Relating Students' Perceptions of Interest and Difficulty to the Richness of Mathematical Problems Posted on the CAMI Website

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Context and Problem Statement

Mathematical problems, problem solving, and problem posing became key concepts in mathematics curricula reform documents worldwide over the past decades. Several aspects, such as the types of problems that should be proposed in classrooms; the type of problems students should solve based on their grade level (elementary, secondary, etc.) and their abilities; the ways in which students can solve problems; and the challenges and obstacles problems may bring, attracted the attention of researchers in mathematics education (Lester, 1994; Schoenfeld, 1994; Silver, 1997). The question of what constitutes a "good" mathematical problem also gained some importance of mathematics educators. Two features that make a problem "good" have been highlighted: namely the mathematical concepts involved in the problem, and either the situation to be investigated or the context in which the mathematics is presented (Hiebert et al., 1996). The latter may be purely mathematical or related to real life (Hilbert, 1900).

In this article, we explore the problems posed on the CAMI (Communauté d'apprentissages multidisciplinaires interactifs) website (www.umoncton.ca/cami) in order to examine their richness and to compare it with the members' perception of how interesting and how difficult the problems are. The main goal of the CAMI website as a learning environment is to propose to students from different school levels and with different abilities mathematical problems that are rich, interesting, and challenging (Freiman & Lirette-Pitre, 2009).

Over the first decade of its existence (2000–2010), the CAMI website has evolved into a virtual community that offers online problem-solving opportunities in mathematics and other disciplines including: science; chess; and, most recently, in social studies, and literacy. To become a member of this community, participants must register and create an account. The account includes an e-portfolio in which all website activities (problems solved, feedback received, etc.) are saved. The structure of the CAMI follows the Problem of the Week model similar to the one used on the MathForum website (Renninger & Shumar, 2004). Every two weeks, four problems in each discipline are proposed. Each problem is assigned a level of difficulty, ranging from easiest (Level 1) to hardest (Level 4). Registered members can freely choose and solve the problems they desire and submit their solutions electronically by means of an electronic form implemented on the website. Once the time frame for the problems is over, pre-service teachers assess each solution received and provide personalized e-feedback to each author (member) of the solutions. This e-feedback is saved in the member's e-portfolio. For each problem, the most interesting solutions and the names of successful problem-solvers are shared with other members on a common e-space. Other activities are also available on the website, such as: a discussion forum, a space for members to submit their own problems to the team of researchers, etc. Teachers may also find their own way to integrate problems into the classroom and beyond.

Following the Design-Based Research paradigm (The DBRC, 2003), we introduced several tools that allowed us to collect multiple data about the problem-solving activities in the CAMI environment to be used for further studies (Freiman & Lirette-Pitre, 2009). For instance, Manuel (2010) studied the mathematical creativity in members' solutions in relation to the richness of the problems *virtual collective solution space* (which contains all submitted solutions to one problem). He found that richer problems generated more original solutions containing multiple correct answers. From a Vygotiskien creative imagination perspective (Vygotsky, 2004), and using DeBlois's (2003) model for assessing students' authentic solutions to mathematical problems, Bélanger, DeBlois and Freiman (2014) found four types of creativity (which they called 'colours'). Each 'colour' puts emphasis on one of the following aspects: domination of personal knowledge, domination of the problems' constrains, building relationships between constrains based on the system of personal knowledge, as well as search for equilibrium between the constrains of the problem and personal knowledge) in the plausibility of the solutions submitted combined with the type of mathematical relationships constructed by the members.

The nature of the problems posted on the CAMI in relation to the members' perceptions was also important to study. Based on the interviews conducted with middle school students actively participating in problem-solving, Freiman and Manuel (2007) found that students and teachers appreciated the problems posted on the virtual environment. However these results were very broad, as they did not take into account the various types of problems posted, and were obtained from a small sample of participants. A more recent paper by Pelczer and Freiman (2015) aimed to connect students' perceptions of a problem's difficulty to the level (varied from 'easy' to 'hard') assigned to the problem by the CAMI-team when each problem was placed into one of the four mentioned above categories. Results obtained from a small sample of problems (one category, two years) suggested that members perceived problems that could be solved step-by-step (algorithmically) and had familiar contexts as easy, whereas more complex problems requiring more sophisticated mathematical modeling were perceived as more difficult. These results did not take into account the richness of the problems when studying the difficulty of the problems. Thus, in this paper, we choose to focus of our analysis on the richness of the problems (Manuel, 2010; Manuel, Freiman & Bourque, 2012) in relation to how interesting and difficult the students perceive them to be (this time we analyse all problems for which the perceptions were collected). By also adding the perception of the problem as *interesting* along with being *difficult*, we aim to diversify determine whether the CAMI members' perceptions of the problems they solved and relate them to the richness of the problems according to the researchers-designers' perspective. Hence, our research questions are as follows:

1) How rich are the problems posted on the CAMI website?
2) How does the richness of the problems according to the researchers of the CAMI website relate to the members' perceptions of interest and difficulty?

We hypothesize that students will perceive richer problems as being more interesting and difficult. However it is possible that, for some problems, the perceptions of the designers of the problems differ from those of the problem-solvers. We were thus motivated to explore the possible gap between creators' and users' perceptions of contents within the CAMI website. Hence, by confronting two perspectives of a mathematical problem, one from the designer's point of view and the other from the problem-solver-learner's point of view, we expect to get a deeper insight into the nature of the mathematical problems we post on the website and how they are perceived by students. Besides the local impact on the quality of problems we create and the learning experiences of our students (which would also be of interest of other teams working on designing virtual content), the paper addresses important (and underexplored in the literature) issues of assessment of the richness of mathematical tasks and confronts it to problem-solvers' perceptions of problem's difficulty and interestingness.

