as genuine problems for students since they were new to them. The following abbreviated version of her lessons on systems of linear equations illustrates this approach.

For the first inquiry cycle, Teacher-A began her unit on systems of linear equations with a word problem. She assigned students the following task by drawing a picture on the white-board as she described it:

> We have two situations. This is a balance, or at least my best attempt at drawing a balance. You have got two cats and three kittens together for a

> mass of 22 kilograms. And then on the other balance, the same three cats and one kitten have a combined mass of 24 kilograms. Assume that all the big cats have the same mass as each other, and all the little kittens have the same mass. What is the mass of a cat and what is the mass of one kitten?

Students worked in groups to solve the problem with no other direction given to them. Teacher-A told them, "you can use whatever way will help you to get to an answer or solution to this question". She circulated, observed, and intervened only if students initiated it by asking her a question. After enough time to arrive at a solution, she facilitated students' whole-class sharing and discussion of their different strategies. The discussion prompted students to reflect on their strategies and the efficiency of their guess-and-check approaches and to question whether other approaches were possible. Teacher-A used this as an invitation to introduce them to "formal strategies" (graphical and algebraic approaches) required by the curriculum.

The second inquiry cycle began with teacher-A introducing the graphical approach to solving systems of equations. The students' task was to work with the cat-kitten problem to try and represent it in a graphical way building on what they did in the first inquiry cycle. After a brief teacher-led discussion to help them to understand the problem, in particular, "finding a way of taking out information on the balance … and representing it in a way that can be turned into a graph" (Teacher-A), the students worked in groups to find a solution using this strategy. Teacher intervention and students' sharing and discussion followed the same pattern as the first cycle. The teacher also facilitated a discussion of the relationship between the point of intersection and the solution. Students then practiced the graphical approach with other problems from the textbook. The pattern continued with the remaining inquiry cycles of the unit dealing with different algebraic approaches and problem solving. For example, the next cycle began with the problem:

> I have a rectangle whose perimeter is 14cm. The width is 1cm less than the length. What are the dimensions of the rectangle?

At the end of the unit students were allowed to use any approach to solve the practice questions or future tasks involving linear systems of equations.

Central features of Teacher-A's thinking

For each teacher, as noted before, the same four central features were identified in her thinking as a basis of her sense-making of her inquiry-oriented approach. These features relate to particular aspects of the teacher's thinking of the nature of the algebra concept, task, inquiry, and peer interactions in her practice. Following is the interpretation of each of these central features for Teacher-A.

1. Algebra concept as problem-solving strategy

In the context of her teaching, Teacher-A's thinking considered the concept of systems of linear equations in terms of strategies to solve contextual problems. Her thinking and teaching emphasized strategies. For example, she explained that as she circulated during students' problem-solving session she was "actually trying to keep track of the different strategies that they're using" to facilitate their whole-class sharing. She emphasized sharing of strategies during whole-class discussion of the concept because "it's important … that kids see that there are a variety of ways of getting to the end". Once students learned the different ways of solving linear systems of equations, she considered these ways as different strategies of equal value, so "it does not matter which one students used to solve a problem" involving this concept. This focus on strategies in her thinking supported her sense-making of the problem-solving approach in teaching this concept.

2. Tasks as situated strategies

Teacher-A's thinking indicated that students should learn the concept (systems of equations) through contextual situations and treated tasks as situated strategies, i.e., situations that embodied the strategies of solving systems of linear equations. Based on experience, she developed the view that contextual tasks are more meaningful for students than straight algebra. She explained:

> I think from my perspective it needs to be contextual because otherwise why would we bother. … Sometimes the context that I find or that I use are not the greatest … but I still think it's better than straight algebra. It allows the kids — like in the cat one — to attach some meaning to the variables and then the equations have meaning because they are situated in a (inaudible) situation. So that's why, as much as possible, I like to use something contextual.

The conflict for her, as she explained:

> Not that I totally agree with that [contextual problems], but it seems to me more of a grabber for kids … to get kids turned onto math.

> I firmly believe that we don't need applications for the mathematics to be wonderful and exciting.

> I always believed that if I provided a nice caring environment for my kids to learn the content that it wouldn't matter that it applied to anything, because it doesn't matter to me personally whether it applies to anything, but I know that that's not the case.

