Using lesson study in mathematics to develop primary school teachers' practices: a case study

Utilizando o estudo de aula em matemática para desenvolver as práticas dos professores do 1.º ciclo: um estudo de caso

Valérie Batteau
University of Geneva & Lausanne Laboratory Lesson Study, HEP Vaud, Switzerland
valerie.batteau@hepl.ch

Abstract. This paper presents a study of primary school teachers' practices in mathematics in the context of a professional development training, lesson study (LS) in Lausanne (Switzerland). The study deals with the following question: what practices will change and what resistance in practices will be observed when primary school teachers engage in LS as a form of professional development? The practices are analyzed in a double didactical and ergonomical approach theoretical framework and the qualitative method used is a case study of one particular teacher. This study underlines the evolution of the teacher's practices concerning the choice of mathematical activities and the interactions between the teacher and the students. The study highlights some limits linked to the difficulty to reinvest and to transfer professional acts, didactical and mathematical knowledge worked during the lesson study process.

Keywords: teachers' practices; lesson study; professional development.

Resumo. Este trabalho apresenta um estudo das práticas do professor da escola primária em Matemática no contexto de uma atividade de desenvolvimento profissional, o estudo de aula, em Lausanne (Suíça). O estudo aborda a seguinte questão: que práticas vão mudar e que resistência nas práticas será observada quando professores primários se envolvem em estudos de aula como uma forma de desenvolvimento profissional? As práticas são analisadas a partir de um quadro teórico com uma dupla abordagem didática e ergonómica e o método qualitativo utilizado é um estudo de caso de uma professora particular. Este estudo destaca a evolução das práticas da professora sobre a escolha de atividades matemáticas e as interações entre a professora e os alunos. O estudo destaca alguns limites ligados à dificuldade para reinvestir e transferir ações profissionais e conhecimento didático e matemático trabalhado durante o processo de estudo de aula.

Palavras-chave: práticas do professor; estudo de aula; desenvolvimento profissional.

(Recebido em dezembro de 2016, aceite para publicação em agosto de 2017)

Introduction

Lesson study (LS) is a field of research and professional development developed principally in Asia, in the US and in Northern Europe (Lewis & Hurd, 2011; Yoshida & Jackson, 2011). This collective and reflexive process involves a group of teachers and facilitators meeting to improve instruction. Therefore, LS is proper to effective professional development: site-based, practice-oriented, focused on student learning, collaboration-based, and research-oriented (see Murata, 2011). Research in the field of LS (for example, Hart, Alston & Murata, 2011; Lewis & Hurd, 2011) highlights the effects on teachers (knowledge for teaching mathematics, beliefs, dispositions), on school (community of practice), and on the system (tools and resources). In a meta-analysis of 643 programs of mathematics professional development in the US, Gersten, Taylor, Keys, Rolfhus and Newman-Gonchar (2014) point out that only two programs, including a LS program (Perry & Lewis, 2011), have statistically significant positive effects on students' mathematics proficiency according to standards from US Department of Education Institute of Education Sciences.

Research on professional development (e.g. Lefeuvre, Garcia & Namolovan, 2009) focus on teachers learning, and the main indicators of this learning concern knowledge, beliefs, and practices. LS process has effects on teachers' practices, but few research works focus on the modification and development of teachers' practices in a LS context.

> (…) many studies identified aspects of the Personal and Practice domains that professional development programs seek to affect, but few studies focused on the processes or mechanisms of teachers' learning; therefore, they have little to say about how teachers develop knowledge, beliefs, or instructional practices. (Goldsmith, Doerr & Lewis, 2014, p. 21)

Our study does not focus on the efficiency of a specific LS process, but rather on how teachers' practices are modified by a LS process over a long period of time, almost three years (September 2013 to May 2016). Furthermore, we focus on effective practices (before, during and after the lessons) - and not only discourses on practices - of a particular teacher involved in a professional development training in Lausanne (Switzerland). This study presents a longitudinal study and an original theoretical and methodological viewpoint by joining a double didactical and ergonomical approach. This study proposes to analyze what will change in the teacher's practices and what resistance in practices will be observed for a primary school teacher. To tackle this question, this paper presents first the theoretical framework, issued from Robert and Rogalski's double didactical and ergonomical approach, then, the qualitative method used and finally the detailed case study of one particular teacher. In conclusion, we provide elements of the answer to the research questions in relation to the findings and the main advances made by this study.

Theoretical framework

Robert and Rogalski (2002, 2005) developed a specific framework based on a double viewpoint, one in the French didactics of mathematics and another in ergonomics with the activity theory (Leontiev, 1975; Leplat, 1997). We chose this theoretical framework in order to analyze effective practices (before, during and after the class time), their evolution, changes, because this double viewpoint is proper to better take into account the complexity of teaching, both as an individual and a professional act. Indeed, this double approach aims at analyzing the relation between teachers' and students' activity in class, but also the constraints on teachers in the context of their profession. In this paradigm, teacher's practices are seen as "all work done by that teacher, whether before, during, or after class time" (Robert & Hache, 2013, p. 25).

In this theoretical framework, we consider two closely linked elements to analyze teachers' practices: students' activities and teachers' management of the class (Robert, 2001; Robert & Hache, 2013; Robert & Rogalski, 2005). In this sense, the authors highlight two specific aspects (components) of teachers' practices in the class: the organization of the tasks for the students, that they name the *cognitive* component, and teachers' interactions with students, that they name the *mediative* component (Robert, 2001; Robert & Hache, 2013; Robert & Rogalski, 2005). Moreover, the *personal* component is linked to professional aspects and describes how the teacher invests his/her leeway, what his/her representations (about mathematics, teaching of mathematics, his/her students) and his/her mathematical knowledge are.

In this framework, teachers' practices are seen as a complex, yet coherent and stable system. Indeed, these authors have shown that teachers' practices get stabilized very early in their career and are therefore difficult to change (Chappet-Paries, Robert & Rogalski, 2013; Roditi, 2013). In our work, we question the stability of practices when teachers engage in a LS process. Teachers' practices are analyzed in reference to levels of development (Charles-Pézard, Butlen & Masselot, 2012; Peltier-Barbier et al., 2004) and teachers' activity is analyzed as a process of modifications between different levels of tasks (Leplat, 1997; Mangiante, 2007). Those core aspects of this theoretical framework are developed below.

Levels of development of practices

Charles-Pézard, Butlen and Masselot (2012) set up and implemented a training engineering in order to study relationships between trainers' actions, their effects on teachers' practices and potential observed resistance. These authors used the terms of "observed resistance" to point out the fact that trainees showed that they had learned some elements during the training and yet did not reinvest them in their following practices. We used their methodology in order to analyze the teachers' practices, their evolution but also the potential observed resistance in the practices in relation to what they learned during the LS process and what the teachers did not apply in their practices.

Teachers' practices are considered as a coherent and stable system of answers to the teachers' obligations (Charles-Pézard et al., 2012; Peltier-Barbier et al., 2004). From elements of the teacher's management in the class, regularities within global teaching strategies were observed among teachers and for the same teacher (Charles-Pézard et al., 2012; Peltier-Barbier et al., 2004). These regularities were observed during three important moments of teachers' activity: the processes of devolution, regulation and institutionalization (Brousseau, 1997; Clivaz, 2015) which are characteristics of teachers' strategies and choices. Teachers' practices are categorized from observed regularities and are analyzed by measuring the difference with one of these categories considered as a reference. This reference is based on the idea that the existence of whole class discussions, synthesis, and institutionalization make for the setting up of collective knowledge as a reference for the class and the mathematics proposed to students are a priori richer and vectors of learning (Charles-Pézard et al., 2012). This reference is available in several levels, which concern the presence of problem-solving, whole class discussion (with formulation and overview of strategies, synthesis, institutionalization), individual work, students' help, and the place of the students' initiative. Specifically, level 1 is achieved when a teacher can establish a 'scholarly peace' including both social peace and students' enrollment to teacher projects. Level 2 is achieved when a teacher proposes problem-solving with actual research time during the lessons. Furthermore, this level is achieved when a teacher establishes a working atmosphere in his/her classroom and does not reduce his/her mathematical requirements neither in the issues in terms of knowledge and learning, neither in the students' help. Level 3 is achieved when a teacher manages a whole class discussion in which students can expose and explain their strategies, with a validation. Level 4 is achieved when a teacher compares and ranks students' strategies during a whole class discussion and manages contextualized synthesis. Level 5 is achieved when a teacher manages institutionalization of the mathematical knowledge or the method at stake. The process of institutionalization (Coulange, 2012) presents characteristics of decontextualization of knowledge (beyond a given situation: knowledge can be used in other situations) and depersonalization of knowledge (beyond a given person: knowledge is recognize with official statute). Levels 1 and 2 are more connected to the process of devolution whereas levels 3, 4, and 5 are more connected to the process of institutionalization (Charles-Pézard et al., 2012). Each level can be achieved independently from the others, which means that there is no strict hierarchy.

