Design-Research-Based Curriculum Innovation

Design research como modo de inovação curricular

Koeno Gravemeijer
Eindhoven School of Education e Eindhoven University of Technology, The Netherlands

Introduction

One of the main origins of design research is in innovative curriculum projects. Professionals working on innovative curricula found that the research base for their design activity was rather limited to non-existent as they tried to create innovative curricula. Gradually, they started to realize that their professional instructional design activity carried the germs of a research activity, and started to expand their work in this direction. One of the places where design research emerged was the IOWO in the Netherlands, the predecessor of the later Freudenthal Institute. Over time design research has been elaborated in various ways, while the methodological underpinning was strengthened. As a consequence, however, the activity of doing design research became detached from the activity of curriculum innovation. Still, an important goal of design research is to support or inform curriculum innovation. In this article we discuss today's design research from the original perspective of serving curriculum innovation. We start by discussing the RDD (Research, Development and Dissemination) approach for curriculum innovation that was en vogue in the 1960s and 1970s. It not only formed the background against which design research emerged, it also re-emerged in the form of evidence-based curriculum innovation that could be considered the antipode of design research. Next we will discuss the general characteristics of design research. We will argue that design research aiming at supporting teachers in supporting students in constructing mathematical knowledge has to focus on developing (local) instruction theories. After discussing RME (Realistic Mathematics Education) as a basis for this type of design research, we will move to the problem of upscaling and teacher professionalization. We close by observing that design-research-based curriculum innovation that aims at putting inquiry-based mathematics into practice is complex and demanding, but worth the effort.

RDD, Research Development and Dissemination

In the 1960s and 1970s the RDD model for curriculum innovation was very popular. RDD stood for, Research, Development, and Dissemination. According to this model, research findings would be the basis for the curriculum development, and the products of that activity would be disseminated and implemented in classrooms. The underlying assumption was that teachers would implement the curricula as intended by following the directions of the developers. Fullan and Pomfret (1977) called this a 'fidelity' approach, which they contrasted with the process that actually took place, for which they coined the term, 'mutual adaptation'. They found that not only the teachers' behavior changed under influence of the new curriculum, but the curriculum was also changed by the teachers who put it into practice. The problematic character of the fidelity approach may be illuminated by the following anecdote in the field of agriculture.

> A firm had produced a new fertilizer that was tested in randomized experimental trials, and proved effective. However, when the fertilizer was brought on the marked, it quickly showed that the farmers were not satisfied. They claimed that the yield was worse than before. So the firm ran a new round of lab experiments, which again proved the efficacy of the fertilizer. Eventually the scientists decided to talk with the farmers. Then they learned that farmers do not read manuals. The fertilizers should have been worked into the ground, but the farmers did not do that. As a solution, the company decided to put the fertilizer in a liquid form, and diluted in water. However, this was no success. The farmers still complained that it did not work properly. Again people of the firm talked with the farmers. They found that the source of the problem was an uneven distribution, as the tractors did not drive with a constant speed.

The message of the story is that it is very hard to package an innovation in such a way that the implementation will correspond with the intentions of the producers. After the publication of Fullan and Pomfret (1977) we saw a growing acknowledgement of the limitations of the fidelity perspective. It lost its popularity and the interest of the education research community shifted towards the problems of curriculum implementation. Nowadays, however, the RDD model is back in the guise of 'evidence-based' curriculum innovation. Current wisdom says that teachers and policymakers should only implement curriculum innovations that are evidence-based (Slavin, 2002). This evidence should be 'hard evidence' obtained by randomized controlled trails (RCT's). Medicine is often pointed to as exemplary for how educational innovation should be carried out. New medication is only introduced when it has proven its efficacy in RCT's, and the same approach should be followed in education. In the mean time, however, resistance has arisen against the evidence-based approach in medicine. Especially, because pharmaceutical industries abuse the idea by manipulating the (interpretation of) research data (Healy, 2012). Another big problem, however, is problem that what is a proven effective approach in general, does not have to be the best option for an individual patient. Healy (2012) reports the following. The American Food and Drug Agency (FDA) obtained the data of all placebo-controlled trials from the companies making antidepressants. This involved 100,000 patients. They ran statistical tests and it showed that the drug did indeed work significantly better. When looking in more detail, however, it showed that while 5 out of 10 patients recovered when taking the drug, 4 out of 10 recovered without the drug. Which means that for 4 out of 10 it might be better to wait, instead of immediately start taking the drug. We may argue that this finding illustrates a more general limitation of RCT's and decision making on the basis of statistical significance-levels. The results of RCT's constitute in fact generalizations over many individual cases, for which the effects may have varied in many ways. Likewise, what works best according to the RCT might not be the same as what would be the best for a given classroom.

In line with the above, (Gravemeijer & Kirchner, 2008) expressed the following, general critique on, randomized controlled trials as being:

– *not feasible*. Organizing double blind trials, with randomized participants, comparable groups, and large numbers of students and teachers is not feasible in educational research.

