Lesson Study: from professional development to research in mathematics education

Lesson Study: du développement professionnel à la recherche en didactique des mathématiques

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Introduction

Lesson study is a collaborative model of teacher professional development which originated in Japan. Since the beginning of 21st century lesson study has received growing international attention in terms of both educational research (see WALS annual conferences[1]) and mathematics education research (for eg. Hart, Alston, & Murata, 2011). In this paper, I first briefly present lesson study and the on-going lesson study projects in Lausanne, particularly focusing on a mathematics lesson study group for primary 3 and 4 in-service teachers. This group's work and on-going research will illustrate reflections on aspects of lesson study, in particular: lesson study as teacher training, as resource development, as teacher research, as mathematics education research and as a way to connect education professionals. The conclusion synthesises these aspects as links between research and practice.

Lesson study and our projects

The Lausanne Laboratory Lesson Study (3LS)[2] was created in 2014. It aims to gather and develop research projects at Lausanne University of Teacher Education (HEP Vaud). The 3LS is hosted by two research and teaching units: teaching, learning and evaluation on the one hand, and maths and science education on the other. Its members are researchers from several units of HEP Vaud, student teachers, teachers, and school administrators involved in lesson study.

According to Murata (2011), in a lesson study:

> teachers organically come together with a shared question regarding their students' learning, plan a lesson to make student learning visible, and

> examine and discuss what they observe. Through multiple iterations of the process, teachers have many opportunities to discuss student learning and how their teaching affects it. (p. 2)

The lesson study process can be described as a cycle. The most common sequencing is based on Lewis et al.'s (2006) description (Figure 1).

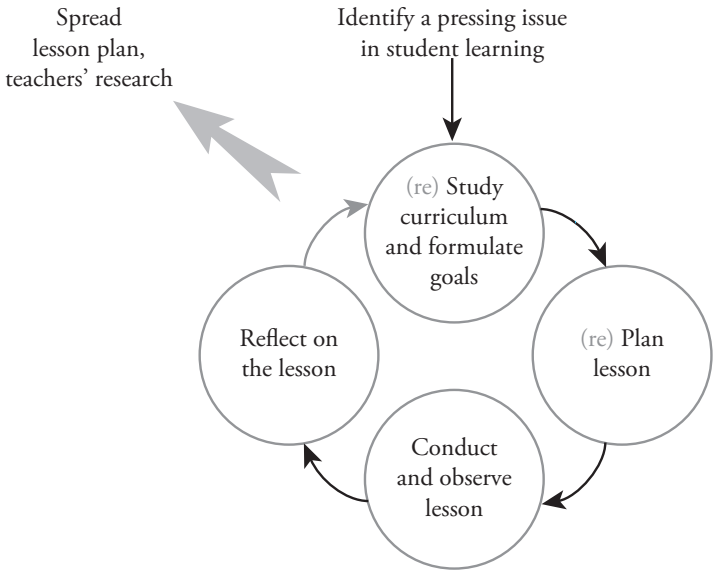

Figure 1. Lesson study cycle, adapted from (Lewis et al., 2006, p. 4)

As we will illustrate, lesson study is at the same time very simple and very complex as stated by Lewis (2002):

> Lesson study is a simple idea. If you want to improve instruction, what could be more obvious than collaborating with fellow teachers to plan, observe, and reflect on lessons? While it may be a simple idea, lesson study is a complex process, supported by collaborative goal-setting, careful data collection on student learning, and protocols that enable productive discussion of difficult issues. (pp. 1–2)

In 3LS, several adaptations have been made of this process. These adaptations include: micro lesson study (Fernandez & Robinson, 2006) for pre-service student teachers (Clerc & Martin, 2011); student teachers' exchange program (Gilles, in press) with Singapore National Institute of Education, Humbolt State University (USA) and HE Galilée (Belgium); art and craft additional training; and more classic professional development groups. Several disciplines are involved (mathematics, science, French language, art and craft, …), but the originality is to combine a transversal "teaching and learning"

approach with a content-specific didactic approach with most groups being coached by one specialist of each approach.