This analysis in levels of development allows to determine in which level are based the teachers' practices and to observe the evolution of practices. This analysis is based mainly on the teachers' practices during the class with indicators concerning mostly the structure of the lesson, teachers' choices, and students' help. In order to understand the observed teacher's practices, we had to complete this first analysis by taking into account the teacher's activity before and after the lesson. This is why we used a second core aspect of the theoretical framework which allowed us to consider teachers' activity before, during and after the class, and to consider the gaps between what the LS group plans to do during the collective meetings and what the teacher does in reality in the classroom with the students.

A process of modifications between tasks

In the ergonomical approach, it is essential to distinguish the task from the activity, which is consistent with the double point of view, that takes into account both the mathematical situation and the subject of the action (Rogalski, 2013). Indeed, teachers "aim to achieve task goals, and their actions are driven by motivations of the activity" (Rogalski, 2013, p. 4). The task is the "goal to be attained under certain circumstances" and the activity is what a teacher "engages in during the completion of the task" (p. 4). The *prescribed* work (the *prescribed* task or what the teacher must do) is distinguished from the effective work (the *effective* or *conducted* task, or what the teacher does in reality). To appropriate the *prescribed* task, the teacher should modify it. A gap exists between the *prescribed* task and the *conducted* task: the reasons can be a lack of teacher's motivation to engage in the desired actions, a lack of the necessary competencies, an inappropriate representation of the task, or even a divergence between the intended and *prescribed* tasks (Rogalski, 2013).

> An *activity* is not a direct response to a *prescribed* task. The task is first redefined by the subject. To complete this task, the subject must form a representation of the task, allowing or forbidding possibilities (not always consciously), lifting or imposing restrictions, and using evaluation criteria that may differ from those of the prescription. This constitutes the *effective* task, to which the subject's activity represents a response. (Rogalski, 2013, p. 5)

Based on this distinction between task and activity, Leplat (1997) adds two tasks: the *represented* task (how the teacher represents the *prescribed* task and that he/she thinks that we expect from him/her) and the *redefined* task (the teacher redefines his/her task according to the *prescribed* task and his/her own professional goals). A teacher combines professional acts and knowledge to represent the *prescribed* task and to redefine a new task. In this study, the professional acts and teacher knowledge are analyzed for the representation of the *prescribed* task and for the redefinition of the *represented* task (Figure 1). Thus, teachers' activity is analyzed as a process of modifications between tasks (Leplat, 1997; Mangiante, 2007). We analyzed the different sources of this process of modifications and so the effects of the LS process on these sources.

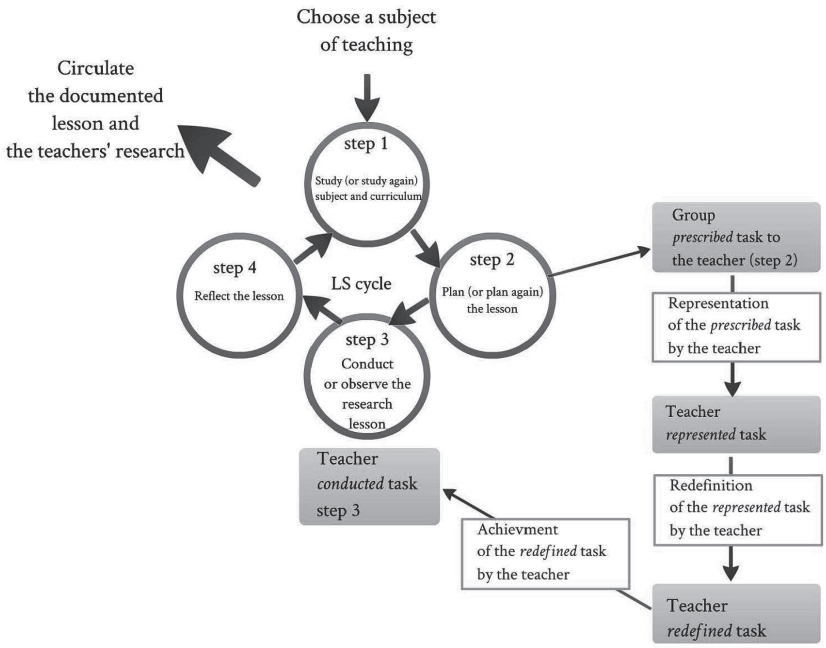


Figure 1. Model of a LS cycle (Lewis & Hurd, 2011) with different levels of tasks (Mangiante, 2012)

These two core aspects of the theoretical framework complete in providing elements at different levels: teachers' practices (all the teacher makes, says, writes, before, during, or after class time) set on a global level whereas teachers' activity set in a local level and corresponds to what a teacher engages in during the completion of the *prescribed* task, including the representation, the redefinition and the achievement of the task.

Methodology

Our first questions: "what will change in the teacher's practices and what resistance in the practices will be observed for a primary school teacher by participating to a LS process?" can be expressed with tools developed in the theoretical framework:

- How can the analysis of the process of modifications between the *prescribed* task and the *conducted* task provide information about a potential development in teachers' practices?
- To what extent can the evolution of practices during the LS process be made apparent? How does this affect the levels of development?

These empirical questions target to understand how the teachers' practices are developed by a LS process.

A LS process in mathematics began in Lausanne (Switzerland) in September 2013 and occurred over two years with two collective meetings occurring per month (Clivaz, 2016). Each meeting lasted for approximately 90 minutes. The LS group consisted of eight generalist primary school teachers with wide-ranging experiences in teaching who were volunteers, and two facilitators. The LS group worked during four LS cycles focused both on one mathematical aspect and one professional act (Clivaz, 2016). During the 2013/2014 school year, Cycle *a* was about numeration and the devolution, while Cycle *b* was about geometry and the institutionalization. Each cycle lasted five months. During the 2014/2015 school year, Cycles *c* and *d* dealt with problem-solving and the help for students. The mathematical subjects were chosen by the group of teachers and facilitators, according to teaching difficulties or students' learning difficulties. During steps 1 and 2 of a LS process, the group studies professional papers on the mathematical subject, along with the official program in mathematics, and the textbooks.

In this paper, we chose to focus on one specific teacher Océane, for several reasons. First her profile, she has seventeen years of experience so her practices are stable and she said during the first collective meeting that she participated to this LS process in order to reinvent her practices, to learn new teaching ways and methods, and to renew herself. Thus, her profile suggests that she was opened to enter in a dynamic of professional development by the LS process. Furthermore, she taught a research lesson during the LS process and she agreed to participate in our doctoral research in addition to the LS process. Her students are eight to ten years old. For this teacher, the data consisted of one lesson before the LS process (about numeration), one lesson during Cycle *a* (about numeration and devolution), the research lesson of Cycle *b* (about geometry transformations and institutionalization), and one lesson after the LS process (about problem-solving) (see Table 1). Data were collected during informal meetings after each lesson observed before and after the LS process. Data were collected from all collective meetings, all written documents produced during each lesson, and students' productions.

Table 1. Océane's data collected and analyzed for this study

	Before LS process	During LS process			After LS process
		Cycle *a*	Cycle *b*	Cycles *c* and *d*	
Mathematical subject	numeration game: "The hidden side"	numeration and devolution: "Strange game of Goose…"	geometric transformations and institutionalization: "In the aquarium"	problem-solving and how to help students represent a problem: "99 squares"	problem-solving: "Fold"
Data	one lesson informal meeting	one lesson collective meetings	research lesson (parts 1 & 2) collective meetings	collective meetings	one lesson informal meeting

All video data (lessons and collective meetings) were transcribed. All video data, written documents, and students' productions were analyzed with indicators, according to the method described below.

