Improving mathematics teaching and developing teachers through Parallel Lesson Study

Promover o ensino da matemática e o desenvolvimento profissional do professor através do estudo de aula paralelo

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Abstract. To understand Chinese students' high performance on international mathematics assessments, researchers have explored Chinese teachers' professional development and argued that engaging in school-based teaching research activities, including conducting Chinese lesson study, has played a critical role in improving teaching and learning in mathematics. In recent years an enriched model of Chinese lesson study, parallel lesson study (PLS) has been implemented nationwide to support the reform-oriented curriculum. PLS follows the same cycle of regular lesson study while there are at least two independent lesson study groups (including mathematics specialists and teachers) that develop research lessons of the same mathematics topic simultaneously. This chapter synthesizes the key findings of two research studies on PLS to describe and discuss how PLS contributed to improvement of teaching that promotes student learning as well as teacher learning. The district PLS addressed developing algebraic reasoning with 7^{th} grade students while the school-based PLS explored learning trajectory-based lessons on division with fractions at 6^{th} grade.

Keywords: Lesson study; Chinese lesson study; core practice; deliberate practice; elementary mathematics.

Resumo. Para entender o elevado desempenho dos estudantes chineses em avaliações matemáticas internacionais, os investigadores têm explorado o desenvolvimento profissional dos professores chineses e têm argumentado que envolver-se em atividades de investigação relacionadas com o ensino, incluindo a realização de estudos de aula chineses, tem desempenhado um papel fundamental na melhoria do ensino e da aprendizagem da matemática. Nos últimos anos, um modelo enriquecido de estudo de aula chinês, o estudo de aula paralelo (PLS), foi implementado em todo o país para apoiar um novo currículo de matemática. O estudo de aula paralelo segue o mesmo ciclo do estudo de aula regular envolvendo pelo menos dois grupos de estudo de aula independentes (incluindo professores e especialistas em matemática) que desenvolvem simultaneamente aulas de investigação no mesmo tópico de matemática. Este capítulo sintetiza as principais conclusões de dois estudos de aula paralelos para descrever e discutir como este processo contribuiu para a melhoria do ensino, promotora da aprendizagem do aluno, bem como da aprendizagem do professor. O estudo de aula paralelo realizado a nível do distrito escolar centrou-se no desenvolvimento raciocínio algébrico em alunos do 7.º ano enquanto o estudo de aula paralelo realizado a nível da escola explorou lições baseadas numa trajetória de aprendizagem na divisão com frações no 6.º ano.

Palavras-chave: Estudo de aula; estudo de aula chinês; prática nuclear; prática deliberada; matemática elementar.

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Introduction

In 1999 Liping Ma's comparative study on teachers' mathematical knowledge in China and the US revealed that Chinese teachers developed a profound understanding of fundamental mathematics (PUFM). Ma (1999) and other researchers (Paine & Ma, 1993; Wang & Lin, 2005; Wang & Paine, 2003) speculated that Chinese teachers studied curriculum and other instructional materials collaboratively in school-based teaching research groups, which promoted their PUFM development. Recently, Chinese students' strong performance on international mathematics assessments (Fan & Zhu 2004; OECD, 2010, 2014) attracted an increasing interest in exploring the characteristics of mathematics instruction in China (Boaler, 2015; Fan et al., 2015; Li & Huang, 2013). However, the question posed by Ma (1999) about how Chinese teachers had developed their PUFM is still a significant educational phenomenon researcher have been investigating nowadays. A unique teacher professional development system in China has been argued to play a crucial role in developing teachers' competence (Huang et al., 2016a). Systematic studies of lessons in school-based teaching research groups including public lessons and exemplary lessons have been considered key professional development activities that are linked to Chinese mathematics teachers' expertise development (Han & Paine, 2010; Hu, 2005; Huang & Bao, 2006; Wang & Paine, 2003).