– *too costly*. According to the Scientific American of November 20, 2014, the cost to develop a new pharmaceutical drug now exceeds $2.5 billion, in education research we can only dream of such amounts of money.

– *too generalized*. The problem, as we already noticed above, is that research on groups is used to produce guidelines for individual cases; moreover, the controlled trial requires an unwanted homogenization of treatments.

– *too slow*. It will take many years to prepare for, organize, and carry out randomized controlled trial in education, which hampers a speedy reaction to an educational problem.

The alternative is to not just focus on 'what works, but also on when, for whom, and how'. Thus, instead of trying to find the silver built that works for everybody everywhere, it would be better to try to take into account the complexity and the messiness of everyday classrooms. For instance, by developing theories that can be – and are meant to be – adapted by teachers to the circumstances of the own classrooms.

Design research

This is exactly what design research tries to do. In relation to this we may refer to the characteristics of design research that Cobb, Confrey, diSessa, Lehrer, and Schauble (2003) discern. Distinctive for design research is the dual goal of developing theory and designing educational interventions. Where the theories come first as the interventions are carried out in service of creating the conditions for developing theories about innovative instruction. The theories developed during a design experiment are humble in that they are concerned with learning processes with a limited scope, such as local instruction theories (Gravemeijer, 2004). Local instruction theories encompass theories about a possible learning process and the means of supporting that learning process. Here the term ‘local’ has to be thought of in comparison with ‘domain-specific’ character of a domain-specific instruction theory. A local instruction theory may, for instance, concern a theory for teaching addition and subtraction of fractions, long division, or percents. In line with the intention of informing practice, the theories developed in design research are to, ‘do real work’, as Cobb et al. (2003, p.10) put it, in practical educational contexts. Another characteristic is the innovative character of design research. The objective is to create new learning ecologies in order to study them. In contrast to many other forms of educational research, one does not investigate a given intervention or treatment that should not be changed during the experiment. Instead, design experiments are iterative in character. During a teaching experiment, conjectures are generated and perhaps refuted, and new conjectures are developed and tried out. This results in an iterative and cumulative process in which we may discern micro cycles consisting of anticipatory thought experiments, followed by instruction experiments, which instigate reflection leading to new thought experiments and so forth. Of course teaching experiments may be iterated also, either to try them in a different setting or to incorporate some new ideas. The iterative process of designing and trying out implies that while design experiments generate theories, it also puts these theories to the test. As a clarification we may add that putting theories to the test, has quite a different meaning than testing theories in (quasi-) experimental research. The aim is not to either accept or reject grand theories, but to revise, refine or improve humble theories.

Supporting students constructing

We should clarify that the kind of design research Cobb et al. (2003) characterize, and to which we subscribe to, is one of many interpretations that is around. In this article, however, we will limit ourselves to this one type of design research, also denoted ‘design research with a focus on learning processes’ (Prediger, Gravemeijer & Confrey, 2015). A key element here is the constructivist orientation of the researchers. The premise that the objective is to support students in constructing their own knowledge has significant consequences for teaching, and in extension thereof, for instructional design. Instead of transferring knowledge to students, teachers will have to support students in constructing, reorganizing, and building on, their own knowledge. This has implications for how teachers can be supported. They will have little use for scripted lessons. They will need support in coming to understand the students’ thinking and in deciding on how to proactively support their students. Hence the rationale underlying the instructional sequence becomes really important to them, which is what is aimed for with a local instruction theory. This fits with the focus on developing theory in the type of design research we elaborate here. Theory that has to do real work in practical educational contexts as we just observed. Before diving deeper into the method of design research, we will give an example of a local instruction theory. We will briefly sketch a local instruction theory for addition and subtraction (Gravemeijer, Cobb, Bowers & Whitenack, 2000), in doing so we will build on an exposition in Gravemeijer (2008).

A local instruction theory for addition and subtraction up to 20

The objective of the local instruction theory for addition and subtraction up to 20 is to support students in developing a framework of number relations that can function as basis for flexible mental computation. What is aimed for is that the students will be able to flexibly solve tasks such as '7+6' by using numerical relationships such as

'7+3=10, 6=3+3, and 10+3=13', or

'6+6=12, 6+1=7, and 12+1=13', or

'7+7=14, 6+1=7 and 14-1=13'.

Mark that as mathematics educators, *we* might perceive these solutions, as applying strategies – such as 'filling up ten' – for the students this is not the case, when they are combining number relations that are ready-to-hand for them.

The envisioned learning process consists of two phases. In the first phase, the students are to develop very basic number relations, and in the second phase, they are expected use these number relations as a basis for deriving new number facts. The first phase involves activities in which the students structure and compose quantities in a wide variety of situations, using counting as a means of establishing quantitative relations. In this sense there is a close relation between cardinal and ordinal numbers; sums and differences are mostly defined cardinally, but are calculated ordinarily (Freudenthal, 1983). Counting strategies, such as counting on and counting back, rely on integrating the cardinal aspect of number (quantity) and the ordinal aspect of number (position/rank). From this it is concluded that it is important that the students connect the first and the latter. An important issue here is that the number relations that the students construct signify curtailments of the procedure for quantifying sets of objects by counting individual objects. In this manner, visual patterns, such as finger patterns, come to embody the results of counting, to use Steffe, Cobb, and von Glasersfeld's (1988) terms. In other words, the students have to construe the procedures for establishing sums or differences – counting on, and counting back – as extensions of the counting procedure that they use for quantifying sets of objects. The importance hereof is shown by research of Gray and Tall (1994), who observe that students, who have come to see the first and the latter as two *unrelated* procedures, do *not* use 'derived facts' strategies.