The numeration game "The hidden side" and the problem-solving "Fold" were chosen by Océane whereas the activities "Strange game of Goose…" and "In the aquarium" were chosen by the LS group during collective meetings of Cycles *a* and *b*. The LS group modified these two activities during collective meetings according to the discussions in order to target students learning.

During the *conducted* task, the teacher's practices are analyzed in comparison with levels of development, as a process of modifications between the *prescribed* task and the *conducted* task. These two core aspects of the theoretical framework are crossed in this qualitative method (see Figure 2).

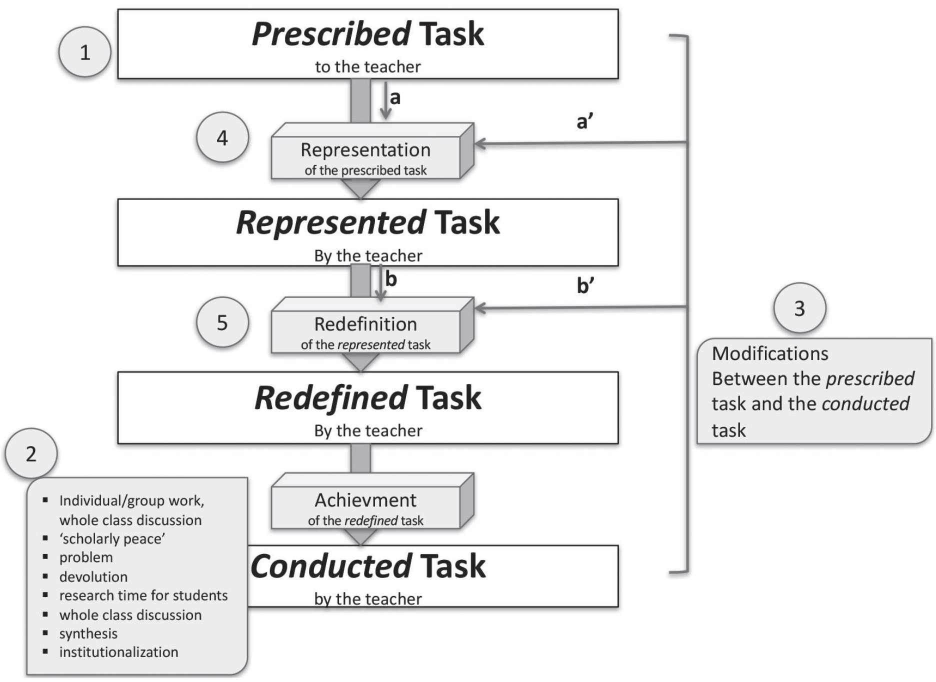


Figure 2. Model of analysis

We detailed each point of this model of analysis: point 1 (Figure 2) corresponds to the *a priori* analysis of the *prescribed* task. *Prescribed* task includes the mathematical problem, especially the mathematical knowledge at stake, the lesson plan, and the planning material (see Table 2). Thus, for each lesson, we conducted an *a priori* analysis of the *prescribed* task that includes an analysis of the mathematical knowledge aimed by the problem, the possible solving strategies, and the didactical variables. The *a priori* analysis provided information about possible students' activity.

Table 2. A priori analysis of the prescribed task

Data	Analysis of
- mathematical problem - mathematical knowledge - lesson plan - planning material	- mathematical knowledge at stake in the problem - possible strategies - didactical variables

We applied methodological research tools: an *a priori* analysis and an *a posteriori* analysis (Clivaz, 2015). We presented for each lesson for the point 1: some elements of the *prescribed* task and the *a priori* analysis, the mathematical problems, and the background.

Point 2 (Figure 2) was the *a posteriori* analysis based on the proceedings and the students' proposed activities during the *conducted* task which allowed to compare the students' proposed activities with the possible students' activity. For this *a posteriori* analysis (see Table 3), we used indicators (Charles-Pézard et al., 2012) in order to categorize teacher's practices in levels of development.

Table 3. *A posteriori* analysis of the achievement of the *redefined* task

Data	Indicators
- interactions between teacher and students during the lesson - students' productions	- individual work, group work, whole class discussion - 'scholarly peace' (social peace and students' enrollment to teacher projects) - problem with mathematical substance - devolution of the problem to students - research: actual time for students - whole class discussion: with presentation of students' answers and strategies, with validation, with explanation of students' strategies - synthesis: ranking of students' strategies by the teacher, contextualized synthesis - institutionalization of knowledge or method

Charles-Pézard et al. (2012) specify that *a posteriori* analysis is not sufficient to decide if levels 4 and 5 are reached. It is necessary to compare the teacher's contextualized choices and the researcher's choices from the *a priori* analysis and the global context. From these indicators, the *a priori* analysis and the global context, we proposed a contextualized synthesis and an institutionalization that it should be possible to manage and then we classified the teacher's practices in levels of development.

In this paper, we develop in point 2 mainly elements of the *a posteriori* analysis, possible contextualized synthesis and/or institutionalization. Moreover, we will develop more general results of analysis using levels of development in the specific section on results.

Point 3 (Figure 2) deals with the analysis of the modifications between the *prescribed* task and the *conducted* task.

Point 4 (Figure 2) is more the analysis of the representation of the *prescribed* task based on these modifications (arrow a' in Figure 2) and the *a priori* analysis of the *prescribed* task (arrow a in Figure 2). To analyze the representation, we need to try to find out what the teacher thinks the LS group expects from him/her. The data used in this analysis are the teacher's speeches during collective meetings enacted during steps 2 and 4 of a LS cycle (see Figure 1 and Table 4), and the research lesson.

Table 4. Analysis of the representation of the *prescribed* task

Data	Analysis from
- teacher's speeches during collective meetings - the research lesson	- *a priori* analysis of the *prescribed* task - modifications between the *prescribed* task and the *conducted* task

For each lesson, we identified the professional acts and the knowledge used by the teacher for the representation of the *prescribed* task, and the teacher's mathematical analysis.

Point 5 (Figure 2) presents the analysis of the redefinition of the *represented* task from these modifications (arrow b') and the *represented* task (arrow b). The data are the teacher's speeches during collective meetings enacted during steps 2 and 4 of a LS cycle, and the research lesson (see Table 5).

Table 5. Analysis of the redefinition of the *represented* task

Data	Analysis from
- teacher's speeches during collective meetings - the research lesson	- modifications between the *prescribed* task and the *conducted* task - the *represented* task

This method is also used for the lessons observed before and after the LS process, but with different data. The data for analyzing the *prescribed* task are made of the problem and the indications from the teacher handbook. The data for analyzing the representation and the redefinition come from the informal meetings after the lessons.

Finally, the process of modifications between the *prescribed* task and the *conducted* task was analyzed for each lesson: the way how the teacher modified the *prescribed* task, the sources of this process of modifications, and the effect of LS on this process. Then, the evolution of indicators was analyzed according to the levels of development and in order to describe evolution of practices during the LS process. In the next part, this method is applied for each lesson in order to analyze Océane's practices: 1. Some elements of the *prescribed* task, *A Priori* Analysis, background; 2. Some elements of the *conducted* task, *A Posteriori* Analysis; 3. Main modifications between the *prescribed* task and the *conducted* task; 4. Representation of the *prescribed* task; 5. Redefinition of the *represented* task; and Process of modification.

In the following part, we present some elements of analysis for the four observed lessons and in the next part the results of these analyses.

Analysis

Lesson before the LS process

1. *Prescribed* task. The lesson before the LS process was based on a game in which students had to form the closest number to a target number using four given digits (Danalet, Dumas, Studer & Villars-Kneubühler, 1999b, p. 95). The knowledge at stake in the problem is the place value, the comparison of numbers, and the estimate or the calculation of the difference between two numbers. Furthermore, students had to apply a strategy to form numbers.

2. *Conducted* task. We highlighted some elements of the *a posteriori* analysis from three characteristic extracts of the lesson. In the extract A, the target number was 3621 and a student formed the number 4189. Océane asked him if he could form another number closer to 3621 with the digits 4, 1, 8, and 9.