One particular group in 3LS focuses on lesson study in maths (LSM) for primary 3 and 4[3] in-service teachers. The group is composed of eight primary generalist teachers and two facilitators (one in teaching and learning and the other a specialist in maths didactic and author of this paper). The lesson study process occurred over two years with a meeting about every two weeks during the school year. All 37 meetings and 8 research lessons were videotaped and transcribed. Since qualitative analysis of these data is still in progress, detailed results are not yet available. Therefore the data is used here to illustrate our reflections on aspects of lesson study, as outlined above.

Lesson Study as teacher training: development of MKT

Citing many research studies, Murata synthetizes that "lesson study incorporates many characteristics of effective professional development programs identified in prior research: it is site-based, practice-oriented, focused on student learning, collaboration-based, and research-oriented" (2011, p. 2). Each of these aspects is present in our LSM group, and we have observed the effects of each of these features on the teacher training process.

One of the most noticeable effects was the de-personification of teaching linked with the focus on student learning. Following the first research lesson, the post lesson discussion was formulated in a "we did that" instead of "you (the teacher) did that" way. The discussion was about teaching (and the way of teaching the group had planned) and not about the teacher. For each observing member of the group the conducting teacher was his/her *avatar*[4] (Cameron, 2009), who personified the collaboratively planned teaching of the group. This allowed very uninhibited and open discussions during the whole process, and also led to ease of opening of all teachers' classrooms for the next research lessons, despite the presence of colleagues, researchers and video camera. This depersonalisation was most probably encouraged by the collaborative work, but also by the focus on student learning, the preparation, and the observation had. Our interpretation is that the focus on student learning allowed attention to turn away from the teacher and to indirectly focus on teaching.

During the first year of meetings, two cycles of lesson study were completed with the research lesson taught once or revised and taught again. The themes of these cycles were: numeration (value aspect of place value) and geometrical transformations. The discussions in the group were a great occasion for developing Mathematical Knowledge for Teaching (MKT) as categorized by Ball, Thames, and Phelps (2008).

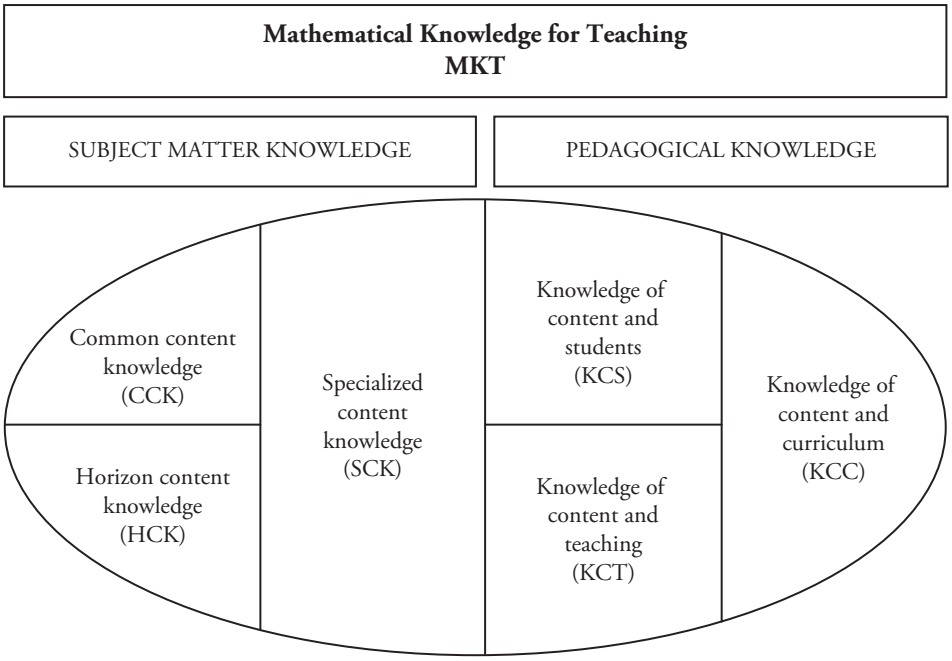

Figure 2. Domains of mathematical knowledge for teaching (Ball et al., 2008, p. 403)

This practice-based division for mathematics of Shulman's (1986, 1987) categories of teacher knowledge provides an effective way for putting the spotlight on some development of MKT during the lesson study process.