Theoretical Perspectives

Working definitions of rich mathematical problems

For many decades, it has been suggested that problem solving plays a central role in mathematics teaching and learning reforms (National Council of Teachers of Mathematics, 1980, 1989, 2000). Furthermore, problem solving is known to be one of the most cognitive demanding activities (Charnay, 1996). Researchers have made numerous attempts to study possible ways of supporting students in becoming better problem solvers and in looking into different factors affecting the problem-solving process including the type of problems. Many authors based their inquiry on Polya's (1945) work in which he proposed four steps to solve non-familiar problems: understanding the problem; creating a plan; executing the plan; and interpreting the results. Since then, other models have emerged thus refining Polya's steps and suggesting other aspects to be considered. One such aspect is the richness of problems, which can be defined as follows:

> Rich tasks can enable students to work mathematically by allowing them to: step into activities even when the route to a solution is initially unclear; get started and explore because the tasks are accessible to pupils of wide ranging abilities, pose as well as solve problems, make conjectures; work at a range of levels; extend knowledge or apply knowledge in new contexts; work successfully when using different methods; broaden their problem-solving skills; deepen and broaden mathematical content knowledge; see and make sense of underlying principles or make connections between different areas of mathematics; work within include intriguing contexts; and observe other people being mathematical or see the role of mathematics within cultural settings (http://nrich.maths.org/6299).

Different scholars view the concept of rich problems differently. For instance, Piggot (2008) saw a rich problem as one possessing many characteristics that can offer a variety of opportunities to meet the needs of learners at various moments in an environment in which the problem is posed. She also suggested that the richness is influenced by the questions asked by teachers and the expectations from students. Manuel (2010) conducted a literature review to identify the features or characteristics in the text of problems that could classify them as being rich. He argued that a problem is rich when it respects as many of the following features as possible: is open-ended (Diezmann & Watters, 2004; Takahashi, 2000); is complex (Diezmann & Watters, 2004; Schleicher, 1999); is ill-defined (Murphy, 2004); contextualized (Greenes, 1997); and has multiple possible interpretations (Handcock, 1995). The following paragraphs present working definitions of each feature used by Manuel (2010) to analyze richness of the problems on the CAMI website.

A problem is *open-ended* if it has multiple correct answers or can be solved using various strategies (Takahashi, 2000). Although some might argue that open-ended problems automatically bring both multiple answers and strategies, those two criteria were considered by Manuel (2010) as disjoint, suggesting that some problems could lead to multiple answers, but could be solved using the same strategy. For instance, consider the following problem:

Find all possible sums of 12 using whole numbers.

This problem has multiple answers. However, it is possible that a student will use the same strategy, for example a systematic approach: finding possible answers with two numbers, then three, and so on.

A *complex* problem is one that respects most of the following criteria: requires more than one step to solve it (Schleicher, 1999); asks solvers to find patterns, generalize results or make mathematical proofs; asks to make choices and justify them; and asks to create other problems or questions for further explorations (Diezmann & Watters, 2004; Freiman, 2006).

A problem is *ill-defined* if it is missing necessary data (information). That missing data can be found either by searching (for instance on the Internet), or can be explicitly defined by the problem solver himself (Murphy, 2004). A problem is also ill-defined if it contains unnecessary data or only presents unrelated information (Kitchner, 1983). Although, it is plausible to suggest that an ill-defined problem can also have characteristics making it open-ended, Manuel classified ill-defined as a different feature since a problem can be ill-defined yet have only one correct answer. For instance, consider the following:

When Bob got on the plane in Paris, the clock indicated 10 AM. When Bob got off the plane in Montreal, the clock indicated 11 AM. How long was Bob's flight?

This problem is considered as ill-defined since the time difference between the two cities is not indicated in the text. The student must search for this information and add the value to the difference between the times indicated the problem. However, Manuel did not consider it as an open-ended problem since there is only one correct answer.

A problem with *multiple possible interpretations* is one that encourages different ways of thinking (can be seen in different ways) about it, in other words has multiple entry points (Handcock, 1995). This feature could automatically qualify it as open-ended since different interpretations lead to different answers. In fact, some problems have multiple interpretations, yet each interpretation leads to a single correct answer; a problem of this type would not be considered open-ended. For instance, consider the following problem relating to the game Snakes and Ladders. Students are given a Snakes and Ladders game board and asked the following questions:

1. Using one die, what is the minimum number of turns you would need to win the game if you always rolled the number you wanted on the die on your turn?

2. How many turns would you need to win the game?

Each question can have two different interpretations depending on whether you begin on cell #1 of the board or you begin outside of the board. However the first question has only 1 correct answer for each interpretation. Thus it would not be considered open-ended.

The *contextualized* problem is one in which the mathematics are presented in real life or in fictive situations (Greenes, 1997). An exercise asking to solve the equation $3x + 5 = 14$, or problems wrapped in a kind of "artificial situation" like referring to a person that needs help to solve an equation from a mathematics textbook are not considered as contextualized problems. They simply have a mathematical context.

Figure 1 gives a visual representation of Manuel's (2010) model of the rich mathematical problem. The rounded rectangles represent the five main features selected in his definition while the ovals represent criteria used to assess different elements of definition for each feature in the text of the problem. These criteria were used to analyze the richness of each problem on the CAMI website.

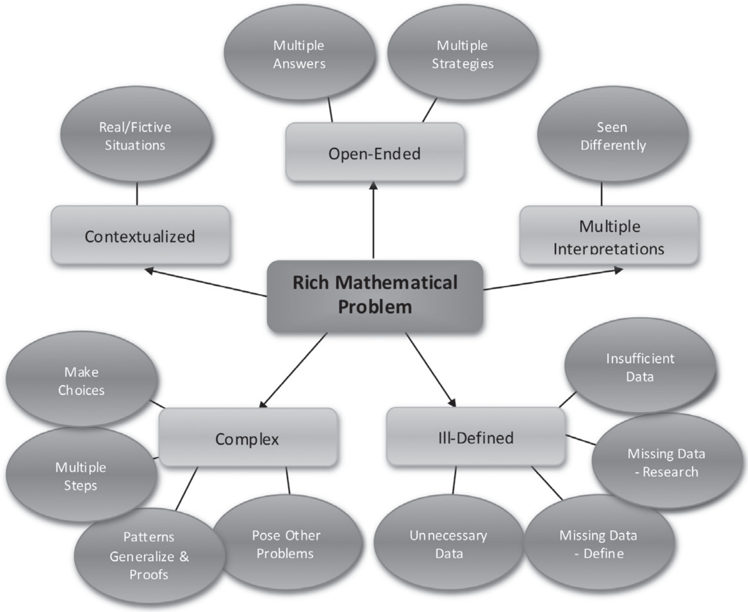

Figure 1. Model of the richness of a mathematical problem (Manuel, 2010).