This conflict in her thinking is resolved by using contextual tasks but attributing little importance to the context beyond holding the concept, or situating the strategy, of systems of equations. For example, she considers "the contextual problem being the start of whatever it is and then the need for the mathematics coming out of that". So it is less about real-world applications. Hence contexts such as in the cat-kitten problem are meaningful situations for students' inquiry of the strategies associated with it. Thus the conflict in her thinking supported her sense-making in using contextual tasks that were more about the concept as strategy and less about the real-world application consistent with her problem-solving inquiry-oriented approach.

3. Inquiry as play

Teacher-A's thinking suggested inquiry as play, i.e., students are expected to play with the problem, to try and solve it on their own. She explained:

> I think what we've got to do is to let go, allow children to experience the problem for themselves. … The kids don't learn if you don't let them play, and I think the strategy that works most effectively is to do some debriefing as a large group, some playing as individuals or small groups and then some sharing afterwards.

She also associated play with challenges and some struggle. For example, if students were off track in their solution to a problem, she explained, "I let them go. I let them play with it. … I want some initial struggle". Her thinking also reflected the importance of supporting students' autonomy during problem solving to allow them to play. For example,

> I try not to make any judgments about their solution. I'm just simply asking questions for them to make the judgment for themselves.

> I am encouraging kids without doing the thinking for kids. … I expect the kids to think for themselves but that I don't expect them to do that without some initial support.

This view of inquiry as play provided another basis to support her sense-making of the focus on strategy (important in play) and students' autonomous problem solving in her inquiry-oriented approach.

4. Peer interactions as source of information

Teacher-A's thinking suggests the importance of peer (student-student) interactions as a source of information to motivate and support students' learning. She explained,

> I think getting information from your peers helps you understand that there are people who are experiencing the same difficulties as you or who can have a perspective that you can share, they can bring something to this situation that you may not have thought of, and it shows the fact that we all need to talk to somebody sometimes.

> I think that the learner needs to be in an environment that is one where they get to work with somebody else … where as a learner I can ask questions, I can test my ideas.

This thinking supported her sense-making of the type of information students shared in her inquiry-oriented approach, *e.g.*, their strategies and experiences in solving the problems to allow each other to get information to develop or check his or her processes and ideas.

Teacher-B's Case

Inquiry as research

The theme "inquiry as research" represents the central features of Teacher-B's inquiry-oriented approach. The approach consists of an inquiry cycle with features that parallel a conventional research method as in the following two examples from her lessons on the concept of systems of linear equations: (1) Students planned an investigation of real-world graphical applications of the concept; gathered and analyzed information in terms of visual representations and meaning; drew and reflected on conclusions; shared, discussed, and applied outcomes in solving a teacher-assigned, real-world problem; extended the investigation to interpret the graphical solution. (2) Students investigated solutions of systems of equations to understand the process; planned their investigation; gathered and analyzed information from studying solved examples; drew conclusions; reflected on and justified their findings through own examples and counter examples provided by teacher; shared findings in a way to convince the audience of their efficiency; developed examples to apply findings. Following is an abbreviated version of her lessons on systems of linear equations to highlight key aspects of the inquiry cycles.

For the first inquiry cycle, Teacher-B asked students to investigate graphs that intersected based on pictures they were to obtain from any source other than mathematics textbooks. The graphs did not have to be straight lines. They could be any graphs that intersected but should represent actual real-life situations. Students' inquiry included determining where to look for graphs, what and how many graphs to select, and meaning and application of graphs. They shared their pictures and knowledge created and engaged in whole-class discourse that included what it means when graphs intersect and why anyone would want to find the intersection. For the second inquiry cycle, Teacher-B assigned the task: "If you have a part-time job in sales, is it better to have a fixed hourly rate or a fixed weekly salary plus commission?" Students were to investigate what information they needed to solve it, provide realistic information, and determine a way to solve it. They shared, discussed, and reflected on their findings and engaged in whole-class discourse that included how the graphical approach could be used to solve the problem, how the point of intersection or break-even point was useful to the analysis and solution of the problem, the relevance of solving systems of two linear equations to the students' real-life experiences and the importance for them to look further at where graphs intersect.