Detailed classifications and analyses are in progress, but we can already stress that Common Content Knowledge and Specialized Content Knowledge of these subjects were particularly developed. In addition, Knowledge of Content and Teaching, Knowledge of Curriculum and Horizon Content Knowledge were also enhanced. We will give some illustrations of each while describing the three lesson study cycles of LSM group.

First lesson study cycle: place value

During the first session, the two facilitators presented the lesson study process to the teachers. They then asked the teachers to identify the mathematics topics their students had difficulties with, what topics they had the most difficulties in teaching, and what content they would like to work on during this lesson study process. Three topics emerged: numeration, axial symmetry and problem solving. The two facilitators solicited the group to enter the lesson study process with the topic they felt that would be the most manageable to begin the process: integers and place value.

The starting point for reflection was to characterize students' mistakes with place value in grades where teachers felt students should master the concept. The group analysed

their students' mistakes they had noted in their own classrooms. The facilitators also brought some typical mistakes from a professional development website (Figure 3).

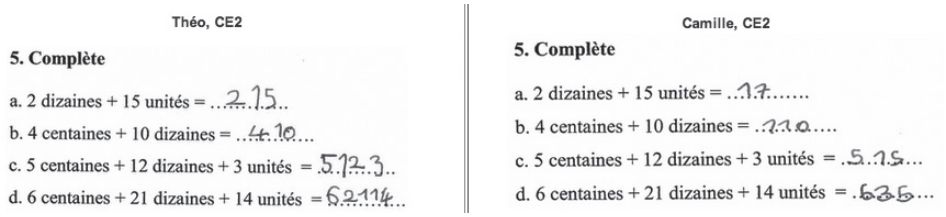

Figure 3. Students errors (Tempier, 2013)

Looking at students' errors and linking them to mathematical knowledge about place value contributed to the development of Knowledge of Content and Student. This website was linked to a doctoral research thesis (Tempier, 2013) and a professional article that allowed the group to reflect more deeply about the distinction between the place component and the value component of our number system (Tempier, 2010). This distinction, facilitated by the external literature, allowed a move from analysing students' error to purely mathematical considerations (Specialized Content Knowledge) and from there to the implications of this distinction for teaching (Knowledge of Content and Teaching). It was also a part of analysis of one of the research lesson that was conducted in one of the teachers' class.

During the post-lesson discussion, we pointed out that many students and also many teachers used a particular reasoning to convert one hundred into tens. For example, explaining to student Amandine why one hundred equals ten tens, teacher Anaïs says[5]:

> Teacher Anaïs: Amandine? One hundred, how many is that? (the teacher shows a square plastic slab composed of 100 small unit squares).
>
> Student Amandine: Hundred.
>
> Anaïs: Hum, hum. Correct. Is that ok? Ten tens, you told me it's hundred. Is that ok?
>
> Student (all together): Yes.

In short, this reasoning would be:

1 hundred is 100 units
and 100 units is 10 tens
so 1 hundred is 10 tens.

This deduction meant that, for many students and also for some teachers, the direct exchange between orders of units was not immediately possible and required understanding of the place value system. The awareness of the links and the differences between place and value was both a development of Specialized Content Knowledge and of Knowledge of Content and Teaching.

From the Horizon Content Knowledge point of view, the future implications of understanding place value for students were also discussed. The importance of this concept was traced into the future of students' learning of long multiplication or of polynomials.

The first research lesson used a task in form of a game from a French textbook (*Un drôle de jeu de l'oie*, in Charnay, Combier, Dussuc, Madier, & Madier, 2007) that was brought by one of the teachers and was new to all members of the group. The task was analysed, used for the research lesson, discussed again, and transformed in many ways for the second research lesson conducted in a second teacher's class. This transformation was the result of both the analyses and observations of the group of teachers. Observation and analysis allowed the group to differentiate students' success in a task from student's understanding and to deeply think about the effects of every detail of a task on students' learning. As the teachers expressed: "the analysis of our research lesson's observations allowed us to check immediately the consequences, positive, negative, useful or useless, of the various interventions and aids by the teacher" (Baetschmann et al., 2015, p. 34). These two observations remained for the rest of the process: the tasks in textbooks are not always "perfect" and succeeding in a task does not necessarily mean understanding the concept. These are two important features of Knowledge of Content and Teaching.