Perceptions of Interest and Difficulty of Problems Posted Online

In the literature, affective components related to learning mathematics in general and to problem-solving in particular are grouped into two main categories: factors related to the perceptions of the task (how the person feels about the problem); and factors related to self-efficacy (how the person feels about his/her capacity or abilities to solve the problem). For example, Montague and Applegate (2000) studied middle school students' perceptions of problem difficulty. They found that students with learning disabilities rated problems as significantly more difficult, and obtain significantly lower results on the tests than both average and gifted students. Their findings suggested that average students rated problems as significantly more difficult than gifted students but their results did not differ significantly.

One of the popular models that may stimulate students to solve rich mathematical problems on a regular basis is the Problem of the Week Model (Renninger & Shumar, 2004) which became popular with the growth of online environments in the 1990s-2000s. A Google-based search for "problem of the week in mathematics" produced 257,000,000 links (May 15, 2015). Reflecting on the growing online resources on problem-solving, it is plausible to assume that creators of these problem-solving activities expect their problems to be attractive, interesting, rich, and challenging while addressing a specific (access is restricted to certain groups) or a general (resources open to all) public. When choosing the content, creators of these environments rely on: specific learning objectives and goals; their experience in and out of classrooms; their work with students with a variety of backgrounds; their research experiences; and their own understanding of what a problem should look like in order to reach particular groups of students.

The creators of the CAMI website in the early 2000s had multiple goals. First, they tried to create rich problems with criteria similar to those presented in Manuel's model (Manuel, 2010). The second goal was to pose problems that would provide students with an opportunity to develop a mathematical reasoning and communication skills for all types of abilities and for all school levels. Solving a problem would require the student to use an authentic process, with or without the help of the teachers, which could potentially allow him/her to go beyond the application of an automatic (algorithmic) solution. Students would thus need to construct their own procedure in order to solve the problem, as well as use appropriate mathematical language to communicate their strategies. The last goal was to propose problems that would suggest meaningful contexts (familiar or attractive) to as many members as possible. These three goals would not only appeal to the mobilisation of cognitive abilities to engage in the problem solving process but also to the emotions and beliefs that reflect feelings and perceptions (affective components). In our study, we analyze the latter by examining how interesting and difficult students perceive the problems to be. This may or may not be related to what the creators and researchers of the CAMI website defined as rich mathematical problems.

Method

This study used a mixed sequential method paradigm (Teddlie & Tashakkori, 2009). It consisted first of a quantitative analysis of the richness of problems and the students' perceptions of the interest and the difficulty of the problems. Second, it consisted of a qualitative study of the problems that received extreme scores for the three variables.

Data

The sources of our data were some of the problems posted on CAMI as well as members' perceptions of interest and difficulty. The problems were selected from the set of 180 problems (posted on the website over the 3-year period) analyzed in Manuel's (2010) study. We did not take into account the problems from the first year in this study since we did not have data on interest and perception at that time.

After the first year of this 3-year period, we added an online survey asking each member to share their perceptions on whether they found the problem interesting or not, and if they found the problem easy or difficult. The two questions were as follows:

1. I found the problem:
    a. Not interesting at all
    b. Somewhat interesting
    c. Very interesting

2. I found the problem
    a. Easy
    b. Medium
    c. Difficult

We collected data on members' perception from 118 of the 180 problems. Those problems were selected for further analysis for this study. The number of solutions per problem varied from 8 to 380 with a mean of 107. There were generally more solutions to problems in the first 2 levels (1–2), which were identified as relatively easy by the researchers. The level 3–4 problems were considered more difficult, many highlighted concepts usually learned in high-school, but could still be solved using other methods from elementary school (mostly grades 5–8).

Analyzing the Richness of the Problems

We used Manuel's (2010) model to investigate the richness of each of the 118 problems. The following rubric (Table 1), used in Manuel's study, assessed if the problem respected each feature and met the corresponding criteria. The shaded parts represented elements taken away after validation of the rubric. The feature "Problems with Multiple Interpretations" and the criteria "Problem contains unnecessary data" (feature ill-defined) were taken out since the coders found them too difficult to assess. After analysis, the criterion "Problem contains insufficient data, which makes it impossible to solve" (feature ill-defined) was also removed because none of the problems fell under this category. The two criteria on missing data (feature ill-defined) were combined into one because the coders found it difficult differentiating between the two (see * in Table 1).

To help us investigate our first question, we used the grid to measure the richness of each problem. We began by evaluating the problem on the basis of each criterion. All criteria were weighted equally: if a criterion was met in the text of the problem, we added one point to the total score. The final score was the number of criteria satisfied by the problem. This metric represented the richness of the problem. Thus the total score for richness varied from 0 (no criterion met) to 8 (all criteria met). We then used normality tests to analyze of the richness of all the problems.

Analysis of Relations between the Richness of the Problem and Members' Perceptions of Interest and Difficulty of the Problem

We first considered the relative frequencies of members' perception of the interest (not interesting, somewhat interesting and very interesting) and the difficulty of the problem (easy, medium, and difficult) from the online survey that each member completed individually after solving the problem. The compilation of data was conducted automatically by the system, so we did not have access to the original data, which guaranteed anonymity for the members.