For the third inquiry cycle, Teacher-B asked students to determine and validate the equations and points of intersection for the graphs they collected involving two intersecting straight lines. Students investigated this in groups, then shared their findings and engaged in a whole-class discussion of the meaning of the algebraic representations in the context of the applications, why they are useful, and the usefulness of an algebraic approach instead of a graphical approach to find the point of intersection. For the fourth cycle, Teacher-B asked students to use graphing calculators to investigate special cases of systems of two linear equations, *e.g.*,

$$x + y = 4 \text{ and } 2x + 2y = 8$$
$$2x + y = 3 \text{ and } 2x + y = 5.$$

Students worked in groups to investigate these and other relationships they created between two linear equations. They shared and discussed their findings and drew conclusions that included "what happens if the lines do not intersect" and "what it means if one line is on top of another".

For the fifth cycle, Teacher-B asked students to investigate "solved examples" of systems of linear equations to understand the structure of the method and be able to teach it to others. Students worked in their groups to investigate the particular solution pattern for the one method Teacher-B randomly assigned to them, tested and validated their understanding through their own and the teacher's examples and counter-examples, planned how to teach their approach to the other groups in a way that was interesting and would help them to understand it, taught and led discussions on the method including making a case that their method was the most efficient, and posed problems that could be solved with the method (must show creativity, cannot select problems from a textbook). The unit continued with other tasks including solving a selection of these problems and "bringing in real-world word problems that they find elsewhere, other than the textbook, to talk about how they used or can be solved by systems of equations".

Central features of Teacher-B's thinking

As previously noted, the same central features identified for Teacher-A were also the basis for determining Teacher-B's sense-making of her inquiry-oriented approach. However, the interpretation for each feature is different in terms of her thinking of the nature of the algebra concept, task, inquiry, and peer interactions in her approach.

1. Algebra concept as useful tool

In the context of her teaching, Teacher-B considered the concept of a system of linear equations from a utility perspective, i.e., as a tool to solve and model real-world situations. As a tool, the concept also embodies mathematical and real-world meanings. Her thinking emphasized applications, real-world connections, and meaning, which were dominant in her teaching of the concept. In particular, regarding applications, she explained,

> I don't think that the teaching of mathematical applications is anything separate from the teaching of mathematics [concepts], so that the applications have to be integrated throughout a course and not taken as some-

> thing separate … so that it's not thought of as a special part of math, but thought of as doing mathematics.

This way of thinking allowed her to make sense of how she integrated applications throughout the unit of systems of equations and engaged students in investigating them as a way of doing mathematics. In general, viewing the concept as a tool was a basis for sense-making of the inquiry-oriented tasks that permeated her inquiry-oriented approach.

2. Tasks as meaningful experiences

Teacher-B's thinking indicated that tasks are meaningful experiences, i.e., situations that embodied the usefulness of linear systems of equations and provided meaningful experiences for students. Thus tasks had to be interesting and relevant to students' real-life, involve real-world situations, and offer students meaningful experiences through variety and unpredictability. She explained that when planning her classes:

> I think about making the experience meaningful for the kids, so that they're not bored, so that they see some use to the problems. … I always try to come up with a variety of things happening all the time, so that people aren't bored, so they come in there and wondering what is going to happen. So they can't predict what to expect.

She also explained,

> I think that when you're teaching and using contextual problems in class the big thing is that they have to be interesting to someone, to those kids, and so a lot of times, the problems could be generated from the class.

So she expected students "to write problems on their own about things that they find interesting to share with others" and to be creative in what they produced, i.e., they could not reproduce a textbook problem. She also expected them to be innovative in how they shared their solutions or findings from an investigation. For example, she would ask them

> to make their presentation interesting because you tend to fall asleep if it's not. You can get pantomimes, plays, interviews, all kinds of things and they embedded the math into it so that … it actually seems fun, and the kids will remember it [the concept].

These ways of thinking supported her sense-making of tasks that motivated and invited students to engage in inquiry to learn the concept and communicate outcomes in meaningful ways.

3. Inquiry as investigation

Teacher-B's thinking suggested inquiry as investigation, i.e., students conduct a study of a situation as a basis of their learning. She emphasized the importance for students to be placed in situations to develop their understanding and make meaning of the concept on their own. She explained that

> learning [of the concept] occurs when you give students activities to investigate the concept so they can understand and can explain the concept in their own words, and knows why a certain process works … or understand why other ways would not work [and] … is able to make meaning about the problems for the concept that has been presented to them, and knows it's sufficiently to teach to someone else, talk about it to someone else.