Lesson Study that is practice-based, collaborative, and research lesson-oriented professional development model has been considered a powerful approach to improving teaching and developing teachers' expertise (Hart et al., 2011; Huang & Shimizu, 2016; Lewis et al., 2009). Akin to Japanese lesson study structurally (Lewis et al., 2009), Chinese lesson study enacts rehearsal teaching of a research lesson and draws on knowledgeable others' input during the process (Huang & Han, 2015; Yang & Ricks, 2013). Chinese Lesson Study differs from Japanese Lesson Study in three aspects. First, Chinese Lesson study is content-focused and strategies-oriented so that the essential goal is to develop an exemplary lesson to demonstrate effective teaching of specific content or using specific teaching strategies. Second, sometimes rehearsal teaching is repeated multiple times. Third, a knowledgeable other is involved in the entire process of lesson study (Huang & Han, 2015; Huang et al., 2017a). Consequently, the Chinese lesson study model pays attention to "both content and pedagogical knowledge and skills, and an open, learner-centered implementation component" (Lerman & Zehetmeier, 2008, p. 139).

Recently, Chinese Lesson Study has evolved into a new form, known as Parallel Lesson Study (PLS), which has been implemented nationwide to support the curriculum reform. The PLS approach is a response to the call of new curricula that requires teachers to creatively and innovatively use their textbooks in their classrooms to provide differentiated instruction for their students (Huang & Han, 2015). To conduct a PLS, a key topic is selected based on extensive discussions among teachers and mathematics teaching researchers. A lesson study group usually consists of a mathematics-teaching researcher from a district educational bureau, a master teacher, a *demonstrating teacher* (who takes the main responsibility for developing and teaching the selected content), and other mathematics teachers. Through a typical process of Lesson Study, at least two independent lesson study groups develop exemplary lessons of teaching the selected content. Then a teaching research activity at the cross-district level is organized, inviting teachers from different study groups to demonstrate their respective lessons. A post-lesson meeting focuses on comparing and contrasting the public lessons. PLS follows the same cycle as regular lesson study, but PLS includes at least two lesson study groups to develop exemplary lessons of the same mathematics topic simultaneously. Both lesson study groups demonstrate their respective lesson at the final stage of PLS activity.

In this article we will introduce the background information about Chinese lesson study in general, and examine critical features of parallel lesson study to explain how PLS promoted teacher learning and improved mathematics instruction in the classrooms through two research projects. Especially, we aimed at answering the following research questions: How did the parallel lesson study activities as deliberate practice enact the repeated cycles of implementing the core practice? What kinds of core practice were implemented during the cycles? At the end we will discuss the implications of Parallel Lesson Study for teacher professional development.

The deliberate practice and parallel lesson study

The Chinese lesson study has been in place since 1950s and played an important role in implementing curriculum, improving teaching and learning, and developing teachers. All in-service teachers have chances to conduct public lessons or exemplary lessons in school-based teaching research groups or at the district level (Han & Paine, 2010; Hu, 2005; Huang et al., 2009; Huang et al., 2016a; Wang & Paine, 2003). Teachers also have an opportunity to attend teaching competition in districts or cities (Li & Li, 2009). In addition, teachers are also involved in curriculum studies, student test results analysis, and collaborative lesson planning activities on a regular basis (Hu, 2005). Yet it is only in recent years that researchers have examined the mechanism of Chinese lesson study and its effects on teachers, teaching and learning in mathematics classrooms (Gu & Wang, 2006; Hang & Bao, 2006; Han & Paine, 2010; Hang & Han, 2015; Hang et al., 2014; Huang et al., 2016b; Yang, 2009). Those studies documented how teachers developed their expertise in teaching through constant enactment, collaboration and reflection, and repeated practices of core aspects of instruction in lesson study activities. To interpret how conducting lesson study in China contributed to improvement of instructional practices, Han and Paine (2010) argued engagement in deliberate practice (Ericsson, 2003, 2005; Ericsson et al., 1993) of teaching public lesson situated the teachers in a cycle of learning.

Ericsson et al. (1993) found deliberate practice has the following features: first, the participants were instructed to improve particular aspects of performance by repeatedly practicing representative tasks that defined the essence of the domain; second, the participants had opportunities to get detailed immediate feedback on their performance; and, third, deliberate practice was done with intense concentration.