> Charles: I don't think so, because if I place one thousand, one thousand four hundred eighty-nine, there are two numbers, it deviates from two thousand. And, if I place eight thousand nine hundred doesn't matter, so it exceeds.
>
> Teacher: All right, I will listen to another student.
>
> Extract A – Lesson Before the LS Process

During this whole class discussion, students could expose and explain their strategies. Charles' strategy used place value and order of magnitude. This strategy allowed him to choose the digits and to form the closest number to the target number. During the lesson, Océane did not use or explicit this strategy, and she did not explain the knowledge: the place value and order of magnitude. Furthermore, she modified the game rules, and she did not use and did not explain another strategy of choice to form number. In the extract B, another student explained a strategy of subtraction to decide if a number is (or not) the closest number to a target number.

Arnaud: I think I must make a calculation.

Teacher: All right.

Arnaud: I think for example, Elodie says like this: four thousand two hundred sixty-six minus three thousand six hundred twenty-one.

Teacher: All right.

Arnaud: And, the number which is the smallest, the smallest, so, it's the closest (*to the target number*).

Teacher: All right, very well. […]

Extract B – Lesson Before the LS Process

After the explanation of these two strategies, Océane concludes by "all right," but she explained that only one of the two strategies allowed to validate that the number was the closest number to the target number, by calculating the difference of two numbers (explained in the extract B). She did not compare these two strategies, and she excluded Charles' strategy without explanation.

From the global context and the *a priori* analysis, it was possible to compare these two strategies and to identify knowledge at stake in the two strategies (subtraction, place value and order of magnitude). Charles' strategy is more efficient than subtraction regarding time and cognitive costs.

For this lesson, it was difficult (but not impossible) to manage institutionalization because of her management of time, and the difficult and long game rules. A possible institutionalization would have been to set up a strategy to choose the closest number, based on place value and order of magnitude (see extracts A and B). For example, from the Charles' strategy, to form the closest number to the target number (or to determine which one is the closest number), one should choose first the digit of thousands.

3. Main modification between the *prescribed* task and the *conducted* task. The main modification was the modification of the game rules: to choose the target number (in rolling dices) first and second, to choose the figures on the tokens to form the closest number of the target number. Océane asked students to choose first the figures on the tokens and second to form the target number with rolling dices. She modified these game rules during the lesson according to Charles' proposition and in order to make game fairer. These new game rules change the mathematical strategies and mathematical learning.

Charles: Miss, we should do in the other way round. First, we pick four tokens and then we roll the dice. Because if we already know the number and then if we take all the numbers which correspond well… So it is a little of cheating.

Teacher: But, in the problem, it is marked before. But, maybe that by seeing this game and by saying alternately, the first ones took the right numbers and then the last one is annoyed. […] Couldn't we make exactly the target number later? It would be more fair…

Students: Yes.

Teacher: For the last person who is going to take every token.

Extract C – Lesson Before the LS Process

The teacher seemed to modify the game rules without taking the effects on mathematical learning into account, but rather aspects of equity (see Extract C).

4. Representation of the *prescribed* task. During informal meetings, Océane explained that she prepares every lesson before teaching and does a mathematical analysis, even if not necessarily written: she identified the learning objective, the place of the activity in a learning sequence. She also plans some pedagogical aspects: the succession of individual or group work, and whole class discussions.

For the representation of the *prescribed* task, Océane did not take the mathematical knowledge (place value) into account (see Extract A). In her mathematical analysis, she took into account a single strategy of validation: the subtraction (see Extract B). Her mathematical analysis was incomplete for this lesson because only a single strategy was taken into account, and every knowledge was not identified (place value).

5. Redefinition of the *represented* task. In the redefinition of the *prescribed* task, **t**he participation of every student is more favored than the explanation of mathematical knowledge (example Extract A "all right, I will listen another student" and the teacher did not explain the Charles' strategy).

The process of modifications between the *prescribed* task and the *conducted* task had its origins in her taking into account the students' activity during the lesson (that's why she modified the game rules). This taking into account unfolded during the lesson. The aims of these modifications were to promote the game aspect and the social aspect at the expense of the explanation of mathematical knowledge.

Lesson during cycle *a*

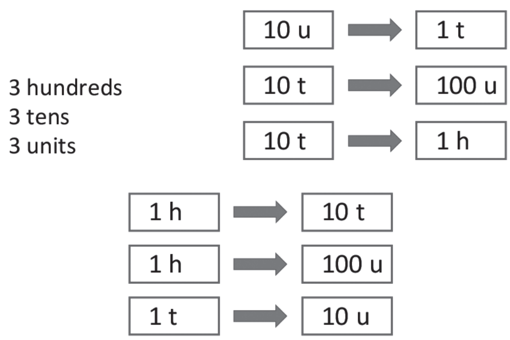
1. *Prescribed* task. This lesson took place during the Cycle *a* of the LS process. The LS group worked about numeration during four collective meetings, then a first research lesson took place in the class of another teacher, observed by Océane and the LS group. Just after the first research lesson, the LS group worked on the observed difficulties from students' activity and the mathematical game "Strange game of Goose…". Then, the LS group decided to teach again a new version of this research lesson and mathematical game. Every teacher like Océane taught the lesson with the liberty to modify the game and/or the lesson plan. The aims were to improve the next research lesson in relation to observed difficulties and problems during the first research lesson.

The mathematical knowledge aimed by the mathematical game is numeration: the value of a digit according to its position (Batteau & Clivaz, 2016; Clivaz, 2016). The LS group planned to manage institutionalization with a decontextualization of knowledge: the two equalities 10 units = 1 ten and 10 tens = 1 hundred.

This game aims to practice trades of cards between players and the banker in order to give exactly the sum of points indicated on the cases. At the end, the players have to change with the banker 1 card "1 ten" against 10 cards "1 unit," or 1 card "1 hundred" with 10 cards "1 ten."

2. *Conducted* task. During this lesson, Océane wrote on the blackboard the contextualized knowledge (representing cards see Figure 3) instead of the decontextualized knowledge planned by the LS group.

Figure 3. Reconstitution of the blackboard – lesson of Cycle *a*



Océane wrote arrows to symbolize the trade between cards (for example, 10u corresponds to 10 cards "1 unit" and 1t corresponds to 1 card "1 ten") and she certainly framed 1t, 1h, 10u, 10t, 100u to symbolize the form of cards. At the end of this lesson (see Extract D), a student, Elodie, asked her "but how come that a ten is equal to ten units, miss?".

Teacher: So… What did you find the most complicated in this game?

[…] Then, is this game too easy? Very easy? I don't think so… because you had a lot of problems with trades… I thought that it was going to be simpler… We should make trades with thousands… And we stop to hundreds. And even then, I observed that there were some problems… We couldn't always find that one ten were ten units… Or one hundred, ten tens.

Elodie: But how come a ten is equal to ten units, miss? […]

Teacher: If we look with the orange material (*see Figure 4*), you're right. We will be concrete. You're right if you take ten units like that.

Elodie: Yes, it's worth…

Teacher: t's worth one ten. (*Océane shows one ten with the material; see Figure 4.*)

Elodie: Yes. […]

Teacher: Ten tens? One hundred.

Elodie: Um...

Teacher: Ten hundred. (*Océane shows a cube; see Figure 4.*)

Elodie: Thousand.

Teacher: One thousand. If you want to change ten tens, you can change with? (*Océane shows a plate of hundred; see Figure 4.*)

Elodie: One hundred. […]

Teacher: And you can exchange ten tens with?

Elodie: One hundred. No, one unit.

Extract D – Lesson of Cycle *a*

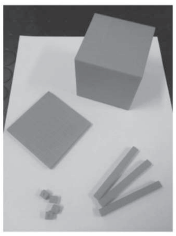


Figure 4. Numeration material used during Océane's Lesson

Océane took the material for numeration (see Figure 4) to make a link between the trades in the game and the material for numeration, but she did not decontextualize the knowledge aimed by the game: the place value. She used the verbs "exchange" or "change" that are contextualized in the game and not "be equal" that is in a mathematical context. Furthermore, her interactions with students during the lesson and what she wrote on the blackboard were contextualized in the game.