Instructional activities in the first phase will be so designed that they foster procedures that make use of 'doubles', 'five' or 'ten' as reference points. Working with finger patterns, for instance, students may be asked to show 'eight' in different ways, and the teacher may draw the attention to the number relations involved, such as, '8=5+3', '8=4+4', or '8=10-

2’. In the second phase the so-called *arithmetic rack* (Treffers, 1991) will be introduced as a means of support. The arithmetic rack is designed to support reasoning with number relations in which five, ten, and doubles function as points of reference. The rack consists of two parallel rods each containing ten beads. As shown in Figure 1, the first five beads on the left of each rod are red (dark), and the second five beads are white.

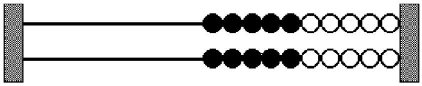

Figure 1. Arithmetic rack.

Students use the rack by moving all beads to the right and then creating various configurations by sliding beads to the left. For example, if a student wants to show eight, he or she may move five beads on the top rod and three on the bottom, or he/she may move four beads on each rod. These ways of acting with the arithmetic rack may facilitate the use of the relations that come to the fore in the first phase. Mark that the intent is not that students will use the arithmetic rack configurations to read off number relations. Instead, students are expected to use the arithmetic rack as a means of scaffolding. Thus when the arithmetic rack is introduced, students already have to have developed five-, ten-, and doubles-referenced number relations.

> For instance, to find the sum of 6 and 7, the students may then use their knowledge that 6=5+1 and 7=5+2 to visualize 6 and 7 on the rack as 5 red and 1 white on the top rod and 5 red and 2 white on the bottom rod. And subsequently realizing that the two fives together make ten, reason, that 6+7 = 5+1+5+2 = 10+3 = 13. Or they may realize that 7=6+1, and 6+6=12, and relate this to of 6 plus 1 on the top rod and 6 at the bottom and reason 7+6 = 12+1 = 13, while pointing to the rack.

Working with the rack, however, is not introduced as as a self-contained activity. Instead, the rack is introduced as a means of keeping track of quantitative changes in some experientially real situation.

> Here one may use a scenario that involves a double-decker bus, which is developed by Van den Brink (1989). Initial tasks in this scenario concern different ways in which a given number of passengers could sit on the two decks of a double-decker bus. Follow-up activities involve situations in which some passengers get on and others get off the bus. The arithmetic rack can be introduced as a means of showing the number of passengers on each deck, and as means of keeping track of the number of people getting on or off the bus.

In this way, the rack can initially function as a *model of an experientially real situation;* (the changes in) the number of passengers on the two decks. In this manner, the students may use the rack as a means of scaffolding when reasoning with basic number relations. Subsequently the students will be asked to develop ways of notating their reasoning with the rack so that they can communicate it to others. Subsequent activities involve developing and negotiating symbolizations, the key criterion being that the other students can understand how the task had been solved. For example, the ways of reasoning with rack about 6+7 might be symbolized as shown in figure 2.

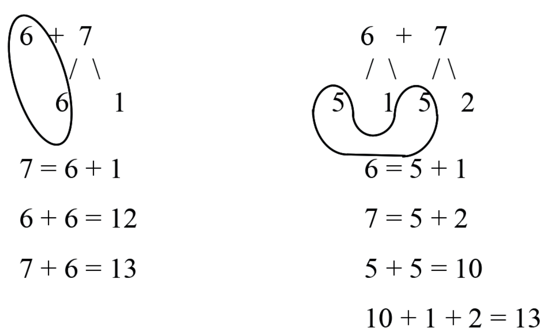

Figure 2. Ways of visualizing arithmetic rack solution strategies.

Then, the drawings of the way of reasoning with the rack, will start to function as *models for more formal reasoning*. Eventually the students will start using number sentences without any auxiliary drawings, while reasoning with numbers as mathematical objects that derive their meaning from a network of number relations.