To investigate the second question of our study, namely to compare data on the richness of the problems with member's perception of interest and difficulty, we first created

Table 1. Rubric used to assess the richness of a mathematical problem

Problem Level (circle the level): 1 2 3 4 **Problem # ________________**		
Feature	**Criterion**	**Respected (X)**
Open-ended problem	Problem has multiple correct answers	
	Problem has multiple appropriate strategies	
Complex problem	Problem requires using multiple steps to get answers	
	Problem asks to make and justify choices	
	Problem asks to create and explore other questions	
	Problem asks to find patterns, generalize or prove results	
Ill-defined problem	*Problem is missing necessary data	
	Problem contains unnecessary data	
	Problem contains insufficient or unrelated information	
Contextualized problem	Problem is centered around a real or fictive situation	
Problem with multiple interpretations	Problem can be interpreted in more than one way	
RICHNESS OF THE PROBLEM (# of criteria the problem respected)		

*There were 2 criteria referring to the missing data in the model. After validation of the rubric, we combined them into one since it was too difficult to distinguish between the cases.

a score for the perception of interest of a problem by multiplying the relative frequency for the option "very interesting" by 3, multiplying the relative frequency for the option "somewhat interesting" by 2, and multiplying the relative frequency for the option "not interesting" by 1. We added those results together and divided the sum by 100. We used the same strategy for the perception of the difficulty of problems, multiplying the relative frequency for the option "difficult" by 3, the relative frequency for the option "medium" by 2 and the relative frequency for the option "easy" by 1, added those results and divided the sum by 100. The scores for students' perception of interest varied from 1.62 and 2.59, while the ones for their perceptions of the difficulty of the problem varied from 1.23 to 2.55. These procedures yielded discrete values for each of our variables. We used a Pearson Correlation to determine possible links between the variables.

In order to conduct a more qualitative analysis of the problems students perceive as interesting (or not) and easy (or difficult), we created 3 categories for each variable depending on the scores we described in the previous paragraphs. We organized the categories so there wouldn't be more than 15 problems in the extreme categories (very interesting, not interesting, difficulty, and easy). The first category (score of 3) was termed *very interesting problems*, and the problems in that category were those that had a score of

2.45 or more. The second category (score of 2) was called *somewhat interesting problems* and the problems in that category had a score greater than or equal to 2 and less than 2.45. The last category (score of 1) was called *not interesting problems* and those problems had a score of less than 2.

The first category of the perception of difficulty (score of 3) was called *difficult problems* and consisted of problems that had a score of 2 inclusively or more. The second category (score of 2) was called *medium problems* and consisted of problems that had a score between 1.5 included and 2. The last category (score of 1) was called *easy problems* and consisted of the problems that had a score of less than 1.5.

As criteria for selecting *extreme cases* with respect to the richness of a problem, we considered as *very rich* a problem that had scores greater than or equal to 6 and less than or equal to 8. Problems having a score from 3 to 5 inclusively were considered as *moderately rich*. All other problems were considered *non rich*. This categorization of the problems permitted us to isolate the most extreme cases on which we performed a qualitative analysis. We called an *extreme case* a problem which had at least two of three aspects with scores categorized as extreme (example: very rich and perceived as very interesting, non rich and perceived as easy, etc.). We qualitatively analyzed problems that were identified as *extreme cases* based on our categorisation according to (a) the type of problem (for example, looking like a 'typical' textbook problem), (b) its original level of difficulty identified by the CAMI researchers, and (c) its mathematical content or (d) other aspects that could be identified.

Results

Normality tests revealed that the scores for the richness of the 118 problems posted on the CAMI website were almost perfectly distributed (Z score of –0.982 for Skewness and –0.452 for Kurtosis). The mean of the richness was 4 with a standard deviation of 1.34. These numbers indicated that the vast majority of the problems were moderately rich.

Looking at the relative frequency obtained for each of the criteria from the grid (see Table 2), it became apparent that most of the problems met with four of our criteria: problems with multiple correct answers, 63.6%; problems with multiple appropriate strategies, 94.1%; problems requiring multiple steps to get to answers, 90.7%; and problems centered around a real life or fictive contexts (contextualized problems), 88.1%. However, the four other criteria (problems asking to make choices and justify them, 16.9%; problems asking to create and explore other questions, 2.5%; problems asking to find patterns and generalize results, 29.7%; and problems that are missing some data or information (ill-defined), 29.7%) were satisfied by less than half of the problems. The differences among the frequencies of the criteria indicated that the vision of the CAMI research team in creating rich problems was somewhat restricted.

In summary, the problems posted on the CAMI virtual community were relatively moderate in terms of their mathematical richness, and this richness was mostly based on three main features: contextualized, open-ended (multiple appropriate strategies), and complex (required multiple steps to find one or more correct answers).

A Pearson correlation test revealed that there was no significant relation ($r = -0{,}003$ $p = 0{,}975$) between the richness of problems and members' perceptions of the interest of problems. In addition, the same test revealed that there was no significant relation ($r = -0{,}047$, $p = 0{,}612$) between the richness of the mathematical problems and the members' perceptions of the difficulty of the problems. The effect size for both cases was small.

Table 2. Relative frequencies of each of the richness of a mathematical problem criteria, N = 118

Feature	Criterion	%
Open-ended problems	Problem has multiple correct answers	63.6
	Problem has multiple appropriate strategies	94.1
Complex problems	Problem requires using multiple steps to get answers	90.7
	Problem asks to make and justify choices	16.9
	Problem asks to create and explore other questions	2.5
	Problem asks to find patterns, generalize or prove results	29.7
Ill-defined problems	*Problem is missing necessary data	29.7
Contextualized problems	Problem is centered around a real or fictive context	88.1

In order to pursue our analysis in a more qualitative way, we looked at the distribution of the data from the online survey with respect to richness, interest and difficulty. Table 3 classifies the problems according to richness, perception of interest and difficulty.