3. Main modifications between the *prescribed* task and the *conducted* task. For this lesson, Océane anticipated modifying the *prescribed* task about the aesthetic game board: she modified the colors of cases on the game board in order to help students to apply the game rules (red, blue, and yellow instead of black, white, and gray). Another modification was: she did not write mathematical equalities but only trades with arrows (see Figure 3 and Extract E).

Océane: Maybe because the cases are black, gray and white. That's only the numbers… without thinking if it's the banker who must give, and the black case, that's it, and the white case, that's it. […] *asked to the teacher who taught the first research lesson*): Well, if you had to introduce colors, it would have been longer or not? If there were colors instead of gray and white, did you need more time to explain or not?"

Extract E – Collective meeting before this lesson

Océane: In relation to the last meeting, I changed nothing. Except that I changed the colors. On the game board, red and blue, in order to distinguish when the players must give cards and when the banker must give cards. Some children made a mistake anyway. So the colors… maybe it helped them, maybe not…

Collective meeting after this lesson

Océane did the modification about colors of the game board according to her understanding of the instructions given by the facilitators for this lesson. But she did another important modification about mathematics and institutionalization without realizing it: "in relation to the last meeting, I changed nothing" (see Extract E).

4. Representation of the *prescribed* task. Océane considered the mathematical knowledge only in the context of the game, thus she did not link trades and mathematical equality (see Figure 3 and Extract D). In Océane's mathematical analysis, the aim of the game is the contextualized knowledge useful for the game (illustrated with the blackboard: Figure 3) and the following extract of a collective meeting: "Because the purpose of the game is that, it's to know, well… one ten are changing against ten units."

Her analysis took priority over the collective analysis of this lesson: indeed, she should have managed institutionalization according to the lesson plan and she modified it without realizing it.

5. Redefinition of the *represented* task. Océane used the material for numeration to explain trades in the game (see Extract D) and she wrote on the blackboard only trades useful for the game (see Figure 3).

The process of modifications between the *prescribed* task and the *conducted* task had its origins in the taking into account of the students' activity during the lesson and her representation of the *prescribed* task. This taking into account unfolded during the lesson. The aims of these modifications were to adapt her teaching to students: she said that trades in the game were difficult for them.

Research lesson – Cycle b

1. *Prescribed* task. The research lesson of Cycle *b* was about the geometric transformations (especially the isometries) and the institutionalization. The research lesson was based on the activity "In the aquarium". Part 1 of this lesson was based on the construction of figures (fish) in a grid and every figure must be in a different position. Part 2 of this lesson was based on a whole class discussion in order to manage institutionalization. This institutionalization was about the names of isometries (symmetry, rotation, translation), with an example for each isometry, and the properties of isometries (superimposition of figures and conservation of measures).

2. *Conducted* task. During this lesson, Océane managed to carry out institutionalization. Indeed, the lesson was observed by the LS group and the institutionalization was expected in the lesson plan.

> Teacher: (*Océane placed two fishes by symmetry*) So, how do we remember? We did something like that, when we placed something in front and it's turned over.
>
> Sam: That's a mirror.
>
> Teacher: Yes, that's a mirror effect.
>
> Student: Axis of symmetry.
>
> Teacher: Right an axis of symmetry. […] You make an axis of symmetry. […]
>
> Do we make just one rotation? Do we make just one symmetry? Or, can we make two things together? I don't know, I wonder… […]
>
> Sometimes, we make a rotation and sometimes we return in other side. We make an axis of symmetry.
>
> Extract F – Research Lesson – Cycle *b*

During a whole class discussion (see Extract F), Océane asked students the names of the isometries and institutionalized it according to the lesson plan. She drew an example for each on the board but she did not institutionalize the properties of isometries as planned (see Figure 5). She used the terms "same figures" instead of "super imposable figures" during the lesson. The terms "same figures" are less specific than "super imposable figures" because "same figures" can name similar or super imposable figures.

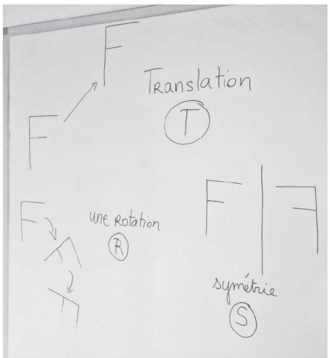


Figure 5. Reconstitution of the board – research lesson of cycle *b*

3. Main modifications between the *prescribed* task and the *conducted* task. The main modifications of the *prescribed* task were: about the problem, Océane asked students to recognize and identify symmetry, rotation, and translation, but also the composites of two isometries (symmetry and rotation, symmetry and translation) (see Extract F). Thus, she took the composite of two isometries into account during whole class discussions and in her mathematical requirements to students (see Extract F). Another modification concerned the whole class discussion: she did not explicit the properties of isometries as planned by the LS group, she did not write the properties of isometries, and she changed the example for the rotation with two images of the initial figure instead of one (see Figure 5).

4. Representation of the *prescribed* task. Océane confused symmetry and its axis of symmetry. During the collective meeting after the lesson, another teacher (Caroline) underlined it, Océane agreed and completed in saying that she rectified it (confirmed notably by the board, Figure 5).

Caroline: There is just some stuff in languages. You don't make an axis of symmetry. You make symmetry. You don't make an axis of symmetry.

Océane: Yes. I said: "an axis of symmetry".

Caroline: Then the students applied: "we made an axis of symmetry."

Océane: Yes and after I corrected it. [*see on the board - Figure 5*]

Extract G – Collective meeting just after the research lesson

Océane realized this mistake during the collective meeting after the lesson. She also confused an isometry and the image of a figure using isometry: she did not realize it certainly because it was also the point of view of the LS group. She identified the composite isometries: symmetry and translation (or rotation). Thus, her analysis deepened those collective. Her knowledge about isometries is in the origin of her representation of the task (see Extract F).

5. Redefinition of the *represented* task. Océane asked students to identify the composite isometries (see Extract F). She built this mathematical knowledge from the analysis of students' productions between the phases 1 and 2 of the research lesson. In her redefinition, Océane confused an isometry and the image of the figure using isometry as the LS group. For example, in the Extract F, when she said "we make a rotation" instead of "we draw the image of a figure by rotation."

The process of modifications had its origins in the taking into account of the students' activity during the lesson and her representation of the *prescribed* task. Océane has taught this problem many times and she said she was satisfied by students' activity: the number of fishes produced by students and the speed of the lesson (Extract H; Edith is another teacher).

> Océane: I never saw as many different fishes produced during one lesson
>
> […]
>
> It's clear that the stencil allowed… the lesson occurred faster. Without this stencil, it would have been more laborious, I'm sure and I'm not mistaking on this.
>
> […]
>
> Edith: At the end, we can say three terms of vocabulary and then to the worst, without the terms of vocabulary, there is the idea of movement. Me… during two lessons I never…
>
> Océane: Yeah… every time I taught this lesson I was frustrated because it didn't progress… and some students made only two fishes.
>
> Extract H – Collective meeting just after the research lesson

This extract underlines the fact that the LS group identified as important elements: the teaching of vocabulary and the idea of movement associated to an isometry, but the group did not underline the lack of the mathematical properties (neither during another collective meeting).

Lesson after LS process

1. *Prescribed* task. During Cycles *c* and *d*, the LS group worked on problem-solving and how to help students represent a problem. The LS group relied on an article (Julo, 2002) in which the main idea was explained during a collective meeting.

> Facilitator 1: (*quoting Julo*) "This help doesn't give clues about the answer, doesn't guide to a strategy and doesn't suggest a modeling." But it's difficult to achieve, it's written just after that. It is an ideal […] but if we don't follow this ideal, it means that we do precisely the part that students have difficulty with.
>
> Extract I – Collective meeting during Cycle *c*

The research lesson of cycle *d* was based on a problem-solving activity: examine the matchstick pattern represented below (Figure 6). How many matchsticks are needed to align 99 squares? (Danalet, Dumas, Studer & Villars-Kneubühler, 1999a, p. 187).