Design research methodology

Developing this kind of local instruction theories is one of the main goals of the type of design research that is center in this article. To characterize this kind of design research, we will briefly describe *the structure of corresponding a design experiment*; we refer to Gravemeijer (2015) for a more detailed elaboration. In regard to this type of design research we may discern three phases, which concern the preparation for the experiment, the actual teaching experiment, and the retrospective analysis. In the preparation phase the researchers identify the starting points and the potential end points of the instructional sequence. They further develop a preliminary local instruction theory and design a draft of the corresponding instructional sequence In relation to this the researchers are expected to explicate their background theories and elaborate the theoretical intent of the design experiment. The second phase concerns the enactment of the actual teaching experiment, during which the researchers design and adjust instructional activities on the basis of the evolving local instruction theory, and assemble data that allow for the systematic analysis of the learning processes of the students and the means by which that learning is generated and supported. This process of designing instructional activities, trying them out and designing or redesigning follow-up activities on the basis of what happened in the classroom creates an empirical basis for the emerging local instruction theory. This is elaborated further in the third phase of the design experiment, the retrospective analysis. Here the researcher analyses the complete data set in order to found out what happened and why this happened; and to check whether conjectures, about what happened and why this happened, consistently represent the data. In part, those conjectures are already formed during the teaching experiment while trying and adjusting the emerging local instruction theory. In this respect, the retrospective analysis aims at improving the empirical grounding of the theory and the alignment of the theory with the available data. The retrospective analysis may further involve overarching issues such as the role of the teacher, the classroom culture, or the role of symbols and tools. Investigating those overarching issues may be part of the theoretical intent that is formulated at the start of the design experiment, but they may also come up during the teaching experiment or in the retrospective analysis.

Design research that aims at developing a local instruction theory for a given topic, will also produce an instructional sequence, which may be considered a *prototypical instructional sequence*. The assumption, however, is not that this instructional sequence can unproblematically be transferred to other classrooms. Whereas the RDD model is based on the premises that similar practices and patterns of learning will emerge when an instructional sequence is enacted in other classrooms. The idea behind design research is to cater for the fact that this usually is not the case – as research has shown repeatedly. Here one may build on the notion of ecological validity. Ecological validity concerns the similarity of the experimental setting and the situation where the findings are used. Instead of aiming at similar conditions, however, one can also elucidate how the learning ecology evolved while the instructional sequence was enacted. In this manner one may enable others to consider how their circumstances differ, and what this might mean for enacting the instructional sequence in their classroom. In relation to this one also speaks of a 'thick description'.

We may further point out that instead of considering the teaching experiment as an example that has to be followed literally, a design experiment may be thought of as a paradigm case; what happened in the experimental classroom is considered paradigmatic for processes that might happen in other classrooms. Thus the theoretical analysis developed when coming to understand this one case may considered to be relevant when interpreting other cases. In other words, the understanding of (the role of) the specific characteristics of the investigated learning ecology may help others in understanding the same phenomena in other settings.

Curriculum Innovation

In the following sections, we will elaborate upon this idea of using local instruction theories as a basis for curriculum innovation. We will take RME theory as an exemplary basis for designing local instruction theories that aim at supporting students in constructing mathematics. In relation to this we will discuss the classroom culture and the teacher knowledge, which are required, and discuss how these requirements can be met by teacher professionalization and teacher support.

Realistic Mathematics Education

The kind of design research discussed above, aims at developing local instruction theories that can function as frameworks of reference for teachers designing hypothetical learning trajectories for their classrooms. Simon (1995) coined the term hypothetical learning trajectory (HLT), when considering how a 'constructivist teacher' might guide the students' thinking in an indirect manner – as the premise is that the students construct their own knowledge. A hypothetical learning trajectory (HLT), involves, anticipating the mental activities of the students when they engage in envisioned instructional activities, and considering whether those mental activities will contribute to advancing towards the intended learning goals. The hypothetical character of the learning trajectory entails that the teacher has to try to infer whether the students perform the anticipated mental activities when the instructional activities are carried out in the classroom. Based on this assessment the teacher has to reconsider and potentially adapt the instructional activities and the HLT for the next day. Mark that Simon (1995) originally introduced the notion of HLT in the context of planning lessons for one or two days. Later others have expanded this to encompass complete instructional sequences. In this article we will stick to the original time frame for the HLT, and use local instruction theory in relation to instructional sequences. This allows us to discuss the relation between the local instruction theories that are the product of design research and the HLT's that the teachers design on a day-to-day basis for their classrooms. The line of reasoning underlying the idea of local instruction theories is that teachers will be able to use local instruction theories as a framework of reference when designing hypothetical learning trajectories. It will be clear that this places high demands on the local instruction theories. It is very helpful therefore to have an instructional design theory at one's disposal that is tailored to this type of instruction. The domain-specific instruction theory of realistic mathematics education, RME, which we will focus on in this article, offers such a theory. RME theory was first articulated by Treffers (1987). Later RME theory was formulated in terms of three instructional design heuristics: guided reinvention, didactical phenomenology and emergent modeling (Gravemeijer, 2004).