Her thinking and actions also emphasized students' autonomy during their investigations. As she explained,

> I get to go around and listen to the groups … and I ask them questions if they're stuck but that's about it. I will simply watch how the groups are working together and if I do see a group is stuck, I will try to come up with a question that will allow them to continue, but I will not give anybody the answer at any time.

This thinking of investigations and how to support it provided another basis to support her sense-making of the research orientation of her inquiry-oriented approach.

4. Peer interactions as collaboration

Teacher-B's thinking suggested the importance of peer (student-student) interaction as collaboration to motivate and support students' learning. She explained,

> I would again stress the importance of having them work with others, so that ideas could bounce back and forth between students and perhaps could spur them on. … Student interaction is really important because I don't see things the way kids see things, and I don't solve problems the way kids solve problems … and many times they can express things in different ways that I haven't thought of, so it's very important. Also, if there's a lot of student interaction, and if you can get kids working together in groups, then they won't always look to the teacher for solutions, they'll look to each other, and I think that's very important, … but also they get to interact more with each other, and can use each other more to enhance their own learning. They have to talk to each other a lot, plan and investigate together, and work together to understand the concept.

This thinking supported her sense-making of group investigations and students' collaboration to inquire with and through each other's thinking as integral aspects of her inquiry approach.

Teacher-C's Case

Inquiry as dialogic discourse

The theme "inquiry as dialogic discourse" represents the central features of Teacher-C's inquiry-oriented approach, i.e., students' inquiry is driven by student-student and student-teacher dialogue during mathematical discourse. The approach consists of a discourse-inquiry cycle based on students' and teacher's questions with central features of: pose question; reflect or investigate; share, discuss and reflect on conclusions. This cycle varies depending on whether there is a predetermined inquiry task or whether the inquiry task flows out of the discourse. For example, in Teacher-C's lesson on systems of equations, teacher or student posed a question that initiated a discourse-cycle; students investigated or reflected on examples of the algebra concepts being studied to identify what they noticed; they verified what they noticed; they made and investigated conjectures; they discussed and reflected on findings. Following is an abbreviated version of her first lesson on systems of equations to highlight key aspects of the discourse-inquiry cycles.

For the first discourse-inquiry cycle, Teacher-C initiated the discourse by asking students to think of and suggest examples of a linear function and a quadratic function. (Students were always seated in groups and were given time to think and talk with each other before contributing to the whole-class discourse.) Students shared examples and agreed to focus their discourse on

$$y = x^2 + 2x + 10$$
$$y = 2x + 3$$

They investigated, reflected on and discussed what they noticed about these two functions. The discussion resulted in a new discourse cycle based on students' noticing and questioning: "why does the quadratic not cross the x-axis?" Teacher-C initiated the discourse by getting them to think about what they knew about integral and non-integral zeros and how they relate to the factored form of the function by considering the two examples: $y = (x+3)(x-7)$ and $y = (2x+3)(3x+4)$.

Students reflected on the examples to identify what they noticed and how it related to their initial question. They shared what they noticed and their conjecture that the reason the quadratic did not cross the x-axis "must have something to do with whether the function is factorable". They checked their conjecture by investigating different examples (including $y = -x^2 + 2x + 11$) with their graphing calculators, shared and discussed their findings. They verified their conjecture and also concluded that "there were zeros except when they (x values) were irrational".

This led to the next discourse cycle initiated by Teacher-C asking, "So then what determines if there are zeros or not, if it is not just whether we can factor it?" Students conjectured that "it must have something to do with the combination of the coefficients and the constant". Teacher-C showed them the discriminant $\sqrt{b^2 - 4ac}$ without explanation about it [a topic she returned to in a future class]. Students investigated it with their examples and discussed in their groups what role the discriminant plays in determining the nature of the zeros. They shared their findings and discussed the relationship between the discriminant, zeros, and existence of solutions. The class then discussed briefly the complex number system and imaginary numbers and why a negative number was problematic and led to no solution. Students talked about radical functions and their understanding of not being able to take the square root of a negative number, making connection to a previous lesson.