Overall, according to our scoring system, 16.1% of the problems were classified as *very rich*, 72.9% of them were *moderately rich*, and 11% were classified as *not-rich*. None of the *very rich* problems were perceived by members as *very interesting*. In fact, 84.1% of the *very rich* problems were perceived as *somewhat interesting*, and 15.9% as *not interesting*. Moreover, 57.9% of *very rich-somewhat interesting* problems were perceived as *moderately difficult*, 15.8% as *easy*, and 10.5% as *difficult*. 5.3% of *very rich-not interesting* problems were perceived as easy, 5.3% as *difficult*, and 5.3% as *moderately difficult*.

Members' perceptions of the *moderately rich* problems were more spread out. 79% of those problems were perceived as *somewhat interesting*, 10.5% as *very interesting*, and 10.5% as *not interesting*. For the perception of difficulty, we noticed that most (61.6%) of the *moderately rich-moderately interesting* problems were perceived as *moderately difficult*; while 8.1% were considered *difficult* and 9.3% easy. Only 1.2% of the *moderately rich-very interesting* problems were perceived as *difficult*, 7% as *moderately difficult* and 2.3% as *easy*.

As for the *non-rich* problems, we noticed that the perceptions were also spread out. The members perceived 15.4% of the *non-rich* problems as *very interesting* and all of them were perceived as *moderately difficult*. Once again, the majority of the problems identified as *non-rich* were perceived as *somewhat interesting* (69.2%). However, none of them were perceived as *difficult*, 53.8% were perceived as *moderately difficult* and 15.4% as *easy*. 15.4% of the *non-rich* problems were perceived as *not interesting*. Among them, 7.7% were perceived as *difficult* and 7.7% as *moderately difficult*.

Table 3. Distribution of perceptions of interest and difficulty for rich, moderately rich, and non-rich problems, N = 118

	Members' Perceptions of Interest and Difficulty (%)								
	Very Interesting			Somewhat Interesting			Not Interesting		
	Difficult	**Medium**	**Easy**	**Difficult**	**Medium**	**Easy**	**Difficult**	**Medium**	**Easy**
Very Rich	0	0	0	10.5	57.9	15.8	5.3	5.3	5.3
Med. Rich	1.2	7.0	2.3	8.1	61.6	9.3	1.2	9.3	0
Non rich	0	15.4	0	0	53.8	15.4	7.7	7.7	0

Qualitative Analysis of Extreme Cases

The first part of our analysis showed that most of our data fell into the *moderately rich* and *somewhat interesting* categories, with perceived difficulty being rather spread out. We decided to analyse qualitatively 12 problems that fell into at least two of the six extreme categories, namely: (a) *very rich*, and perceived as *difficult* (2 problems); (b) *very rich*, and perceived as *easy* (2 problems); (c) *very rich*, and perceived as *not interesting* (2 problems); (d) *non rich*, and perceived as *very interesting* (2 problems); (e) *non rich*, and perceived as *not interesting* and *difficult* (2 problems); and (f) *non rich*, and perceived as *easy* (2 problems). The problems have been translated from French.

(a) Very rich and perceived as difficult

Problem 1:

Tanya is solving a puzzle with her brother, Yvan, during their family trip to Grand-Sault. The goal of the puzzle is to place all numbers from 1 to 9 in a 3 by 3 grid so that the sum of the numbers in each row, each column, and each diagonal is the same. Can you solve this puzzle?

After solving the puzzle, Tanya and Yvan wonder if it is possible to place the numbers in a such a way that the sum of the numbers in the four corners would also be equal to the other sums. Is this possible? Explain your answer.

Bonus question: Invent a new game using this grid and explain how it works.

This problem is a modified version of the very famous 'magic square' puzzle. It is possible to tackle the problem using basic arithmetic (addition), thus making it accessible to very young children. This explains why we placed it as a Level 1 problem on the website. This problem also has multiple answers and a rich potential for searching for patterns and for making generalizations. Therefore this possible investigation can foster the development or algeabric reasoning. The last question (bonus) made the problem even more open-ended because it allowed members to create other problems. The context, children solving a puzzle during a local family trip, added a familiar flavour.

Problem 2:
Mr. Deschamps's workshop produces a set of Acadian Flag stickers. The employees place the stickers on a standard sheet of paper. On each sheet of paper, they make rows of stickers without leaving any space between them. They do not leave any space between the columns either. Knowing that the stickers are all the same size, and that the width of each sticker is half of its length, how many stickers can be placed on one sheet of paper?

This problem also belonged to the *very rich* category and was perceived as *difficult*. The problem satisfied our richness criteria since it is open-ended and ill-defined (you must consider stickers of different sizes and you must see how it is possible to place the stickers without leaving empty spaces — hence different strategies and solutions may be considered). It also presented a familiar local context (Acadian Flag) and stickers, objects which children are very familiar with. In addition, the size of a standard sheet was not given, thus requiring a search for additional information. When solving this problem, members used different mathematical content related to the curriculum: measurement (size of the sticker, size of the sheet of paper, etc.), arithmetic (counting, addition, multiplication, division, proportionality), and more general algebraic work involving variables. From the point of view of difficulty, this problem was classified as Level 3, which coincided with the students' perception of it as also being difficult.

(b) Very rich, and perceived as easy

Problem 3: (one of the first problems posted in 2008)
On January 1st, 2008, Joshua made a New Year's resolution. Every day, he wanted to read at least 5 pages with his parents. If Joshua follows his resolution, how many pages will he have read by the end of the year? How many books can he read by the end of the school year?

Bonus question: If he had challenged himself to read 5 pages on January 1st, and then one extra page per day after that, on what date would he read 100 pages? What about 200 pages?

At first look, this problem seems to be a standard textbook arithmetic task. The first question can be answered quite straightforwardly and only has one correct answer. The problem was still considered ill-defined since members needed to verify if 2008 was a leap year, determine if "at least 5 pages" was always 5 pages or could be more, and also if January 1st should be taken into account, making it more open-ended since two different interpretations were possible. But if one looks at the second question, another assumption is to be made regarding the average number of pages in one book. This aspect made the problem ill-defined because members would need to define that average or research the average number of pages in a book. The notion of the type of book (children's books vs. novels) could also influence the interpretation. Moreover, the bonus part could lead to further questioning thus requiring the use of algebra and adding both complexity and richness to the problem. Yet this problem was classified as Level 1 by the researchers; it was also perceived as 'easy' by the members. The context of this problem was familiar (*contextualized problem*) to the students since many teachers encourage their students to read at home.