In respect to *guided reinvention*, Freudenthal (1973) suggested that the instructional designer would design a route along which students could reinvent the intended mathematics. Here the starting points should be experientially real for the students, that is, the students should be able to act and reason sensibly in those situations. Similarly, the end points should be experientially real as well. In relation to this Freudenthal (1991) speaks of growing 'common sense' to describe the envisioned process of mathematical learning. He argues that one could use the history of mathematics as a source of inspiration. Alternatively, one might look at the informal solution procedures of students as they might carry the germs of the more formal mathematics. The *didactical phenomenology* may briefly be described as asking the designer to look at the phenomena that are organized by a certain mathematical concept or tool, and to analyze how this concept, tool, or more general, 'thought thing', organizes those phenomena. This then will inform the instructional designer about what phenomena the students should be asked to organize, in order to be put in the position to invent the organizing tool/thought thing. As part of the analysis the various phenomena that can be organized are inventoried, and the designer is advised not to limit the organizing tasks to one phenomenon, but to use all of them, thus providing for a broad phenomenological basis, which later on may facilitate applications. The *emergent modeling* design heuristic is modeled to the historical process in which mathematical symbolizations and meanings evolved together in a reflexive process. In this process, working with certain symbolizations fostered the development of new meaning, which in turn created the need for new symbolizations etc. As a result of this process, formal mathematical symbolizations and meanings emerged, which are too abstract for most students. To make these abstract symbolizations and meanings accessible to students, the emergent modeling design heuristic aims at supporting students in going through a process similar to the dialectic process in which symbolizations and meanings have been reinvented. This process can be characterized by models, which develop from a *model of informal mathematical activity* into a *model for more formal mathematical reasoning*. First, the models refer to contextual situations that are experientially real for the students. The models support the informal strategies that can be used in the situation of the contextual problem. Next related problems will be solved and the attention will be focused on the mathematical relations and strategies that are used. In this manner the teacher supports the students in constructing a network of mathematical relations. The result of this is that the students come to see the model in a different manner, as it starts signifying mathematical relations instead of contextual situations. In this manner the model may become a basis for more formal mathematical reasoning. Mark that the construction of the network of mathematical relations and the construction mathematical objects that derive their meaning from this network is key in this process. In this respect we may speak of the construction of some new mathematical reality. We may further refer to the Van Hiele levels, to which both Freudenthal (1973) and Treffers (1987) refer.

Up-scaling

Given the fact that design research has its basis in teaching experiments in actual classrooms, we may presume that it has the potential to bridge the gap between theory and practice. We should not, however, underestimate the difficulty of making the transition from an experimental classroom to a multitude of ordinary classrooms. In relation to this we may refer to an observation of Dewey (Dewey, 1901, cited in Brown, 1992). He gave the following sketch of how 'up-scaling' plays out in practice. The whole process starts with reports about problems in schools. Then researchers start to work on this problem, typically collaborating with one or two teachers. After a lot of hard work they are eventually successful, and they get appreciation in academia. Next, the new approach is implemented widely. However, pretty soon reports appear about serious problems. And the circle starts anew. The problem, Dewey (ibid) argues, is that the new teachers have not had the benefit of working on the innovation with the researchers. And as a result, the innovation is not enacted as intended. Mark that Dewey made this observation in 1901, however, since then little seem to have changed.

We may argue with Cobb and Jackson (2015) that this discrepancy between experimental and everyday classrooms is especially problematic in mathematics education today. They point out that the assumptions that guide current design research in mathematics education are at odds with typical forms of mathematics instruction in many countries. They list a series of assumptions of the former that do not fit the latter. These concern the students' learning goals, the role of the students, the way students learn, and the role of the teacher. Student *learning goals* that fit 'inquiry mathematics', but are not common in ordinary schools, concern the goal of students acquiring conceptual understanding of central mathematical ideas, and the goal of making connections among multiple representations. The *students' role* involves the expectation that students will be communicating and justifying mathematical ideas. The *students' learning* is construed as an active constructive process. The *teacher's role* is one of introducing cognitively challenging tasks, monitoring solution strategies, and orchestrating productive whole-class discussions. They conclude that there is a need for substantial teacher learning. For many teachers will have to reorganize their instructional practices, and to change their views on how students learn mathematics and on what students' learning goals should entail.

Supporting teachers

A manner in which the learning demands of teachers who want to shift from conventional teaching to teaching in line with RME theory, can be addressed is described by Stephan, Underwood-Gregg and Yackel (2014). They link RME to Cobb and Yackel's (1996) emergent framework. Yackel and Cobb (1996) found that enacting RME type instruction requires a classroom culture that differs significantly from that in common classrooms. They developed an interpretative framework, denoted 'emergent perspective', in which they try to coordinate a social and a psychological perspective. From the social perspective, they discern social norms, socio-mathematical norms and mathematical practices. These are correlated with individual beliefs that are situated in the psychological domain. In traditional classrooms the norm is that the teacher and the textbook know how mathematical problems should be solved, and the students are expected to adapt their behavior to this basic principle. They have to try to copy and apply the procedures that are shown to them. What is expected from students in an RME classroom is quite different. They are expected to invent solutions themselves, and to explain and defend those solutions in classroom discussions.

In relation to this Yackel and Cobb (1996) speak of classroom social norms, which encompass the mutual expectations and obligations of teacher and students. Crucial here is that the students' beliefs, about what is expected from them in a regular mathematics classroom, are shaped by years of experience. As a consequence they will not easily sift to another set of beliefs or classroom norms. Moreover, teachers also have to adjust their thinking and behavior. Instead of explaining everything to the students, they are now expected to pose tasks and ask questions that may foster the students' mathematical understanding – either by revising, or building on, their current thinking.