Returning to the initial example

$$y = x^2 + 2x + 10$$
$$y = 2x + 3$$

Teacher-C initiated a new discourse cycle by asking "there may not be a solution but is there a relationship?" Students reflected on the example, developed conjectures, and shared their conjectures which included: it has to do with distance; one is above the other when graphed; they are not equal. This triggered a new discourse cycle that focused on "not equal" and reflecting on the inequality statements:

$$x^2 + 2x + 10 > 2x + 3$$
$$2x + 3 < x^2 + 2x + 10$$

Teacher-C then asked students for an example of a linear function and a quadratic that opened down. The pattern of the lesson continued with the students determining that the system had two intersections, which led to a discussion about the equality and the inequality in the graph and how to state it. For homework, the teacher asked them to consider, "how can you tell what function is which, like if there are two of the same function on a graph" which became the task to start the first discourse cycle in the next lesson. Teacher-C eventually got to applications using contextual problems and including discourse around "how the concepts exist in the world".

Central features of Teacher-C's thinking

For teacher-C, as for Teacher-A and Teacher-B, the same central features were the basis of determining her sense-making of her inquiry-oriented approach. However, the interpretation for each feature is different in terms of her thinking of the nature of the algebra concept, task, inquiry, and peer interactions in her practice.

1. Algebra concept as pattern and connection

In the context of her teaching, Teacher-C's thinking emphasized the concept of systems of equations in terms of patterns and connections. This is related to her general view of the beauty of mathematics in terms of its structure, meaning, connections, and challenges or complexities. She explained,

> I want my students to look for patterns and connections and interconnectedness in mathematics, trying to see its connection to the world and where it is.

> I want my students to understand that mathematics … is complex and complicated…. I want them to understand that there are patterns, but there are also no answers, there is not certainty.

This way of thinking about the systems of equations provided a basis for her sense--making of what students attended to and how they attended to it during discourse of the concept in her inquiry-oriented approach.

2. Tasks as mathematical structures

Teacher-C's thinking suggested that tasks are mathematical structures, i.e., situations that represent systems of equations in terms of particular structures based on patterns and connections. She explained,

> The inquiry kind of work that we are doing, I am finding that it is the structure of the mathematics and the patterns and the connections that seems to keep coming up as an entry point for me to be able to start to look at something to do with the kids to learn the concept.

This way of thinking supported her sense-making of focusing on examples of the algebra concepts studied that allowed students to inquire and reflect on their structure and mathematical connections to understand the concepts in her inquiry-oriented approach.

3. Inquiry as reflection

Teacher-C's thinking suggested inquiry as reflection, i.e., students are expected to think deeply and systematically in noticing and unpacking structures of the concepts. This is related to Dewey's (1933) notion of reflective thinking as reasoning methodically and logically and thus is a major instrument in discipline-based inquiry. Teacher-C described inquiry as "a way of thinking, complicated, nonlinear, unpredictable, ambiguous, abundant and uncertain". She expected students "to wonder, be curious, question, and ask why", "to think and make sense for themselves", "to critically enter a topic, looking at the connections", "to reflect and notice for themselves", "to be thoughtful in their discourse", "to look at and think of things in ways that I would have never ever considered". She explained,

> I am constantly amazed at their thoughtfulness; at times they seem so much smarter than I. … The students are continually seeing and thinking of things in ways I never imagined. There are ways they look at things that I would have never ever considered.

Her thinking and actions also emphasized students' autonomy during their reflection and discourse. Her students had autonomy in determining the direction of their reflection in terms of what they noticed and discussed. She built on their thinking and allowed them to take the lesson outside of her predetermined focus. As she explained,

> What I have noticed of late is the openness of my students to think and go places they have not before. As I open a topic, I never know where it will go. More often than not we end up in territory way beyond the 'curriculum' for that grade. The grade 10's, in a conversation about the sine and cosine of supplementary angles, ended up describing the unit circle.

Her way of thinking about inquiry allowed her to make sense of her dialogic-discourse approach in which students' inquiry was often about reflecting on patterns and connections through dialogue in the lesson on systems of equations.