Second lesson study cycle: geometrical transformations

For the second cycle, the group worked on geometrical transformation. The first task was to define more precisely the subject topic. Over one hour the group discussed all the difficulties linked with geometrical transformations. The discussion went in a variety of directions almost without the facilitators' guidance. For the next session the facilitators rearranged the ideas and presented it to the group to distinguish between the different topics and make the links between them. This moment of presentation allowed the group to distinguish between the main topic of a lesson and all the connected mathematical notions. The geometrical transformations had to be distinguished from the properties of the figures, with reproduction tasks, and with measurement, but also linked with these topics. The conclusion of this moment was that it is necessary for a teacher, especially in geometry, to know what the main mathematical notion in a lesson is ("you do not teach everything in one lesson") but also to be aware of the links with the other topics. This Knowledge of Content and Teaching was formulated jointly with Common Content Knowledge about isometrics (what are the isometrics? what are their characteristics? why is axial symmetry "special"?). The teachers welcomed these "reminders" about the properties of isometrics before beginning the planning of the lesson. Other preparatory work before actual planning was the exploration of mandatory textbooks and of the official curriculum (for current grades, but also for previous and following grades) concerning geometrical transformation. This explicit work on Knowledge of Content and Curriculum was one accent of this lesson study cycle.

The focus of the planning and of the research lesson itself was not so much on the task (*Aquarium*, in Danalet, Dumas, Studer, & Villars-Kneubühler, 1999) which had been used previously by most of the teachers, but on the 'pooling moment' where stu-

dents explained and justified their solutions and on the institutionalization practices of the teacher (Brousseau & Warfield, 2014). Articulating what the student should remember after the lesson and the detailed choice of examples in order to fulfil that objective was another main point of this cycle. In particular, teachers discussed which figures to incorporate within the institutionalization, the equilibrium between prototypical examples and more unusual ones. This led to give two carefully chosen examples for every isometry on an institutionalisation sheet to every student (Figure 4).

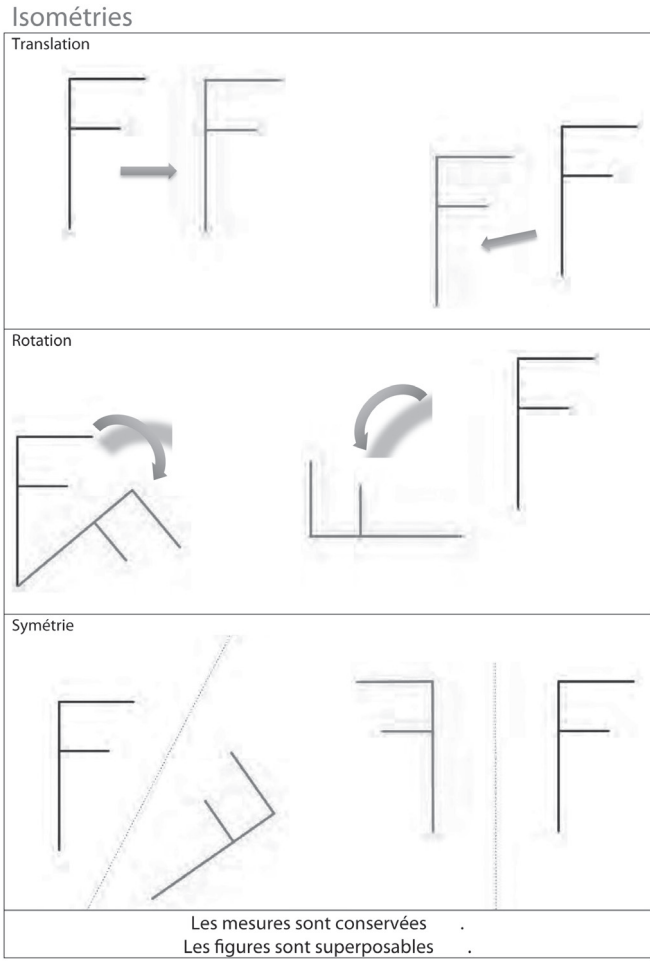

Figure 4. Institutionalisation sheet about isometries

The reflection on the mathematical implications of choosing a particular case to illustrate a general concept was a work on Specialized Content Knowledge with implications on Knowledge of Content and Teaching.