In addition to the social norms, which define the roles of the teacher and the students in a general way, they also have to develop socio-mathematical norms; beliefs about what mathematics is. In RME classrooms those beliefs have to be consistent with RME. This is where Yackel and Cobb (1996) introduce socio-mathematical norms, which basically refer to, what mathematics is, and what it means to do mathematics in this classroom. Here we may think of issues such as, what is a mathematical problem, what counts as a mathematical argument or a mathematical solution, and what is a more sophisticated solution. The latter is especially important if we want to grant the students agency in the reinvention process. The last category Yackel and Cobb (1996) put forward is that of the mathematical practices. These refer to taken-as-shared ways of reasoning, and symbolizing. Over time the forms of reasoning and the symbolizations used may change. For instance, because certain inferences do not have to be justified anymore, or because more advanced symbolizations are adopted as replacement for earlier symbolizations. In other words, the mathematical practices change when the learning process of the classroom advances. Note that this does not mean that every student will have appropriated the level of thinking or symbolizing that corresponds with the established practiced. The taken-as-shared character of the mathematical practices implies that there will be variation in the students' beliefs. Just as there will be variation in beliefs in respect to the actual classroom norms.

Not surprisingly, establishing and cultivating RME-specific classroom norms are a significant element in the practices 'guided reinvention teachers' have to adopt according to Stephan et al. (2014). For those norms constitute a prerequisite condition for enacting RME-type instruction.

They discern the following classroom teaching practices:

1. initiating and sustaining social norms
2. supporting the development of socio-mathematical norms
3. capitalizing on students' imagery to create inscriptions and notations
4. developing small groups as communities of learners
5. facilitating genuine mathematical discourse.

The first two teaching practices more or less speak for themselves. We may add that when discussing the second teaching practice, Stephan et al. (ibid) point to the criterion that the students' explanations are to describe actions on objects that are experientially real to them. Mark that this reflects the criterion of experientially real starting points, and the notion that the goal is to support students in developing networks of mathematical relations and mathematical objects that derive their meaning from those networks (see also Gravemeijer, Bruin-Muurling, Kraemer, & van Stiphout, 2016). They add that the explanation has to be understandable for the other students in the classroom. Otherwise the collective learning process would break down. The third teaching practice connects with the emergent modeling design heuristic. The teacher has to play a proactive role in creating inscriptions that can be used as tools, either by adopting and adapting inscriptions students invent, or by introducing inscriptions (see also Rasmussen & Marongelle, 2006). Here the teacher may present a problem creating the need for a new inscription annex tool and ask the students for solutions before choosing or introducing an inscription annex tool of which they can judge the viability as a solution to the problem (Gravemeijer, Bowers, & Stephan, 2003). In relation to the forth teaching practice of developing small groups as communities of learners, it is noted that the classroom norms also apply to the groups. In addition, the groups will be required to come up with solutions that are shared and understood by all group members. Stephan et al (2014) emphasize that the fifth teaching practice, facilitating genuine mathematical discourse, is the most demanding. They list the following methods to facilitate genuine and meaningful discourse about mathematics:

> (1) introducing mathematical vocabulary and tools to record students' inventions, (2) asking questions that promote students' strategies, (3) restating students' solutions in clearer or more advanced ways, and (4) using students' strategies during exploration time to orchestrate an effective whole-class discussion (Stephan et al., 2014, 44).

The authors stress the importance of a good understanding of the learning trajectory and the mathematics. In relation to this, we may refer to research in the Netherlands that showed that teachers and textbooks break off instructional sequences before more advanced conceptual mathematical goals are reached (Gravemeijer et al., 2016). Gravemeijer et al. (2016) relate this to the fact that such goals are not part of the mandated curriculum goals. Another explanation they offer is in what they call, 'task propensity', which they define as: 'The tendency to think of instruction in terms of individual tasks that have to be mastered by students' (ibid, p. 26). A consequence of this task propensity is that teachers and textbook authors focus on procedures that can quickly generate correct answers, instead of supporting students in coming to understand the underlying mathematics.

Next to the teaching practices, Stephan et al. (2014) discern planning practices, which encompass preparation, anticipation, reflection, assessment, and revision. We may note that these practices resonate with Simon's (1995) idea of a mathematical teaching cycle and the corresponding notion of a hypothetical learning trajectory (HLT). As we may recall, the line of reasoning underlying the idea of local instruction theories is that teachers will be able to use local instruction theories as a framework of reference when designing hypothetical learning trajectories. This implies that the teacher has to come to grips with the local instruction theory under consideration, and with developing HLT's on the basis of local instruction theories. Here we may note that the background theory of socio-constructivism and the domain-specific instruction theory, RME, are integral parts of the justifications of local instruction theories. Local instruction theories developed in design research are not just theories on how to teach given topics. The local instruction theories are theories about how to teach those topics within the framework of a given philosophy of mathematics education. An important aspect will therefore be to understand how the local instruction theory, and the HLT for that matter, fit this broader philosophy.