Recently, some researchers drew on the theoretical lens of deliberate practice to investigate teacher learning and redesign teacher education programs to promote teacher learning (Bronkhorst et al., 2011, 2014; Deans for Impact, 2016; Han & Paine, 2010; Lampert & Graziani, 2009; Lampert et al., 2013; Van Gog et al., 2005). Lampert and Graziani (2009) identified a list of more than 10 instructional activities that were used to develop novice teachers' competence in the core practice. Han and Paine (2010) argued that teaching public lessons as deliberate practice allowed the teachers to recurrently refine their competence and skills in three representative teaching tasks. Lampert et al. (2013) studied rehearsal as a pedagogy to prepare novice teachers around the core practices and argued that the clinical nature of rehearsal, an appropriation of practice, supported novice teachers to do adaptive teaching while developing their knowledge and skills in repeated cycles of enactment and investigation. Ericsson and his collaborators (2016) in education synthesized five key principles of deliberate practice from the science of expertise that are highly relevant to developing teacher skill (Deans for Impact, 2016). The five principles are: pushing beyond one's comfort zone; working toward well-defined, specific goals; focusing intently on practice activities; receiving and responding to high-quality feedback; and developing a mental model of expertise.

Parallel Lesson Study activities have an explicit goal of improving the teaching of a topic. Each research lesson is observed, commented on, and reflected upon by a group of teachers and/or experts. Lesson Study is an embedded part of teachers' practice in a professional community. The cycle of learning allows teachers to teach authentic rehearsal lessons in a community of practice where knowledgeable others such as experienced teachers, teaching research specialists and university experts shared their feedback and collaborated with the teachers to improve instruction.

In what follows we drew on two research projects to elaborate on how PLS improved teaching and student learning as well as teacher learning. The first research project studied one district-level PLS that addressed how to develop algebraic reasoning with 7th grade students. The other research project was a school-based PLS that explored learning trajectory-based lessons on division with fractions at 6th grade.

Research methods of the two studies

In the first research project two lesson study groups at the middle school level in China were involved in the district wide PLS activity (Huang & Han, 2015). Each group was constituted by four teachers and one district mathematics teaching research specialist. Both groups focused on teaching an activity-based lesson in 7th grade to explore patterns in calendars and express the patterns in algebraic expressions. It was the first time that the students learned algebraic expressions formally. Each group first independently studied how to teach this topic through the cycle of lesson study. Then a teaching research activity at the cross-district level was organized, inviting both groups to demonstrate their respective final public lesson. There were more than fifty mathematics teachers who attended the final public lessons and debriefing meetings. The background information of the participant teachers in both groups is summarized in Table 1.

Table 1. Background of the participant teachers

	LSG A	**LSG B**
Demonstrating teacher	Mr. Wu: Beginning teacher with master degree in mathematics	Miss Han: Experienced teacher with bachelor degree in mathematics
Teaching researcher	Mr. Zhu: Former excellent teacher; Experienced teaching researcher	Mr. Hu: Former excellent teacher; less experienced teaching researcher

The data of the first study included five data sources: (1) all the lesson plans created during the process; (2) the teachers' reflection reports; (3) videotapes of the final public lessons; (4) interviews with the two demonstrating teachers, and (5) interviews with the two teaching researchers. These data were collected in Fall 2012 over three months.

We first examined all the lesson plans to identify the major changes in the plans over time. With the identified changes, we triangulated with the demonstrating teachers' reflection reports, the transcribed videos of the final research lessons and the transcribed interview data. Once the preliminary list of the changes was created, we re-examined the list to modify the major changes. Eventually, three dimensions of major changes were identified, including instructional objectives, mathematical tasks, and instructional procedure. In addition, we discussed these major changes with the two teaching researchers, who confirmed these findings.