Problem 4:
Marianne's parents are building a new house for the family. Marianne's father told her that their house will have a rectangular shaped playroom that will have an area of 24 square meters, and that she can choose the dimensions for that room. What are the possible dimensions of this playroom? What should be the most beneficial dimensions for Marianne and why? Be sure to clearly explain your process.

The mathematical content of this problem is clearly related to finding relationships between the area and the perimeter of a rectangle. But there are several hidden aspects that need to be clarified when attempting a solution. For example, are the dimensions of the rectangle required to be whole numbers? If so, the problem consists of decomposing 24 into a product of two factors. Otherwise, how do we express the dimensions? Would students suggest some solutions or attempt to express the relationship in a more general way? In other words, finding the (multiple) answers consists of finding all possible factors of 24. The last question, related to finding the most plausible dimensions, is very open: it could use reasoning about the harmony of the shape (possibly related to the golden ratio), or it could use a more pragmatic reasoning based, for example, on what objects need to fit in the room, or what activities Marianne wants to do in the room, etc. This problem, while meeting several of the richness criteria, was classified as Level 1 (easy), and was perceived as such by the students.

(c) Very rich, and perceived as not interesting

Problem 5:
The Grade 8 students from the 'Happy People' school are decorating their classroom. In order to create a garland, each student takes a standard colored sheet of paper and cuts out a triangle such that there is as little paper left as possible. In what way(s) can these triangles be cut?

This problem requires exploration related to measurement, more specifically the area of triangles. For example, taking one side as the base and placing a vertex on the opposite side implies the area of the triangle is the same as the area of the remaining part. Some other more creative solutions could be discussed, such as cutting out the parts (like for example, alongside the diagonal) and connecting the two triangles together to form another triangle — thus using the whole area of the rectangle without leaving any leftovers. The type of triangle used (scalene, rectangle, etc.) may also come into play in this problem. Not surprisingly, the problem met several of the conditions to be identified as *very rich*. Yet, according to students' perceptions, it was not seen as interesting, despite being wrapped up in a context of decorating the classroom.

Problem 6:
Patrick wants to buy batteries for his electronic devices. However, he wants long-lasting batteries. He conducts a search on the Internet to find the best quality and finds 3 types of batteries: Alpha, Beta, and Delta. The description of the batteries reveals that the mean life of each of the 3 types of batteries is 12h. However, their median life is 15h for the Alpha, 12h for the Beta and 9h for the Delta. Which battery is the best buy?

This problem was particularly interesting for our analysis since it was the only problem that was categorized as extreme in all three cases: being classified as *very rich*, perceived to be *not interesting*, and *easy*. The last aspect (difficulty) is particularly revealing because originally, it was posted on the website as a Level 4 problem, thus potentially being difficult since it related to the normal curve. Usually, a smaller number of students attempt to solve problems of this level. However, we found that almost 100 members solved the problem and, more surprisingly, a high percentage of correct solutions. Yet, students did not perceive this problem as interesting, perhaps because its text explicitly put some standard mathematical terms related to the statistic module (mean, median) in the foreground.

(d) Non rich, and perceived as very interesting

Problem 7:
Here is a game with numbers

1. *Think of a number.*
2. *Multiply it by 2.*
3. *Add 8 to the result you got in step 2.*
4. *Divide the result you obtained in step 3 by 2.*
5. *Subtract the number you thought of in step 1 from the result obtained in step 4.*
6. *Your final answer is 4.*

Try the same steps with another number. What do you get as a final answer? Do you get the same answer with other numbers? Can you mathematically explain why this game works this way?

Despite having a lower score on the richness scale, this problem required some fine-tuning with numbers (arithmetic) and also when trying to generalize and to prove the pattern. A high percentage of members identified this problem as very interesting. Is this because it was wrapped-up in a game-like situation even though the context is purely mathematical?

Problem 8:
Observe the following grid. If each letter has a different numerical value, what is the value of the missing sum?

D	B	C	A	26
B	A	C	C	31
A	A	C	A	38
C	D	C	B	?
26	28	32	33	

Similar to the previous problem, this one has quite an explicit solution, which can be obtained either by trial and error or algebraically, and may be perceived as a game. This type of puzzle was typical in weekly math challenge newspapers, along with other categories of puzzles, games, and enigmas, thus making it interesting for students.

(e) Non rich, and perceived as not interesting and difficult

Problem 9:
Veronica likes to invent new mathematical terms. She defined « magic equation » as an equation that has 0 as a unique solution. Which of the following equations are magic?

a) $x + 0 = 0$

b) $3 - x = 0$

c) $-5 + 5 = m$

d) $x(x - 1) = 0$

e) $b/0 = 0$

f) $0/c = 0$

g) $d/1 = 0$

h) $n = 0$

i) $(x)(x) = 0$

j) $3x - 4 = 2(x - 2)$

This problem is composed of many equations that can be solved quite easily, yet it contains several hidden challenges: division by 0, finding all roots, handling algebraic expressions correctly, etc. This problem may therefore appear as a set of exercises from a textbook and thus look less interesting to the students. It belonged to the Level 4 category (more difficult) and was perceived as such by the members.

Problem 10:
Manon likes to play a game where she gains or loses marbles. At the end of one day, she had 12 more marbles than she had the day before. However, she did not have a good start to the day as she lost 4 marbles that morning. What happened during the afternoon?