So we are back at linking theory and practice. Stephan et al. (2014) report and elaborate on a long-term process of coaching in which the expert teacher and the novice teacher discuss classroom episodes and background ideas. We may envision a type of teacher learning that Oonk (Oonk, Verloop & Gravemeijer, 2015) denotes as a process of 'enriching practical knowledge with theory' in the context of pre-service teacher education:

> Through reasoning in discourse, STs [teacher students] can gradually learn to generalize and objectify concepts, ideas, and beliefs about teaching mathematics, not only because they are meaningfully rooted in the narratives but also because the narratives enable one to recall the meanings of subjects if necessary (Oonk et al., 2015, p. 563).

We may add that such a process can also be part of in-service teacher education. In this respect we may refer to a classroom design research project that Stephan (2015) conducted with a team that mainly consisted of classroom teachers. In physics education, a similar approach was followed by Kock, who involved a group of teachers in the design and enactment of a course on electricity (Kock, Taconis, Bolhuis, & Gravemeijer, in press). More generally we may refer to the Japanese lesson-study model that according to Hiebert and Stigler (2000) proved more effective, than top-down research-based innovations in the US. In this respect, we may argue that instructional designers, and design researches have to broaden their scope to instructional design in order to facilitate, lesson-study type, study groups of teachers.

Conclusion

We started by discussion the problematic character of the RDD model for curriculum innovation known from the 1960s and 1970s, and its successor, the evidence-based approach'. We discussed how design research emerged as a means to fill the need for supporting the design of innovative curricula. We elaborated upon the characteristics of design research, which we narrowed down to design research in service of the learning process (Prediger et al., 2015), which tries to meet the requirements of supporting students in constructing mathematical knowledge. We presented the RME instructional design heuristics as a basis for this kind of design research. And we followed Stephan et al. (2014) to elucidate what this means in terms of learning demands for teachers. These, we finally argued can be met by coaching or by creating communities of learning in which teachers who collaboratively work on their professionalization – by using the lesson study model for instance. In reflection we may note that design-research-based curriculum innovation aiming at inquiry mathematics, such as exemplified by RME, is complex and demanding. We claim, however, that it is worth the effort.

If one subscribes to the constructivist premises that students do construct their own knowledge, we may observe with Cobb (1994) that the issue is not that we should not interfere with the students' constructions. For, if students do construct their own knowledge, they will do so independent of the pedagogy of the teacher. The real issue is: What do we want students to construct? Or to be more precise: What do we want mathematics to be for the students? If answer is real, meaningful, mathematics, some form of inquiry mathematics seems to be the designated approach. And RME that builds on Freudenthal's (1973) adagio of mathematics as a human activity, offers an adequate basis for this. We may further add that such an approach would also be in line with the obligation of preparing students for the 21st century, which requires mathematics education that emphasizes problem solving, communicating and conceptual understanding.

References

Brown, A. L. (1992). Design experiments: theoretical and methodological challenges in creating complex interventions in classroom settings. *The Journal of the Learning Sciences*, *2*(2), 141-178.

Cobb, P. (1994). Constructivism in mathematics and science education. *Educational Researcher*, *23*(7), 4-4.

Cobb, P., & Jackson, K. (2015). Supporting teachers' use of research-based instructional sequences. *ZDM*, *47*(6), 1027-1038.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational researcher*, *32*(1), 9-13.

Dewey, J. (1901, June). The situation as regards the course of study. In *Journal of Proceedings and Addresses of the Fourteenth Annual Meeting of the National Education Association* (Vol. 39, pp. 332-348).

Fullan, M., & Pomfret, A. (1977). Research on curriculum and instruction implementation. *Review of educational research*, *47*(2), 335-397.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht, the Netherlands: Reidel.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Dordrecht, the Netherlands: Reidel.

Freudenthal (1991). *Revisiting Mathematics Education*. Dordrecht, the Netherlands: Kluwer.

Gravemeijer, K. P. E. (2004). Local instruction theories as a means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning, 6*(2), 105-128.

Gravemeijer, K. (2008). RME Theory and Mathematics Teacher Education. In D. Tirosh & T. Wood

(Eds.), *International handbook of mathematics teacher education: knowledge and beliefs in mathematics teaching and teaching development* (Vol.1, pp. 283-302). Rotterdam, the Netherlands: Sense Publishers.

Gravemeijer, K. (2015). Design Research as a Research Method in Education. In A. Pereira, et al. (Eds.), *Entre a teoria e os dados (III): investigar práticas em contexto* (pp. 5-20). Setúbal: Escola Superior de Educação, Instituto Politécnico de Setúbal.

Gravemeijer, K.P.E., & Kirschner, P.A. (2008). Naar meer evidence-based onderwijs? [Towards more evidence-based education?] *Pedagogische Studiën*, 84, 563-571.