After the research lesson and the subsequent discussion, the group made some modifications to the task and to the lesson, but decided not to teach the lesson a second time for a second research lesson. The argument was not that the lesson was perfect, but rather that the group had already modified a task taught many times and would therefore not sufficiently benefit from it in terms of professional knowledge development. Teachers decided they would benefit more from moving to the third cycle.

Third lesson study cycle: problem solving

Problem solving was the subject that teachers wanted most to explore. But, since it seemed the most complicated one, the two facilitators asked not to do it before year two. It was also the subject that allowed most development of teachers' research perspective within lesson study, as we will explore in section 5. This topic was less focused on a particular subject and allowed mostly to work on the Pedagogical Content Knowledge part of MKT classification. This cycle provided teachers with opportunity to reflect on: the dialectics between teaching through problem solving and teaching for problem solving that are present in textbooks and in the curriculum (Knowledge of Content and Curriculum), about the benefits for the students' learning and attitudes towards mathematics (Knowledge of Content and Student), and how to scaffold students building a representation of the problem without helping them too much (Knowledge of Content and Teaching).

These developments of MKT were linked with the school tasks used in class and were reflected in the released lesson plan, making resource development a significant aspect of LSM group work.

Lesson Study as resource development

Discussion about tasks occupied a significant part of discussions in the LSM group. The starting question "what are the problematic points for your students in learning maths, what are common difficult subjects for you in teaching maths?" led to answers such as "problem solving", "geometrical transformations" or "numeration" as we have seen above. But quickly the discussion turned into naming problematic mathematical tasks. So, the group looked to tasks that were representative of the subject and, at the same time, generally problematic. Within the discussion teachers analysed these tasks, after and before using them in the research lesson, transformed them, evaluated their effect on students' learning and worked on the scaffolding students needed. An evolution of the kinds of interventions teachers made around these tasks can be observed. At the beginning, they were mostly focused on the implementation of the task and on classroom management issues linked with the task. Progressively, these interventions focused more of the mathematical content embedded in the task, on the way to use the task to promote conceptual understanding, and on the discussion if succeeding the task guarantied understanding the concept.

After the lesson study cycle, sometimes with several teachings of the lesson, the LSM group produced a *lesson plan*, which was made available online[6]. This plan documented the layout and conduction of the lesson but also contained comments about the process of designing the lesson and the *a priori* analysis of the task. While the writing of this lesson plan is a powerful motivation for the group to formalize and synthetize their discussions, it is important to note that this is not the goal of the process. In the same way that for students succeeding in solving a school task and communicating the solution is the apparent goal but is in fact just the means for learning maths, the lesson and the lesson plan are only the means within the lesson study process. The awareness of this duality between doing and learning, both for students and for teachers, is a characteristic we observed with LSM teachers (Clivaz, 2015b). As a teacher expressed in one session by the end of year:

> What really changed for me is that I don't just see the task as "do it" anymore. Now, I'm aware that I'm really looking for the stakes. What is there behind the task? […] I'm choosing this task: why? why now?

The work on testing textbooks' problems to see their effect on student's learning, transforming them and spreading the new version also brings lesson study close (but not identical) to design research (Cobb, Confrey, Lehrer, & Schauble, 2003) as pointed by Sack and Vazquez (2011).