4. Peer interactions as mirrors
Teacher-C's thinking suggested the importance of peer (student-student) interactions as mirrors, i.e., they allow students to see themselves and understand their thinking. She explained,

> Conversation and dialogue are fundamental. … I think the only way we start to know what we know is when we start to see it through somebody else. So when they start to talk about things, that's the way they can start to process and understand who they are through what they are talking about and start to understand also the math concepts.

This thinking supported her sense-making of discourse in her inquiry approach being about the students and their sense-making of the concept.

Discussion

Central features of inquiry-oriented approaches

The three teachers' inquiry-oriented approaches have a surface structure that is consistent with theory discussed in the section on perspectives of inquiry, i.e., a cycle of: begin with a question, investigate, discuss, and reflect. The other aspect of the three approaches consistent with theory involves an emphasis on learning and not on teaching, *e.g.*, support for students' autonomy and initiative and students' engagement in interactions with peers and the teacher; an emphasis on students' experience (prior and current) in learning; and allowing students to inquire into self (what they know or not know, can do or cannot do) to determine direction of learning. Thus, the three teachers were able to develop their practice oriented towards an inquiry perspective to different degrees. They also displayed abilities, particularly in the cases of Teacher-B and Teacher-C approaches, that are considered essential for supporting inquiry-based learning, for example, (based on the National Research Council, 2000), the ability to embrace uncertainty, foster student decision-making by balancing support and student independence, recognize opportunities for learning in unexpected outcomes, maintain flexible thinking, and tolerate periods of disorganization. They were able to pose questions that direct discussions to another level of understanding and, for Teacher-C in particular, to "let go" and listen to the students for direction. While not a focus in this paper, it is worth mentioning that the teachers did experience challenges, particularly in relation to students' initial resistance to inquiry. But each teacher had a different approach to help students make the transition to become more autonomous in their learning.

While the above common features of the teachers' approaches classify them as inquiry-oriented, the differences in how these features were interpreted and enacted in the teachers' classrooms determined the nature and type of inquiry orientation involved. This resulted in each teacher's approach consisting of central features specific to it that suggested three possible approaches to engage students in inquiry in learning the algebra concepts. These approaches emerged from the data as oriented towards three related perspectives of inquiry.

Teacher-A's approach has central features that orient it as a *problem-solving approach* (*e.g.*, Polya, 1945). In this approach, students are given a problem; work in groups to understand problem, plan and carry out plan to a solution; come together to share and reflect on solution; identify gaps in their knowledge that leads to new problem. Teacher-B's approach has central features that orient it as a *research approach* or "scientific inquiry" (Schwab, 1966). In this approach, students are given an inquiry (research) task that requires them to plan and carry out an investigation that involve collecting and analyze information; draw, discuss, and validate (determine whether makes sense or not) conclusions about a mathematical concept or process; and communicate results and experiences in meaningful ways. In Teacher-B's case, investigation included studying a real-world mathematical situation or a mathematical process involving the concept of systems of equations to learn more about it by discovering its meaning, use, and process, and proposing explanations based on evidence derived from the work. Schwab (1966) considered this type of inquiry learning as "stable scientific inquiry" used to fill a particular space of growing knowledge for students. It is stable inquiry because there is an accepted body of knowledge students must understand.

Teacher-C's approach has central features that orient it as a *dialogic-discourse approach* or "dialogic inquiry" (*e.g.*, Wells, 1999). According to Wells, this approach involves co-construction of meaning through discourse that is dialogic (engaging in reflection and discussion). He describes dialogic inquiry as "A willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them" (p. 121). He explains it can be achieved through telling stories, developing explanations, making connections, and testing conjectures, through action and/or the creation of further symbolic representations. This approach also involves reflective thinking which according to Dewey (1933) requires active, persistent, and careful consideration of a situation in light of evidence that supports it. In Teacher-C's case, students are given a task to initiate the discourse; they reflect on what they know, do not know and want to know or what they notice about the task to contribute to the discourse through, for example, explanations, examples, questions, and conjectures; they check and verify what they notice; they investigate conjectures, and discuss findings; pose new questions. In this approach, both teachers and students' questions guide discussion and direction of the lesson which may deviate from what is intended by the teacher. Investigations are not always predetermined, but can evolve from the discourse.