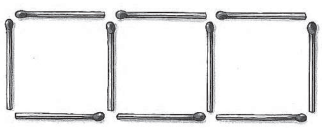


Figure 6. Matchstick problem

The mathematical function at stake was $u(n)=3n+1$, where n is a whole number and corresponds to the number of squares.

The LS group worked on this problem focusing on how to help students to represent and model this problem. During the lesson observed after the LS process, Océane chose a problem-solving lesson. She explained it during a collective meeting at the end of LS process (Anaïs is another teacher).

Océane: There is a lot of problem-solving which I think oh I don't dare to try [...] I think, this year with my students, I take the textbook and I do a lot of things I never did before.

Anaïs : Oh, you dared.

Océane: Yeah, I did.

Extract J – Collective meeting during Cycle *d*

This lesson after the LS process was based on the following problem:

Fold a strip of paper in half. Here are two parts.

Fold a strip of paper in half, then a second time. Here are four parts and so on.

How many parts are there with a folded strip of paper ten times?

(Danalet, Dumas, Studer & Villars-Kneubühler, 1998, p. 96)

The aim of this problem is to develop reasoning capacities and research strategies. In this problem, the students should go from handling to representation in order to predict the result of acts. With the "expert" strategy, to find the number of parts when the strip of paper is folded n times, the students should calculate the product $2\times2\times\ldots\times2$ with n factors 2.

If we folded the strip of paper 1 time: there will be 2 parts.

If we folded the strip of paper 2 times: there will be $2\times2=4$ parts.

> If we folded the strip of paper 3 times: there will be 2×2×2=8 parts.
>
> And so on, if we folded the strip of paper n times: there will be 2×2×….×2 parts (with n factors 2).

The function is power of two but it is not accessible for these students (8/9 years old). However, the product 2×2×….×2 with n factors 2 was knowledge used in this "expert" strategy and was accessible for these students, with n = 1, …,10.

This problem presented similarities with the research lesson of Cycle *d*: two functions explained the mathematical structure of these problems, these functions were not used and not aimed for the students. It was necessary to model these two problems to anticipate the result of an action.

2. *Conducted* task. Océane proposed her own mathematical modeling to students: she ordered students to complete a table. Thus, she took over the modeling of this problem instead of leaving students find their own, while it was the aim of the problem. She did not explain neither her modeling during the lesson, neither the links between different mathematical strategies. Furthermore, she reduced the problem to calculations of doubles of numbers as in this characteristic extract of the lesson. The extract K took place during a whole class discussion, after a research moment.

> Teacher: Doubles. Here, we double. We double every time. The double of two, four. The double of four, eight. The double of eight, sixteen. The double of sixteen, thirty-two. The double of thirty-two, sixty-four. The double of sixty-four? All right? So Nadège, the double of sixty-four is? It folds in seven times. […] It's as if we calculate sixty-four plus sixty-four. Is it? (*Nadège looks at all the folds in her strip of paper*.)
>
> Nadège: One hundred twenty-six… one hundred twenty-eight.
>
> Teacher: Great. […] Next, Luc?
>
> Luc: Two hundred fifty-six.
>
> Teacher: Very well. Yes? If we fold it nine times, it should be?
>
> Romuald: Five hundred six.
>
> Extract K – Lesson After the LS Process

When Océane prepared her lesson, she did not identify the mathematical knowledge: the product of 2×2×2×…×2 with n factors 2 when the strip of paper is folded n times (neither power of two). She validated students' strategies only with calculations (see extract K), and she did not link strategies together. In this extract, she said, "the double of sixty-four is? It folds in seven times." However, she did not explain why it is necessary to multiply by two when the strip of paper is folded half. Her strategy of doubling could not allow to solve directly the problem. With her strategy of doubling, in order to find the number of parts with a strip of paper folded ten times, it is necessary to know the answer when the strip of paper is folded nine times (and eight times, …, until twice). With the "expert" strategy, one only needs to calculate directly the product 2×2×….×2 with ten factors 2. During an informal meeting, Océane explained that she chose this problem to introduce the multiplication, whereas she reduced the problem to calculation of double of numbers with addition ("the double of sixty-four is? It folds in seven times. […] It's as if we calculate sixty-four plus sixty-four") or multiplication by two.

For this lesson, it was possible to manage institutionalization: for example, if we double a number or we add twice the same number, we will obtain the same result. The teacher did not highlight it, neither on the blackboard nor even orally, during the lesson (see Figure 7). Another institutionalization could have been to say that to determine the number of parts when the strip of paper is folded 10 times, one should calculate 2×...×2 with ten factors 2.

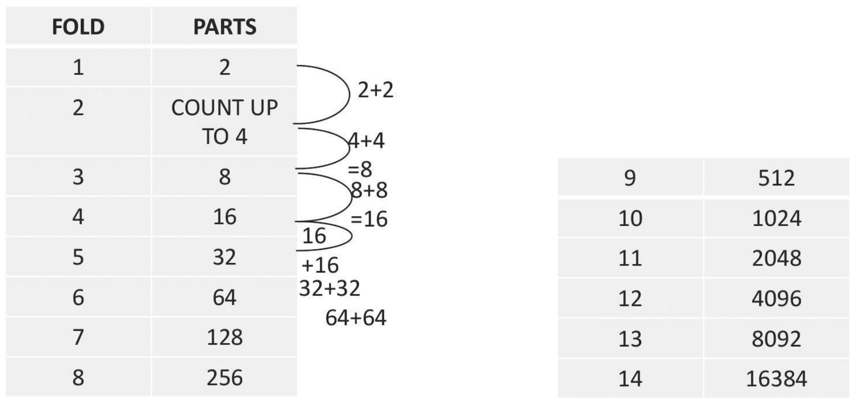


Figure 7. Reconstitution of the blackboard – lesson after the LS process

3. Main modifications between the *prescribed* task and the *conducted* task. Océane took over the modeling of the problem: she realized a two-column table, then the students had to complete it by calculating doubles (see Extract K). Thus, she modified the aim of the problem and also the problem itself.

4. Representation of the *prescribed* task. The issue of the problem, the modeling is taken over by the teacher. The mathematical knowledge at stake in the problem is multiplication and doubling of a number for Océane (see Extract K). For this lesson, her mathematical analysis was in contradiction with the teacher handbook about modeling.

Using a similar problem-solving activity than the research lesson of the Cycle *d*, Océane did not identify the mathematical function at stake (the power of two). Instead of helping students model the problem themselves as worked during the collective meetings, she took over this task.

To conclude, concerning the representation of the *prescribed* task, Océane took the freedom in relation to the institutional constraints from the mandatory textbooks.

5. Redefinition of the *represented* task. Océane took over the modeling of the problem before this lesson. During this lesson, she taught some vocabulary ("double of") and she focused on calculations (see Extract K).

The process of modifications between the *prescribed* task and the *conducted* task had its origins in her representation of the *prescribed* task and not in students' activity. The issue of the problem is not the same for Océane (involving the multiplication) and for the designer of the problem (modeling). She has been taught this problem for the first time the last year, and a second time during this lesson, that is why she decided to prepare by anticipation a two-column table to fill in by students.

Main results

We summarized the results of the analysis in levels of development (see Table 6).

Table 6. Results of analysis in levels of development of practices

	Before LS process	During LS process			After LS process	**Levels of development of practices**
	Lesson	Lesson cycle *a*	Research lesson cycle *b*		Lesson	
			part 1	part 2		
Scholarly peace (social peace and students' enrollment to teacher projects)	yes	yes	yes	yes	yes	Level 1 achieved
Problem with mathematical substance	no	yes (chosen by the LS group)			yes	Level 2 achieved
Actual time of research for students	yes	yes	yes	yes	yes	
Whole class discussion with presentation of students' answers and strategies	yes	yes	yes	yes	yes	Level 3 achieved
with validation	yes	yes	yes	yes	yes	
with explanation of students' strategies	yes	yes	yes	yes	yes	
Whole class discussion with the ranking of students' strategies by the teacher	no	no	no	no	no	Level 4 not achieved
Contextualized synthesis	no	yes	yes	yes	no	
Institutionalization of knowledge or method at stake	no	no	no	yes	no	Level 5 not achieved

The level 1 is achieved because in Océane's speeches there were few reminders (between 1% and 8% of the working time), the students adhered to the teacher projects, and the students and teacher speaking was respected. For the level 2, Océane managed research moments for students (up to 50% of the working time). She dared to teach problem-solving after the LS process. Furthermore, she provided collective helps to students during research moments in particular with the two-column table to fill in (it was a help to represent the problem according to the teacher). She changed her way of teaching and also her representation of teaching (see Extract L; Valentine is another teacher).