Lesson Study as teacher research

During the lesson study process, teachers adopt a research stance. They develop questions about teaching a particular subject, they study existing literature about this subject, they make hypothesis about possible solutions, they design a lesson for testing these hypothesis, they collect data during this research lesson, and they confront their observations with their hypothesis. In our research, LSM teachers said they felt this researchers' attitude develop after the two first lesson study circles about numeration and geometrical transformation. Interestingly, they said they were somehow frustrated to "discover" things they felt the two facilitators already knew. This impression changed during third lesson study circle about problem solving, where the "solutions" for how to teach for and through problem solving where not already available. For the research lesson, the whole group read a research paper about problem solving (Julo, 2002), and tried original approaches suggested in the paper in a research lesson. The reactions of the teachers to the reading of this rather difficult paper were particularly of note since in their initial reading they reported "I don't understand anything" but by the end they remarked that "it's what we are experiencing".

Teacher 1: For me, this kind of things, this kind of text… there is no start (laughter)! Or very, very few… some pictures at the end. It's really off-putting for me. I'm

> really not open to these, these… It's de-natured. I know these are really serious people who write those things, but it's… it's difficult for me to understand, to go into, to make links with my work as a practitioner. This is too difficult. That's it.

[about 5 minutes of discussion between teachers]

> Teacher 2: As for the content of the text, I think it's still interesting, since it's a research. That's true, it doesn't give any solution, but it goes into many details about situations we are really dealing with and, in fact, it allows a reflection for us. A meta-reflection. That's true, it's ultra-theoretical, but it's still what we are experiencing on a day-to-day basis!

This raising of awareness of a link between research and practice is a characteristic of some of teachers' dialogue within the LSM process.

Lesson Study as mathematics education research

Lesson study is gaining interest in the mathematics education community. A sign of this new interest is the book about *Lesson Study Research and Practice in Mathematics Education* (Hart et al., 2011), another one is the presence of two articles concerning lesson and learning studies (Runesson, 2014; Shimizu, 2014) in the *Encyclopedia of Mathematics Education* (Lerman, 2014). This interest is paralleled by a demand of more solid theorization of the lesson study process. Among the frameworks used to conduct and/or to analyse lesson study, some are general education ones (for eg. variation theory (Ling & Marton, 2011), sociocultural theory (Vygotskiĭ & Kozulin, 1934/1997)), some are linked to professional development or interaction between teachers (for eg. improvement science (Langley et al., 2009) used by Lewis (2015), community of practice (Wenger, 1998)) used by Corcoran (2011)), and some of them are subject specific, in our case specific to mathematics education as we will elaborate now.

We see specific interactions between lesson study and mathematics education research mainly in two directions. The first one is for mathematics education research to use lesson study as a structural basis for research. When conducted with a maths education point of view, as in LSM, lesson study is a powerful methodological tool to study math teaching and learning. It can be done through several theoretical frameworks, for example MKT (for eg. Ní Shúilleabháin, 2015), anthropological theory of didactics (Chevallard and Sensevy (2014), used for eg. by Miyakawa and Winsløw (2013)), theory of didactical situations (see Brousseau and Warfield (2014), used for eg. by Miyakawa and Winsløw (2009), or in LSM project), design research (Cobb et al. (2003) used for eg. by Sensevy, Forest, Quilio, and Morales (2013) or Ponte, Quaresma, Baptista, and Mata-Pereira (2014)) or double approach (Vandebrouck (2013) used for eg. by Batteau (2013)). In all these uses, the data generated within lesson study is especially unique since it provides researchers not only to actual lessons but to teachers' reflection and planning practices about one lesson. This use of lesson study as a methodological tool

brings lesson study close to didactical engineering (Artigue, 1994) despite the many differences between these models (Clivaz, 2015a; Miyakawa & Winsløw, 2009). We advocate that lesson study could lead French speaking *didactique des mathématiques* to produce new and more practice-oriented forms of didactical engineering, in the direction pointed by Perrin-Glorian (2011) about "didactical engineering for development and training" also called "second generation didactical engineering" (Clivaz, 2015a, p. 255).

In another direction, didactics of mathematics theoretical framework could be a tool to explicitly examine teachers' principles for lessons, which are often implicit in lesson study, "as regards what aspects of mathematical knowledge are at stake and how different elements in the lesson design could affect students' learning" (Miyakawa & Winsløw, 2009, p. 217).