The second research project we drew on included two lesson study groups of 6th grade teachers in an elementary school of China. Two district teaching research specialists and one university expert participated in the activities of both groups. Though the two groups still studied the same topic — division with fractions, they chose a different interpretation of division when teaching the topic of numbers divided by fractions — partition and measurement. The teaching research specialist and university expert guided the teachers to incorporate the theories of learning trajectory and teaching and learning through variation into their lesson design.

A hypothetical learning trajectory is what teachers predict as to how student learning might develop on a certain mathematics topic. Learning trajectories include sequences of tasks and activities aimed at the progressive development of mathematical thinking and skill (Clements & Sarama, 2004; Simon, 1995). Drawing on the theory of learning trajectory, the teachers planned lessons based on how their students might construct the new mathematical knowledge. Different from the traditional way of deciding scopes and sequences of mathematics curriculum, the theory of learning trajectories considers students' thinking that grows in response to instructional experiences.

The theory of variation (Huang & Li, 2017; Marton, 2015) argues that patterns of variation in concepts and procedures, and invariants in certain aspects will help children discern critical aspects of the new learning. The critical features represent the differences between the new learning objects and others. When students learn the algorithms of division with fractions, they need to discern the relationship between dividend and divisor in a specific problem situation and identify the appropriate visual representation of the relationship for conceptual understanding of the algorithm. Invariance is emphasized in the variation theory because learners need a background of invariance to recognize critical features of the new learning. Conceptual variation aims at highlighting the essential features of a concept while procedural variation helps students develop competences in problem solving, including solving extended problems, using multiple ways to solve problems, and applying solution methods to more similar problems (Huang & Li, 2017). With the conceptual and procedural variations in mind, the group of teachers in the second research project chose and modified mathematical tasks to develop the students' mathematical proficiency in the topic of division with fractions.

In the second study four elementary teachers in an Eastern coastal large city of China formed a lesson study group on a voluntary basis. One university professor and two teaching research specialists —Mr. Sao and Mr. Ren from the school districts — were involved in the lesson study activities as experts. The four participant teachers were three experienced teachers — Ms. Shao, Ms. Han and Ms. Tang —, and less experienced teacher — Ms. Lu — who had five years of teaching experiences. The current paper focus on the Research Lessons taught by Ms. Shao and the related teaching research activities. Table 2 shows the timeline and organization of the Research Lessons taught by Ms. Shao.

Table 2. Timeline and Organization of Research Lessons

Topic of Research lessons	**Rehearsal Research lesson 1 Class: 605**	**Rehearsal Research lesson 2 Class: 603**	**Final Public Lesson 3 Class: 606**
A fraction divided by a whole number	Date: 10-9-2014	Date: 10-10-2014	Date: 10-15-2014
A number divided by a fraction	Date: 10-11-2014	Date: 10-14-2014	Date: 10-17-2014

The second study was conducted in 2014 and lasted about three months. The major data sources used in this study were students' post-lesson tests, videotaped lessons and student interviews. The post-lesson quiz had five word problems about division with fractions and asked the students to justify their solution methods with words, drawings and symbols. The quiz was given right after each Research lesson. Each quiz was about 20 minutes. All completed quizzes were collected. The one-on-one interviews were conducted with 10 of the students on a voluntary basis at the end of the study.

We read through all the data sources to code the common terms that emerged. With the labeled common terms, we examined the data across the different data sources to seek any emerging themes regarding conceptual variation and procedural variation, which are critical for student learning. The triangulation of the data sources produced several themes, including: (1) the teachers enriching visual representations, (2) the teachers revising story contexts, and (3) the teachers reorganizing student learning to allow free exploration that promoted conceptual and procedural variations.

Meanwhile, we read through all the students' post-tests to roughly identify what common understandings among the students were about division with fractions and what common mistakes or misconceptions were. We used the concept of mathematical proficiency to analyze all the tests in the three dimensions, including procedural fluency, conceptual understanding, and strategic competences. Then we compared the results of each test in the aforementioned three dimensions to seek changes in student learning. The analysis of the student tests data was triangulated with other data sources such as student interview data, transcribed lesson videos, etc.