These three inquiry-oriented approaches as practised by the teachers afford different types of leaning (knowledge construction) of the concepts. For example, Teacher-A's students had opportunities to create knowledge of different strategies to solve problems involving the concept and develop problem-solving thinking and procedural understanding in relation to the methods to solve systems of equations. Teacher-B's students had opportunities to create knowledge of real-world sources, meaning and usefulness of the concepts, to develop problem posing, problem solving, collaborative and inquiry skills, to develop algebraic thinking and conceptual and procedural understanding in relation to structural and applied meanings of the concepts, and to develop an inquiry disposition. Teacher-C's students had opportunities to create knowledge of the mathematical structures and meanings of the concepts, to develop algebraic thinking and conceptual and procedural understanding in relation to structural meanings of the concepts, to become critical thinkers, and to develop collaborative skills, an inquiry disposition, and reflective thinking.

Central features of teacher thinking

The three inquiry-oriented approaches emerged not only from the different styles of the teachers' practice, but more importantly, were supported by central features of their thinking. Table 1 summarizes these features that contributed to each teacher's sense-making of her approach. These features emerged as one way of viewing how the teachers' thinking helped to define and shape the way the three approaches were enacted in the classroom.

The teachers posed tasks that reflected how they made sense of the algebra concept (systems of equations in this case) and what students were to learn about and through it. Teacher-A emphasized the concept as *strategy*, Teacher-B as *useful tool* and Teacher-C as *pattern and connection*. This allowed them to make sense of the tasks they posed for students' inquiry as *situated strategies* (Teacher-A), *meaningful experiences* (Teacher-B) and *mathematical structures* (Teacher-C). The teachers also engaged students in ways influenced by their thinking about inquiry and peer interactions. For inquiry, Teacher-A wanted students to *play* with the problems, Teacher-B wanted them to *investigate*, and Teacher-C wanted them to *reflect* (think deeply). Supporting students' autonomy and initiatives were important to them to facilitate these processes as were their sense-making of peer interaction as *source of information* (Teacher-A), as *collaboration* (Teacher-B), and as *mirrors* (Teacher-C).

Studies on the relationship between teachers' thinking and practice (*e.g.*, Leder, Pehkonen, & Torner, 2003; Ponte & Chapman, 2006) indicate the importance of this relationship in understanding practice. This study supports this in terms of specific features of the teachers' thinking that reflect their sense-making of their practice. Through these

Table 1 — Central features of teachers' thinking to make sense of their inquiry-oriented approaches

Teacher	**Teacher-A**	**Teacher-B**	**Teacher-C**
Inquiry-oriented approach	As problem solving	As research	As dialogic discourse
Central features of thinking	Concept as problem-solving strategy	Concept as useful tool	Concept as pattern and connection
	Tasks as situated strategies	Tasks as meaningful experiences	Tasks as mathematical structures
	Inquiry as play	Inquiry as investigation	Inquiry as reflection
	Peer interactions as source of information	Peer interactions as collaboration	Peer interactions as mirrors

features, it highlights one way of interpreting key aspects of the teachers' thinking central to inquiry teaching to explain their sense-making of their inquiry-oriented approaches. Since the aim is to understand the sense-making aspect of the teachers' thinking and to make sense of what they are able to do in using an inquiry approach, discussing deficiencies in their approaches or thinking is not the intent of this paper. Also outside of the scope of this paper is the relationship between their "specialized mathematics knowledge" (Ball, Thames, & Phelps, 2008) for algebra that teachers need to hold and their inquiry-oriented approaches.

Conclusion and implications

Inquiry-based teaching could be a challenge for high school mathematics teachers who are accustomed to teacher-centered classrooms because it requires teaching differently from how they were taught and different skills from the traditional classroom. This study provides examples of teachers who were able to adopt an inquiry-oriented approach to their teaching. Based on the practice of these three teachers, the study offers insights about three ways in which inquiry can be used in teaching high school algebra. While these approaches vary based on the teachers' sense-making, they have at their core key features of inquiry-based teaching. In particular, the study shows how these approaches can be used to engage students in the learning of systems of equations with different learning opportunities for each approach based on actual practice of teachers in actual classroom contexts. Thus, the description of these approaches could be used to provide opportunities for other teachers to learn about inquiry-oriented teaching based on practice and to reflect on their own practice to understand differences and similarities in terms of the central features discussed. Although these approaches were identified in the context of systems of equations, they could form a basis for consideration for inquiry teaching of other algebra or high school mathematics concepts. In fact, the three teachers used aspects of all three approaches in their overall practice, but the focus here was on what was dominant in teaching systems of equations. In particular, Teacher-B was consistent with her approach throughout her practice while Teacher-A and Teacher-C sometimes used the research-oriented approach.