Facilitator 1: (*quoting the teachers' guide*) […] students must seize the mathematical situation without anything given to them about the way to answer. And solutions should be discussed and evaluated by the students themselves [...]. We often hear that, with the goal for students to solve problems, the teacher shouldn't act, shouldn't say anything to the students. You heard that, I guess.

Valentine: Yes, of course.

Océane: It must come from the students.

Extract L – Collective meeting (during the Cycle *c*)

Océane said, "It must come from the students." In the French-speaking part of Switzerland, the pre-service training (followed by Océane) and the mandatory textbooks are based on socio-constructivist approach. Thus, in her representation of teaching, the teacher should not act during research moments, the mathematical knowledge must result from students, and the students should work independently in a group (or individually) with little help from the teacher. After Cycles *c* and *d*, Océane realized that the teacher should help students represent the problem. Representing a problem means that the students should understand the problem and the teacher should provide hints that help students start the solving process. Level 2 is achieved and is marked by the evolution of her practices.

Regarding levels 3 and 4, Océane managed whole class discussions during each lesson. For the lessons before and after the LS process, she did not organize overview, summary of mathematical strategies with a comparison and a ranking of them, neither institutionalization. For the lessons of Cycles *a* and *b*, she managed contextualized summaries. However, during the whole class discussions, she did not rank students' mathematical strategies.

Océane did not identify the mathematical knowledge used in the students' strategies during the whole class discussions. She validated students' strategies principally with calculations. She did not compare and explain links between students' strategies, she did not act directly on the mathematical strategies, but she acted and controlled only the solutions during the whole class discussions. The extracts A, B and K are examples of these characteristics that were observed in her practices. Level 3 is achieved but not level 4.

Regarding level 5, Océane managed an institutionalization for the research lesson and contextualized synthesis (level 4) for the lessons planned by the LS group, and she did not manage the ranking of students' strategies. In her practices, she said that she did not manage institutionalization but only oral assessments. Furthermore, we analyzed that these oral assessments were without decontextualization of knowledge for the lessons before and after the LS process, and for the lesson of Cycle *a*.

This analysis in levels of development of practices highlighted that Océane's practices are categorized in level 3 of development of practices. She did not manage neither whole class discussions with ranking of students' strategies, neither institutionalization (apart from the institutionalization planned for the research lesson). Our conclusion is that this is due to some resistance in terms of professional acts for managing institutionalization: the LS group focused on the importance of the institutionalization in teaching during a whole cycle (8 collective meetings and 1 research lesson) and the LS group chose to manage it during a research lesson. Océane did not apply this professional act during the observed lesson after the LS process. This resistance to change in the teacher's practices to manage institutionalization was also observed in other researches (Charles-Pézard et al., 2012; Peltier-Barbier et al., 2004). The analysis in levels of development highlighted the resistance in her practices especially for the professional acts during whole class discussions with ranking of students' strategies, contextualized synthesis and the process of institutionalization.

To explain and understand the result of the analysis in levels of development, we added elements about her practices before and after the lesson. Does this observed resistance in Océane's practices come from her representation and/or from her redefinition of the *prescribed* task?

We summarized the results of this analysis. For each lesson, Océane modified the problem at different levels (game rules, aesthetic elements, aim of the problem) for observed lessons. She represented the *prescribed* task according to her mathematical analysis. Before teaching each observed lesson, she prepared her lesson and realized mathematical analysis. We underlined gaps between her representation of the *prescribed* task and the *prescribed* task for each lesson. These gaps came from her mathematical analysis. For the lessons of Cycles *a* and *b*, we could suppose that there were fewer gaps than for the other lessons because the *prescribed* tasks were elaborated and explained by the LS group. It should not be possible to explicit all what is expected by the *prescribed* task during the collective meetings before the lesson. Then, the LS process was seen as cycles in which research lessons are not an aim in itself or an end. It means that the *prescribed* task was also discussed after the lesson, according to the modifications of the lesson plan and students' strategies or difficulties that occurred during the lesson.

In her redefinition of the *represented* task, Océane modified the mathematical problem according to her taking into account of students' activity, her mathematical analysis and her representation of the task. In her redefinition of the *represented* task, she encouraged the social dimension between students (equity between students and time of speaking for each group) rather to decontextualize knowledge (for example see Extract A).

The process of modifications between tasks had its origins in her taking into account the students' activity and her representation of the *prescribed* task. This taking into account unfolded during the lessons (for the lesson before and during the LS process) and by anticipation, before the lesson (for the lesson after the LS process). During the lesson after the LS process, Océane did not take into account students' activity, she took it into account only when she taught this problem for the first time. Then, she adapted her teaching when she taught this problem for the second time: she took over the modeling and imposed a two-column table to fill in. The LS process modified this source, Océane did not take into account the students' activity during the lesson but by anticipation in order to help students represent the problem for the lesson after the LS process. The LS process modified the taking into account of this source (students' activity) in Océane's practices.

Conclusion

This case study proposed an analysis of particular teacher's practices before, during and after a LS process. Océane is an experienced teacher; she even regularly has trainees in her class. Therefore she can be seen as a reference in terms of practice for the pre-service training institution (Charles-Pézard et al., 2012; Peltier-Barbier et al., 2004), but also for the facilitators in this LS process. This case study focused on practices before, during, and after lessons. In the paradigm of our theoretical background, teachers' practices are culturally embedded and form a complex, coherent, and stable system. The LS process provides elements on teachers' practices which highlight this complex, coherent, and stable system of teachers' practices. In particular, our study highlighted that the process of LS as a professional development caused changes in Océane's practices and had an impact on the stability of her practices. Therefore, we provide elements to answer to the research question, "to what extent can the evolution of practices during the LS process be made apparent?" In this case, the LS process developed the *cognitive* and *personal* components in Océane's practices: self-confidence to teach problem-solving. Level 2 of development is achieved and marked by the evolution of practices: Océane dared to teach problem-solving during and after the LS process, and she explained it during a collective meeting. The LS process developed the *mediative* component: she provided collective students' help during research moments. We considered it as an evolution because in the Swiss school system, the in-service training and the mandatory mathematics resources are based on a socio-constructivist approach in which Océane has the belief as the teacher should not intervene during the devolution process and the research time for students. Regarding the research question, "how does the LS process affect the levels of development?" we could observe that the LS process supported level 2, but not levels 4 and 5 of development of practices. It was necessary to have the required mathematical knowledge in order to analyze, compare, and rank students' strategies during the lesson, but also to identify the mathematical knowledge aimed by the problem before the lesson and used in students' strategies during the lesson. This study underlined that Océane realized pertinent mathematical analysis before teaching the lesson: it was the case for the research lesson during the LS process about the composite of isometries. However, for the lessons observed before and after the LS process, Océane could not identify the knowledge aimed by the problem, neither in students' strategies (place value in Charles strategy for the lesson before the LS process and the strategy $2 \times 2 \times \ldots \times 2$ with n factors 2 when we folded n times the strip of paper for the lesson after the LS process). Thus, we did not observe that the LS process supported the evolution in Océane's practices in relation to her mathematical analysis with the identification of knowledge aimed by the problem and also used in students' strategies. Crahay (1989) has underlined that it was more difficult for a teacher to change his/her practice during the class than during his/her preparation. We can suppose, following Crahay, that changing practices during the lesson by analyzing students' strategies and managing institutionalization was more difficult than during the preparation of the lesson with a mathematical analysis of the problem and the identification of knowledge aimed by the problem. The Swiss primary school mandatory textbooks lack mathematical analyses for teachers to use while planning lessons. Thus, this resource did not promote a change in the practices even during the preparation of the lesson for a primary school teacher like Océane.