Teacher learning through PLS

Our studies on both of the PLS cases revealed that the participant teachers embarked on the following core instructional practices through rehearsal lessons and with their colleagues' feedback. The first core practice was task selection that set the tone of a lesson. The second core practice the teachers had collaboratively worked on was the ways of organizing student learning in the lessons.

All the participant teachers collaborated with their colleagues to discuss and modify the mathematical tasks in the lessons. When they revised the tasks in the lessons, they considered several dimensions to make a decision. The tasks used to review prior knowledge should not only activate prior knowledge, but also have close connections with the new knowledge and pave the way for the students to understand the new knowledge.

Task selection sets the tone of the lesson that could focus student learning on high-level cognitive demand and engage students in thinking (Stein et al., 2009). Student engagement is an important aspect teachers need to consider when planning lessons and assessing student learning. Strong engagement in learning will support students to develop disposition of persevering in problem solving. With her colleagues' and teaching researcher's suggestions, Miss Han changed one of the mathematics tasks in the lesson. Her colleagues and the district teaching research expert thought that the second task in the lesson did not prepare students for solving the following major tasks. With the feedback, Miss Han designed a game in which the students were given the sum of the three consecutive numbers in a row on a calendar, and needed to figure out the three numbers. In class, when the students were surprised at her magic in finding answer immediately and she challenged the students to explore what strategy she used to find the three hidden numbers so quickly. The game motivated the students to think and discover patterns.

For designing the major exploratory task (e.g., exploring the pattern among the numbers in a 3×3 grid), Miss Han tried different approaches, such as selecting two grids from six grids or selecting simple one from given grids. She and her colleagues finally provided the students with two grids to explore. However, she eventually realized that the final selection constrained student thinking. In addition, Miss Han and her colleagues also created a challenging task, *arrangement of odd number,* which required students to apply the thinking methods to a novel situation. In the interview, she reflected:

> Beyond the calendar context, more application problems do not require the natural numbers, but a number series with same difference (in high school, the arithmetic number series). So, I designed the arrangement of an odd-number series to ask students to explore similar problems using similar methods. Thus, it can develop students' ability to apply what they learned; it emphasizes what they learned not only applied to calendar but also other situations.

In the second study, teaching and learning through variation generates multiple experiences of learning new knowledge. In the 6^{th} grade PLS the teachers kept modifying the tasks of the lessons to enhance students' understanding of algorithm of division with fractions. The teachers in the lesson study designed and revised the tasks for the purpose of making the connection between the students' prior knowledge and the current critical aspects of the new knowledge in the lessons. They created two tasks in the three research lessons on fractions divided by whole numbers. Task One was "Two persons equally shared 2 liters of juice. How much did each person get?", and Task Two was "Two persons equally shared 1 liter of juice. How much did each person get?". The two tasks in the review section aimed at linking partition division with whole numbers to partition division with fractions. With the connection, the students were expected to transfer their prior knowledge of partition division to the learning of new knowledge in the lessons.

In the three lessons on numbers divided by fractions the teachers revised how to review the students' prior knowledge. In the first lesson Teacher Shao simply reviewed the reciprocals of five numbers. However, reviewing reciprocals did not help the students understand the algorithm from the perspective of proportional relationship. Teacher Shao decided to start with reviewing how many ½ hour, ⅓ hour, ⅕ hour and ⅒ hour respectively has one hour, which would prepare the students' conceptual understanding of multiplicative relationship in division with fractions.

Thus, the final review tasks were how many ⅓ hour in ⅔ hour, how many ¼ hour in ¾ hour, how many ⅓ hour in one hour, how many ⅕ hour in one hour, and how many ⅒ hour in one hour. The new numbers — ⅔ and ¾ — in the review tasks were aligned with the numbers in the example problem that followed in the final lesson.