This study also draws attention to the importance of teacher thinking in framing their practice. It suggests that helping teachers to adopt inquiry-oriented approaches must take into consideration their thinking on an ongoing basis throughout the process. Simply exposing teachers to theory about inquiry may not lead to inquiry-based teaching depending on the thinking they use to support the actual practice of it. Professional development needs to expose and build on the teachers' thinking and not offer theory independent of it. This study highlights at least four possible central features of teacher thinking that should be addressed in teacher professional development.

Finally, this study could provide a basis for researchers to build on. The nature of inquiry-oriented approaches as presented here is based on the practice of three teachers, so it is likely limited in scope. The three approaches and central features of teachers' thinking presented in Table 1 are not intended to be exhaustive, but to provide examples of them to address an underrepresented area of research in mathematics education. Further research of other classrooms is needed to develop a deeper understanding of these and other ways teachers' thinking can shape inquiry in practice. Such research could build on this work by studying other teachers' inquiry practice.

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Abstract. Although there are suggested guidelines about teaching algebra from a reform perspective, what happens in the classroom depends on the teacher. This study investigated three experienced high school teachers' inquiry-oriented approaches to teaching algebra with a focus on teaching systems of equations. It addressed two questions: (1) What are the central features that characterize the teachers' inquiry-oriented approaches? (2) What are central features in their thinking that allow them to make sense of their approaches? The findings show that the teachers' approaches had common features consistent with theory of inquiry that classified them as inquiry-oriented. However, the differences in how these features were interpreted and enacted in each teacher's classroom resulted in three different perspectives of their inquiry approaches with central features specific to each that afforded different types of learning of the algebra concepts. Four central features of the teachers' thinking associated with the nature of the algebra concept, task, inquiry, and peer interactions were identified as bases of their sense-making of their inquiry-oriented approaches. Although the key features of the approaches and the teachers' thinking were identified in the context of teaching systems of equations, they could form a basis for consideration of inquiry teaching of other algebra or high school mathematics concepts.

Keywords: Inquiry-based teaching, algebra, systems of equations, high school mathematics teachers, mathematics instruction, teacher thinking

Resumo. Embora existam recomendações para o ensino da Álgebra, numa perspetiva reformista, o que acontece na sala de aula depende do professor. Neste estudo, procurou-se investigar abordagens de ensino exploratório (*inquiry-based teaching*) da Álgebra de três professores experientes, do ensino secundário, com foco no tópico dos sistemas de equações. O estudo pautou-se por duas questões de investigação: (1) Quais são as características centrais das abordagens de ensino exploratório dos professores? (2) Que aspetos centrais do seu pensamento lhes permitem dar significado às suas abordagens? Os resultados mostram que as abordagens dos professores revelam aspetos comuns, consistentes com a teoria da inquirição, e que as identificam como abordagens de ensino exploratório. No entanto, as diferenças no modo como estes aspetos foram interpretados e emergiram na prática de cada professor, na sala de aula, conduziram à identificação de três perspetivas diferentes das suas abordagens de ensino exploratório, com aspetos centrais específicos de cada uma e que proporcionam diferentes tipos de aprendizagem de conceitos algébricos. Foram identificados quatro aspetos centrais do pensamento dos professores como base da atribuição de significado às suas abordagens de ensino exploratório, os quais se associaram à natureza do conceito algébrico, à tarefa, à inquirição e às interações entre pares. Apesar de terem sido identificadas, no contexto do ensino de sistemas de equações, as características chave das abordagens e do pensamento dos professores, elas podem ser consideradas no ensino exploratório de outros conceitos algébricos ou de conceitos de outros tópicos matemáticos do ensino secundário.

Palavras-chave: Ensino exploratório; álgebra; sistemas de equações; ensino secundário, prática de ensino da matemática; pensamento do professor

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