Gravemeijer, K., Bowers, J., & Stephan, M. (2003). A hypothetical learning trajectory on measurement and flexible arithmetic. *Journal for Research in Mathematics Education. Monograph*, *12*, 51-66.

Gravemeijer, K., Cobb, P., Bowers, J., & Whitenack, J. (2000). Symbolizing, modelling, and instructional design. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Communicating and symbolizing in mathematics: perspectives on discourse, tools, and instructional design* (pp. 225–273). Mahwah, NJ: Lawrence Erlbaum Associates.

Gravemeijer, K., Bruin-Muurling, G., Kraemer, J. M., & van Stiphout, I. (2016). Shortcomings of Mathematics Education Reform in the Netherlands: a Paradigm Case?. *Mathematical Thinking and Learning*, *18*(1), 25-44.

Healy, D. (2012). *Pharmageddon.* Berkely (CA): University of California Press.

Hiebert, J., & Stigler, J. W. (2000). A proposal for improving classroom teaching: lessons from the TIMSS video study. *The Elementary School Journal*, 3-20.

Kock, Z.-J., Taconis, R., Bolhuis, S., & Gravemeijer, K. (in press). Supporting teachers to transform their classes into a context-based learning environment: Inquiry as a context. In R. Taconis & P. den Brok (Eds.), *Teachers creating context-based learning environments in science.* Rotterdam, The Netherlands: Sense.

Oonk, W., Verloop, N., & Gravemeijer, K. P. (2015). Enriching practical knowledge: Exploring student teachers' competence in integrating theory and practice of mathematics teaching. *Journal for Research in Mathematics Education*, *46*(5), 559-598.

Prediger, S., Gravemeijer, K., & Confrey, J. (2015). Design research with a focus on learning processes: an overview on achievements and challenges. *ZDM*, *47*(6), 877-891.

Rasmussen, C., & Marongelle, K. (2006). Pedagogical content tools: Integrating student reasoning and mathematics in instruction. *Journal for Research in Mathematics Education. 37* (5), 388-420.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, *26*(2), 114-145.

Slavin, R. E. (2002). Evidence-based education policies: Transforming educational practice and research. *Educational researcher*, *31*(7), 15-21.

Steffe, L. P., Cobb, P., & von Glasersfeld, E. (1988). *Construction of arithmetical meanings and strategies*. New York, NY: Springer-Verlag.

Stephan, M. L. (2015). Conducting classroom design research with teachers. *ZDM*, *47*(6), 905-917.

Stephan, M., Underwood-Gregg, D., & Yackel, E. (2014). Guided Reinvention: What is it and how do teachers learn this teaching approach?. In Y. Li, E. Silver & S. Li (Eds), *Transforming Mathematics Instruction* (pp. 37-57). Dordrecht: Springer International Publishing.

Treffers, A. (1987). *Three dimensions. A model of goal and theory description in mathematics education—The Wiskobas project.* Dordrecht, the Netherlands: Kluwer Academic Publishers.

Treffers, A. (1991). Het rekenrek 1&2. *Willem Bartjens*, *8*(3), 151–153; *8*(4), 199–200.

Van den Brink, F. J. (1989). *Realistisch rekenonderwijs aan jonge kinderen* [Realistic mathematics education for young children]. Utrecht, the Netherlands: OW&OC.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 458-477.

Abstract. In this article design-based curriculum innovation is presented as an alternative for the RDD approach that was in vogue in the 1960s and 1970s (and which seems to have resurfaced in the form of evidence-based curriculum innovation). Limitations of the RDD model are discussed and design research is offered as an alternative. It is noted that current design research in mathematics education in general involves a constructivist orientation. This complicates matters, as it requires a specific classroom culture and a deep understanding of how the mathematics that has to be learned might be constructed by the students. With RME as a domain-specific instruction theory that offers heuristics on how to design instruction that aims at supporting students in constructing mathematics, issues of teacher professionalization and teacher support are discussed.
Keywords: curriculum innovation; RDD; design research; RME; teacher professionalization.

Resumo. Neste artigo é apresentada a inovação curricular baseada em design como uma alternativa para uma abordagem RDD que esteve em voga nos anos 1960s e 1970s (e que parecem ter ressurgido na forma de inovação curricular baseada em evidência). São discutidas limitações do modelo RDD e é oferecida como alternativa o design research. É notado que, em geral, design research em educação matemática envolve uma orientação construtivista. Isto complica as coisas, porque requer uma cultura específica de sala de aula e uma compreensão profunda de como a matemática que tem de ser aprendida pode ser construída pelos alunos. Com RME como uma teoria de ensino de domínio-específico que oferece heurísticas em como desenhar o ensino que visa apoiar os alunos na sua construção da matemática, questões de profissionalização dos professores e de apoio ao professor são discutidas.
Palavras-chave: inovação curricular; RDD; design research; RME; profissionalização do professor.

▪▪▪

KOENO GRAVEMEIJER
Eindhoven School of Education e Eindhoven University of Technology, The Netherlands
koeno@gravemeijer.nl

(recebido em abril de 2016, aceite para publicação em agosto de 2016)