> T: I can also express in this way- how many times one hour is of ⅓ hour.
> S: Three times.
> T: Next problem.
> S: There are five copies of ⅕ hour in one hour.
> T: How to express this relationship in a different way?
> S: One hour is five times of ⅕ hour.
> T: How about ⅒ hour? How many ⅒ hour are there in one hour?
> S: Ten.
> T: Express it in another way? Let's say it together.
> S: One hour is ten times of ⅒ hour. (Lesson on Oct.17, 2014)

The teacher also revised the numbers in the example tasks and the visual representation for the task deliberately. For example, the first example problem is — two persons shared ⅘ of a liter of juice, how much each got. Students got answers using multiple strategies such as diagrams (e.g., area model or length model), meaning of equal sharing ($\frac{4}{5}\div 2=\frac{4}{5}\times\frac{1}{2}$) and equation ($\frac{4}{5}\div 2=\frac{4\div 2}{5}$). In the first research lesson, the example was followed by a word problem that can be solved by the expression ⅘÷3. However, students had difficulties with visual models and equations to solve the problem. Then a scaffolding task was added that shared the same problem structure, but with a different number — how much each person got when two persons equally shared ⅕ of a liter of juice. The new problem helped the students to realize the limits of one method of division with fraction — dividing numerators: $\frac{1}{5}\div 2=\frac{1\div 2}{5}$

The second core practice the teachers had collaboratively worked on was the ways of organizing student learning in the lessons. The ways in which a teacher organizes students to learn in class reflect a teacher's professional vision on teaching and learning mathematics, and decide student's opportunities to learn and think. From both PLS cases we found that the participant teachers expanded their professional visions and shifted from teacher-dominated instruction to student learning-centered instruction.

In the first study the teachers' instructional procedures were transformed to be more consistent and coherent. For example, Miss Han's lesson proceeded more smoothly by adding daily introductory questions such as how many days in a week, and adding some transition exercises between activity 1 and activity 2. She realized that using variation problems helped students develop flexibility. To have a better closure of the lesson, Miss Han shifted from focusing on solving problems to ways of discovering patterns and relevant mathematical thinking methods. To ask the students what they have learned from the lesson, she added a summary that recapped the key methods and steps of discovering patterns.

Miss Han mentioned her appreciation of the teaching research specialist's comments that her research lesson should focus student learning on building connections among different concepts and digging deeply into the thinking methods of discovering patterns rather than the mastering of using letters to represent numbers and seeking patterns. Another teacher, Mr. Wu reflected that he learned how to transfer his role from a knowledge transmitter to an organizer, guide, collaborator, and co-learner of students' learning by listening to students.

In the second study Teacher Shao changed her instruction from teacher-dominated instruction to the students' exploration and investigation. In all the six lessons, Teacher Shao distributed more instruction time to having the students explore how to solve the problems both independently and collaboratively. In each of the two last lessons on the two different topics respectively, she presented the problem and linked the meaning of division with fractions to that of division with whole numbers. Then she had the students investigate the solution methods and explain their solution methods in number sentences, pictures and words. Building upon the students' ideas, she guided the students to compare and discuss their different solution methods.

At the debriefing meeting she mentioned that she was surprised to see one group of students convert the fractions into decimals and divide, e. g.,$\frac{1}{5} \div 2 = 0.2 \div 2 = 0.1$. The district teaching specialist Mr. Sao commented at the meeting, "…when a teacher gives students an opportunity to freely explore, the students' mind would be open and have no boundary. Otherwise, in the first lesson the students' thinking was confined to one visual model and one solution method presented by the teacher" (The debriefing meeting on Oct. 10, 2014). In the second lesson Teacher Shao focused her students' learning on the first problem — $\frac{4}{5}$ liter of juice equally shared by two persons. But the district specialists suggested that the second example problem — $\frac{1}{5}$ liter of juice equally shared by 2 persons — was cognitively difficult for the students. Teacher Shao should organize the students to explore this second problem in small groups and spent more time on discussing their solution methods to this problem and elaborating on their understanding of the algorithm through the second problem.