Lesson Study as a link between education professionals

As stated above, lesson study intertwines research, professional development, and practice aspects but also links didactic and pedagogic aspects. In our 3LS research centre, it allows the interaction between student-teachers, teachers, principals, educational advisers, teacher educators, and researchers. This interlacing is also particularly powerful since every actor in the process keeps their role when working collaboratively on the same central, visible object: a lesson. This observable object enables actors to discuss their practices together at different levels (in Japan, from school based lesson study to district based or even nation based lesson study). It also makes possible international congresses like WALS where teachers, teacher educators, school administrators, and researchers from all over the world gather to discuss lesson studies and also to observe live research lessons in local schools.

Conclusion

As expressed by Lewis (2002), lesson study is a simple idea, but a complex process. In this paper, I have presented some of its complex aspects and wish to highlight the link between research and practice that lesson study allows.

Research is at the centre of the whole process: lesson study "is *research*, a search for a solution to a teaching-learning problem" (Takahashi & McDougal, 2016, p. 7). As I described, this research stance includes the use of research results when studying the teaching-learning problem. Moreover, it includes a kind of clinical research attitude from the participants, making practice be research. In the same way as medical clinical science, "in lesson study, practice *is* research. […] Both medical practice and teaching are clinical sciences, primarily concerned with improving clients' well-being and secondarily concerned with generating knowledge to be applied elsewhere" (Lewis & Hurd, 2011, p. 95).

This clinical attitude emerged during the teacher professional development described in this paper. This attitude also was the base of the task's analysis and development. By

making changes and observing the effects it allowed the development of the awareness of the role of the task's details and of the importance of the teacher's intervention on students' learning.

The professional development and research community that emerged through the lesson study group described here may then become a research subject for us, as mathematics education researchers. Thus, the spreading of lesson study in Europe, particularly in mathematics, brings many opportunities. It also brings many questions: What are the necessary adaptations from the original Japanese model? How can lesson study professional development be sustainable? What is the role of the facilitators? What are the necessary theoretical works that need to be done from a didactics point of view…? These questions, and others, make lesson study topic a particularly interesting and challenging theme for future researches.

Notes

[1] *World Association of Lesson Study* annual congress brings together researchers, school leaders and teachers from various countries around the world to share their experience and practice of lesson studies. See http://www.walsnet.org.

[2] See www.hepl.ch/3LS

[3] 8–10 years-old students.

[4] A reference to the *avatar* in J. Cameron's motion picture by the *20th Century Fox*.

[5] Original dialogues occurred in French. All translations from French are the author's.

[6] http://www.hepl.ch/cms/accueil/formation/unites-enseignement-et-recherche/did-mathematiques-sciences-nat/laboratoire-lausannois-lesson-st/plans-de-lecon.html, retrieved Mai 15th, 2016.

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Abstract. Lesson study is a collaborative model of teacher professional development, which originated in Japan and has spread around the world. The aim of this paper is to present my reflections on some aspects of this multifaceted model. I present lesson study as teacher training, as resource development, as teacher research, as mathematics education research, as a way to connect education professionals and as a link between theory and practice. I illustrate my reflections using lesson study projects conducted in Lausanne Laboratory Lesson Study (3LS), in particular focusing on my work with a math lesson study group for primary 3 and 4 in-service teachers.

Keywords: Lesson study, teacher professional development, mathematical knowledge for teaching.

Résumé. Les lesson study sont un modèle de développement professionnel originaire du Japon et qui s'est répandu dans le reste du monde. Le but de cet article est de présenter nos réflexions à propos de ce modèle aux multiples facettes. Nous présentons les lesson study en tant que formation des enseignants, en tant que développement de ressources, en tant que recherche menée par les enseignants, en tant que recherche en didactique des mathématiques, en tant que manière de connecter les professionnels de l'éducation et en tant que lien entre théorie et pratique. Nous illustrons nos réflexions en utilisant les projets de lesson study menés au Laboratoire Lausannois Lesson Study (3LS), plus particulièrement notre travail mené en mathématiques avec un groupe d'enseignants des degrés primaires 3 et 4.

Mots-clés: Lesson study, développement professionnel des enseignants, connaissances mathématiques pour l'enseignement.

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