This problem, unlike the previous one, makes explicit references to a game; yet it did not appear to be attractive to students who perceived it as being not interesting and rather difficult. In fact, the problem has quite a simple solution based on elementary operations of addition and subtraction (Level 1), yet the task was not so simple since it required some preliminary analysis of the story that surrounding the mathematical content. In fact, the words "more than" could have been an obstacle. One must understand how, after losing some marbles, a person could still finish the day with more marbles than they began with. The question also demanded some deeper understanding since, instead of requiring a numerical answer, it required an explanation of what happened. This would be quite an unusual type of question for a textbook! In fact, out of 245 solutions submitted, only 65 were correct. The low level of success seems to place the problem as rather a difficult one for a Level 1 type problem.

(f) Not rich and perceived as easy

Problem 11:
Mark puts 5 apples on the table. He cuts 3 of them in half. How many apples remain on the table?

This simple problem was identified as Level 1 by the researchers. It was perceived as easy, and had a high success rate: 95 of the 115 submitted solutions were correct. Neverthless, this problem has the potential to inspire further investigation since it can be interpreted in different ways. For instance, cutting apples in half does not automatically change the number of apples; on the other hand, we can count only the whole apples. This problem is a typical riddle and is often used in recreational mathematics.

Problem 12:
Tristan must solve problems 1 to 6 on page 77 of his math textbook as homework in order to understand the mathematical idea of relationship. He does not understand question 6. Can you help him solve this task?

Question 6: Each member of the Pignon family celebrates his or her birthday today with the exception of the son. Mr. and Ms. Pignon have the same age. Mr. Pignon's age is 6 times the age of his daughter and is four times the age of his son. If today is January 1st 2008 and the daughter was born on January 1st 1999, what is the exact age of each member of the family?

The straightforwardness of the procedures required to solve this problem are hidden by the complexity of its formulation in terms of language used to describe the context. Once extracted from the text of the problems, the chain of calculations beginning from the daughter's age, which can be calculated more or less automatically: the age of the other family members can be determined almost instantly using basic arithmetic operations. It is also important to note that this problem was not considered as contextualized according to Manuel's (2010) model. Could this have had an influence on the level of student interest as they perceived the problem to be very interesting? It is also worth mentioning that this problem was one of the few we analyzed which was created by students during one of our school workshops.

We observed some general trends in the problems we analyzed. First, problems with recreational elements (puzzles/enigmas/riddles), such as 1, 7, 8, and 11 were perceived as rather interesting (*very interesting* or *somewhat interesting*) disregarding their identification as rich or not rich. Second, Problems which, on the surface, looked like typical school-like tasks (even though some had 'hidden' thought-revealing aspects which made them rich) were perceived as *not interesting* and *easy* (problems 3, 6, 9, 10, and 12).

Third, problems related to geometry (measurement) belonged to the *very rich* category, yet members' perceptions varied. For instance, problem 2 was considered *difficult*, while problem 4 was considered *easy*. Both problems were considered as *somewhat interesting*. However, problem 5 was perceived as *not interesting* even though it was somewhat similar to problem 2. Perhaps the difference in perception resided in the fact that problems 2 and 4 involved numbers and required calculations, while problem 5 did not explicitly contain any data.

Fourth, problems originally identified as *rather difficult* (levels 3 or 4) were perceived as *not interesting* by students (problems 6 and 9) or *somewhat interesting* (problem 2). There were two extreme cases which were also surprising. The researchers considered problem 1 as easy, but members considered it difficult. On the other hand, the researchers considered problem 6 as difficult, but the members found it easy. Both problems were identified as being *very rich*.

Discussion and Conclusion

In the context of the CAMI online community, we created an important number of mathematical problems which were posted on the website on a bi-weekly basis. Members submitted over 30,000 solutions between 2007 and 2012, most of which came from New Brunswick French minority school settings.

In this paper, we returned to the original concept of the website: as a place where students could interact with a rich mathematical content and be motivated to solve rich, interesting, and challenging problems for which they hopefully need to mobilize their knowledge, meta-cognitive and communication skills to create and share new and original solutions (Freiman & Lirette-Pitre, 2009). The first question we investigated was to determine whether the problems on the CAMI website were rich (open-ended, complex, ill-defined, and contextualized) following Manuel's (2010) model. The second question was to investigate possible relationships between the richness of a problem and members' perceptions of interest and difficulty for the problem.

Regarding the first question, our results show that, in general, the problems were moderately rich. This is not surprising because three of the main factors which influence this online community: a range in the audience since the website doesn't aim for any specific school or ability level; the mathematical content and methods articulated in school curricula; and the teaching and learning context in which the resource can be used (classroom teaching, home schooling, individual work, etc.). On one hand, problem solving is an explicitly stated goal of the mathematics curriculum:

> Mathematics instruction should provide students the opportunity to explore a broad range of problems and problem situations, ranging from exercises to open-ended problems and exploratory situations. It should provide students with a broad range of approaches and techniques (ranging from the straightforward application of the appropriate algorithmic methods to the use of approximation methods, various modeling techniques, and the use of heuristic problem solving strategies) for dealing with such problems (Mathematical Association of America, cited in Schoenfeld, 1992, p. 33)

On the other hand, there are issues related to the teacher practice, as mentioned by Suurtamm et al. (2015):

> Teacher practices that promote inquiry can be challenging to implement, as they cannot be prescribed. Promoting inquiry requires that teachers ask good questions to prompt student thinking. It is equally important that teachers listen and respond to student thinking in order to develop students' mathematical thinking and confidence as mathematicians. (https://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/WW_SpaceThinkMath.pdf)

Given the conditions, is it reasonable for all problems be rich? This remains an open question.

Regarding the second question, we found no statistically significant relationship between problem's richness according to the researchers and member's perceptions of interest and difficulty. Moreover, regardless of the level of richness the researchers assigned to the problems, the members generally perceived them at the moderate level of both interest and difficulty.

We cannot provide a clear explanation for this finding. Further research is needed such as interviewing students to determine what they find interesting (or not), and difficult (or not) in problems. Butler (2009) informed us that, "students appreciating more than one type of solution scored consistently higher in problem-solving measures and frequency of use of higher-order internal representations" (p. ii). This may give us a cue to further investigate our criteria of richness which would take into account possible student's perception of the problem.