Implications for teacher professional development and learning

We argued that PLS improved teaching and teacher learning through intensively, repeatedly practicing core aspects of instruction with support and feedback from colleagues. The core aspects of mathematics instruction identified from the two studies included design of mathematical tasks and ways of organizing student learning in a lesson. Meanwhile, we proposed that in a reform context PLS juxtaposed and explored different instructional approaches by bringing in learning theories such as learning trajectory and theory of variation into practice. Thus the participant teachers collaboratively formed their localized theory of teaching a certain topic in action.

How to teach and learn the topic of division with fractions was prescribed by the textbook and teacher manual the teachers used in the second PLS case. However, the teacher and the experts brought in the learning theories of learning trajectory and variation into practice to explore a better route to student learning and a better approach to instruction. Drawing on the two theories, the teachers outlined the trajectory of student thinking and learning in the specific mathematical topic — division with fractions. During the lesson study activities, the teachers improved their teaching that promoted student learning by adopting the two theories. The learning trajectory was refined, as the lesson was improved. Meanwhile, the two groups employed two different division interpretations in the lessons on numbers divided by fractions — partition division and measurement division. The task of measurement division was *how many* $\frac{2}{5}$ *L glasses there were in 3L of milk*. The task of special partition division asked, *Xiaohua walked 3 kilometers within* $\frac{1}{2}$ *hour. Xiaoming Walked 5 kilometers within* $\frac{2}{3}$ *hours. Xiaohong walked* $\frac{21}{8}$ *kilometers within* $\frac{3}{4}$ *hours. How many kilometers did each of them walk in one hour?* The special partition of seeking one whole in a story context of distance, time and speed posed a challenge for the students to understand the standard algorithm. Therefore, the teachers decided on using proportional relationship to help the students understand it, which was different from the approach the other lesson study group adopted. Thus the participant teachers not only collaboratively formed their local theory of teaching division with fractions in action through creating and refining the learning trajectory, but also juxtaposed their different approaches to supporting the students to make sense of the standard algorithm of division with fractions.

Investigating the status of teacher development in the US, Darling-Hammond and her colleagues (2009) concluded that effective teacher professional development was intensive, ongoing, connected to practice, and focusing on teaching and learning of specific academic content. The PLS studies we discussed in this chapter evidenced the conclusion of Darling-Hammond and her colleagues. PLS offers a teacher an opportunity of intensive, ongoing learning that is tightly connected to their instructional practice through studying how to teach a specific mathematics topic. We argue that PLS situates a teacher in a community of practice where the teacher collaborates with colleagues and experts to conduct research lessons as deliberate practice. Deliberate practice (Han & Paine, 2010; Lampert et al., 2013) poses three key features to develop teachers:

identifying core instructional practices in mathematics education, intense rehearsal of practicing core aspects of instruction, and providing immediate feedback by knowledgeable colleagues and experts. PLS intensifies the key features when the participant teachers interact with more colleagues and compare their different perspectives of and approaches to teaching the same topic.

Meanwhile, we propose that PLS is a way to empower teachers who learn together how to respond to reforms and theorize their local instruction. In reform-context teachers can engage in PLS to explore diverse ways to fulfill the reform expectations, such as how to implement new standards and curriculum, etc. PLS activities help teachers respond to educational reforms through a more intense learning cycle of planning, rehearsal, observation, reflection, revising, and comparison. Two projects (Huang et al., 2017b; Prince, 2016) on the implementation of Chinese lesson study in the US setting have showed promising results. Huang et al. (2017b) documented how the repeated cycle of enactment and reflection with immediate feedback from knowledgeable others helps high school mathematics teachers improve their teaching practices related to mathematics problem solving. Meanwhile, the teachers also developed positive attitudes toward teacher professional development and student learning, and planned to adopt the high-leverage practices in their future teaching. In his dissertation study, Prince (2016) studied a group of middle school mathematics teachers who participated in a lesson study activity that emphasized repeated teaching of the same topic and knowledgeable others' input. The study argued that the participant teachers enhanced their understanding of mathematical teaching practice, fostered positive attitudes toward reform-oriented teaching and learned to adopt innovative ways to teach mathematics.

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