Research has shown that use of the Internet in mathematics classrooms positively affects students' perceptions. For instance, Loong and White (2004) found that 93% of their participants agreed that answering questions on the Internet was more fun than answering the same questions directly from the textbook. This result is similar to our previous study using face-to-face interviews with a small number of local students and teachers (Freiman & Manuel, 2007). Thus it is plausible that solving a problem posted online positively affects students' motivation, thus making the problem more attractive than if it were presented in paper-and-pencil format.

Moreover, we do not know how content (algebra, geometry) and context (real-life, game, standard textbook task, etc.) affect students' perceptions. However, our analysis did provide a deeper insight into possible links between the richness of the problems and how students perceive them. Our analysis also shows the limitations of defining richness based solely on cognitive aspects.

A descriptive analysis on our data on the problems identified six types of *extreme cases*. This shows that a problem we identified as not rich can be perceived as interesting when it is presented within a recreational context (as a game or enigma). For example, Namakashi et al. (2015) stated that magic squares "present a great opportunity to make historical connections and explore problem solving in a fun way" (p. 372). At the same time, very rich problem can be perceived as not interesting. The two examples that fell into this category did not demonstrate any common pattern. However, one of these problems (making garlands) can be considered as too open (not clear what mathematics is involved) and the real-life context is not helpful. The role of the context in mathematical problems needs more discussion. Bates and Wiest (2004) reported that personalizing problems — wrapping them up in "kids-related stories" — could positively affect performance. In fact, this can bring up further discussion about the role of context (wrapping up the mathematical content). The second problem that was considered rich and difficult looked "too traditional", and is explicitly related to school content which may affect the interest. This adds more complexity to the notion of 'richness' which could be viewed in a broader sense (example, the problem presenting more familiar context can increase students' engagement in the problem-solving process), or in focusing more on mathematical content or structure behind the 'wording'

Another interesting finding related to the *extreme cases* is that a very rich problem can be perceived as either difficult or easy. A large amount of work is required to produce a solution (as in the case of the sticker problem), and even more work is required when it comes to deeper investigations (as in the magic square problem). At the same time, the members perceived both problems as easy. In fact, those problems contain questions that could lead to challenging, in-depth investigations which are not that easy from the researchers' point of view. For example, consider Marianne's playroom problem. Some definition of 'best room' can be produced based on a variety of assumptions; the ratio between the sides of the rectangle can be close to the golden ratio (Manuel et al. 2011). Perhaps students' perceptions of these two problems as easy were based on the first question, which, in both of these cases, allows for a straightforward solution. These findings need further investigation.

The last group contained problems that were *not rich*, *not interesting* and *easy*. All problems were related in some way to school algebra or arithmetic where the solutions routinely require the use of standard procedures. Yet, a more careful look reveals these problems can require the use of richer mathematics. Example of this are the algebra problem (questioning why 0 cannot be a solution to the equation $c/0 = 0$) and the apple problem (considering different interpretations and solutions). Teachers can help lead students through this enrichment process thus contributing to the emergence of the culture of mathematical questioning and investigation and, eventually, to the development of deeper mathematical thinking in the context of problem solving.

Overall, our study allows for deeper insight into the richness of mathematical problems and problem solving. Our study also allows for deeper insight into (the art of?) problem-posing and its possible impact on students' perceptions and beliefs although more research is needed in this area.

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Resumo. O CAMI constitui uma comunidade virtual concebida para alunos francófonos, do pré-escolar ao 12.º ano, em particular de New Brunswick, Canadá, mas também do resto do mundo. O principal objetivo deste ambiente de aprendizagem é alargar as oportunidades de aprendizagem da Matemática dos seus membros, propondo-lhes problemas ricos, interessantes e desafiadores. Embora já tenham sido realizados vários estudos sobre o website CAMI, nenhum deles explorou a questão de como a riqueza dos problemas está relacionada com as perceções dos membros relativamente ao seu interesse e dificuldade. O presente artigo responde a essa falta, investigando a possibilidade de ligações entre as perceções dos membros acerca do interesse e dificuldade dos problemas matemáticos propostos e a riqueza desses problemas segundo os criadores do CAMI. Usando uma metodologia mista sequencial (Teddlie & Tashakkori, 2009) e o modelo proposto por Manuel (2010), começámos por estudar a riqueza de 118 problemas relativamente aos quais conduzimos questionários online que interrogaram os alunos sobre o interesse e dificuldade que encontraram nos problemas. Em seguida, estudámos o modo como a riqueza dos problemas se relaciona com as perceções dos alunos. Embora os resultados não tenham revelado qualquer relação significativa entre a riqueza dos problemas, segundo os investigadores, e as perceções de interesse e dificuldade manifestadas pelos alunos, o aparecimento de algumas tendências sugere a necessidade de aprofundar este estudo.

Palavras-Chave: Problema matemático rico; Resolução de problemas online; Perceções do interesse; Perceções da dificuldade.

Abstract. The CAMI website is a virtual community designed for francophone students from K-12 school levels in New Brunswick, Canada, and elsewhere in the world. The main purpose of this learning environment is to increase its members' opportunities to learn mathematics by proposing rich, interesting, and challenging problems. Although few studies have been conducted on CAMI website, none explored if richness of the problems is related to how interesting and difficult the members perceived the problems to be. The present article addresses this lack by investigating the possibility of links between members' perceptions of interest and difficulty of the mathematical problems posted, and the richness of the problems according to the creators of CAMI. Using a sequential mixed method design (Teddlie & Tashakkori, 2009) and Manuel's (2010) model, we studied first the richness of 118 problems for which online surveys were conducted that questioned students about how interesting and difficult they found the problems to be. Then we studied how the richness is related to students' perceptions. Although the results showed no significant relation between the richness of the problems according to the researchers and the students' perception of interest and difficulty, some tendencies, however, prompt the need for further analysis.

Keywords: Rich mathematical problem; Online problem solving; Perceptions of interest; Perceptions of difficulty

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