We provide elements of the answer to the research question, "how can the analysis of the process of modifications between the *prescribed* task and the *conducted* task provide information about a potential development in the teachers' practices?" We argue that the resistance (to change the practices for managing whole-class discussion with ranking of students' strategies and institutionalization) linked to the representation of the *prescribed* task, and the representation of the *prescribed* task relied on the mathematical analysis. Indeed, it could be difficult for Océane to manage institutionalization in relation to her mathematical analysis. Indeed, for the lesson of the Cycle *a*, she only considered the knowledge in the context of the game without decontextualization. For the lesson after the LS process, she did not identify neither the knowledge used in students' strategy (place value), neither the expert strategy that could be institutionalized. She only identified one strategy of validation with subtraction and she changed the issue of the problem (multiplication instead of modeling). We deduced that with her mathematical analysis realized before and during the lessons, it could be difficult for Océane to manage institutionalization, but also to compare and to rank students' strategies. Thus, the absence of institutionalization originated from her mathematical analysis and her representation of the *prescribed* task. In that sense, the results of analysis in levels of development with the observed resistance (for managing institutionalization) could be explained by this analysis in the process of modifications of the *prescribed* task. This study included a didactical and ergonomical approach and highlighted that the gaps between the representation of *prescribed* task and the *prescribed* task could explain the observed resistance in Océane's practices.

Océane was coherent between her discourses on practices and her practices about teaching by problem-solving or managing oral assessments without institutionalization, for example. When the discourses on practices are coherent with the practices, this means that the evolution of practices fits to a professional development, and one of the levers can be a collaborative work (Roditi, 2011). In the case of Océane's practices, this study showed effective professional development in her practices over a long period of time (almost three years). This case study provided elements about how the teacher's practices are modified by a LS process, how the professional development occurred on a long time. In that sense, this study provided elements of understanding about evolution of teacher's practices and professional development, and can contribute to a lack of research in LS field highlighted by Goldsmith, Doerr, and Lewis (2009, 2014). This case study showed some effects of the lesson study process: the teacher dared to teach problem-solving and to intervene collectively during research time in helping the students. But this case study also underlined some difficulties to transfer professional acts worked during the lesson study process, about whole class discussions for contextualized synthesis, rankings of students' strategies, and institutionalization. In order to reinvest and transfer professional acts, didactical and mathematical knowledge worked during the lesson study process, the teachers need useful resources with mathematical analysis of the problems and the identification of knowledge aimed by the problems. The practices can be developed under some conditions such as long-term period and useful resources.

Notes

[1] We translated some elements of the research works (Charles-Pézard, Butlen & Masselot, 2012; Leplat, 1997; Mangiante, 2007, 2012; Peltier-Barbier et al., 2004), but also some extracts of collective meetings and lessons into English.

[2] The devolution is "the activity of the teacher in attempting to induce the student to take on responsibility for a Situation" (Warfield, 2014, p. 16).

[3] "It is the Situation in which the teacher takes the ideas the class has developed, reviews them, shapes them up and if necessary provides them with labels. When it is pertinent, she [the teacher] provides the bridge between the class's production and the concepts and terms accepted by the world at large and in particular the standard curriculum" (Warfield, 2014, pp. 66).

[4] "Mathematical knowledge or method at stake" means the mathematical knowledge or the method aimed by the activity (or problem).

[5] Professional acts are defined as elementary activities taking part to the teacher's activity. The professional acts enable to describe the way how a teacher realizes the process of devolution, regulation, and institutionalization, to describe the different actions which allows him/her to do it, and the mobilized knowledge (Charles-Pézard et al., 2012).

[6] We use the term "knowledge" in the common sense: we distinguish the mathematical and didactical knowledge.

[7] In this particular LS process, the two facilitators were also knowledgeable others in mathematics education and in teaching and learning (Clerc-Georgy & Clivaz, 2016).

[8] Students eight to ten years old correspond to 3rd Grade - 4th Grade (American Elementary School) and 5HarmoS - 6HarmoS (Swiss School System).

[9] See https://www.hepl.ch/files/live/sites/systemsite/files/laboratoire_3ls/plan-lecon-6h-aquarium-v10-labo-3ls-2014-hep-vaud.pdf

[10] See https://www.hepl.ch/files/live/sites/systemsite/files/laboratoire_3ls/plan-lecon-6h-99%20carres-v08-labo-3ls-2015-hep-vaud.pdf

[11] In the French-speaking part of Switzerland, teachers in the primary and secondary schools have mandatory mathematical textbooks and teachers' handbooks.

References

Batteau, V., & Clivaz, S. (2016). Le dispositif de lesson study: Travail autour d'une leçon de numération. *Grand N, 98*, 27-48.

Brousseau, G. (1997). *Theory of didactical situations in mathematics* (N. Balacheff, M. Cooper, R. Sutherland & V. Warfield, Trans.). Dordrecht: Kluwer.

Chappet-Paries, M., Robert, A. & Rogalski, J. (2013). Stability of practices: What 8th and 9th grade students with the same teacher do during a geometry class period? In F. Vandebrouck (Ed.), *Mathematics classrooms. Students' activities and teachers' practices* (pp. 91-116). AW Rotterdam, Netherlands: Sense Publishers.

Charles-Pézard, M., Butlen, D. & Masselot, P. (2012). *Professeurs des écoles débutants en ZEP. Quelles pratiques? Quelle formation?* Grenoble: La pensée sauvage.

Charnay, R., Combier, G., Dussuc, M.-P. & Madier, D. (2007). *Cap Maths CE2. Manuel de l'élève*: Hatier.

Clerc-Georgy, A. & Clivaz, S. (2016). Evolution des rôles entre chercheurs et enseignants dans un processus lesson study: quel partage des savoirs? In F. Ligozat, M. Charmillot & A. Muller (Eds.), *Le partage des savoirs dans les processus de recherche en éducation* (pp. 189-208). Série Raisons Educatives, n°20. Bruxelles: De Boeck.

Clivaz, S. (2015). French Didactique des Mathématiques and lesson study: A profitable dialogue? *International Journal for Lesson and Learning Studies, 4*(3), 245-260. doi: 10.1108/IJLLS-12-2014-0046.

Clivaz, S. (2016). Lesson study: From professional development to research in mathematics education. *Quadrante, XXV*(1), 97-111.

Coulange, L. (2012). *L'ordinaire dans l'enseignement des mathématiques. Les pratiques enseignantes et leurs effets sur les apprentissages des élèves.* (Habilitation à Diriger des Recherches), Université de Bordeaux.

Crahay, M. (1989). Contraintes de situation et interactions maître-élèves: Changer sa façon d'enseigner est ce possible? *Revue Française de Pédagogie, 88*, 67-95.

Danalet, C., Dumas, J. P., Studer, C. & Villars-Kneubühler, F. (1998). *Mathématiques. Livre du maître 3P*. Neuchâtel, Switzerland: COROME.

Danalet, C., Dumas, J. P., Studer, C. & Villars-Kneubühler, F. (1999a). *Mathématiques. Fichier de l'élève 4P*. Neuchâtel, Switzerland: COROME.

Danalet, C., Dumas, J. P., Studer, C. & Villars-Kneubühler, F. (1999b). *Mathématiques. Livre du maître 4P*. Neuchâtel, Switzerland: COROME.

Gersten, R., Taylor, M. J., Keys, T. D., Rolfhus, E. & Newman-Gonchar, R. (2014). Summary of research on the effectiveness of math professional development approaches. *Regional Educational Laboratory Southeast*, 3-15. Retrieved from http://files.eric.ed.gov/fulltext/ED544681.pdf

Goldsmith, L. T., Doerr, H. M. & Lewis, C. (2009). Opening the black box of teacher learning: Shifts in attention. In M. Tzekaki, M. Kaldrimidou & C. Sakonidis (Eds.), *Proceedings of the 33rd conference of the international group for the psychology of mathematics education* (Vol. 3, pp. 97–104). Thessaloniki, Greece: PME.

Goldsmith, L. T., Doerr, H. M. & Lewis, C. (2014). Mathematics teachers' learning: A conceptual framework and synthesis of research. *Journal of Mathematics Teacher Education, 17*, 5-36.

Hart, L. C., Alston, A. S. & Murata, A. (Eds.). (2011). *Lesson study research and practice in mathematics education*: Springer.

Julo, J. (2002). Des apprentissages spécifiques pour la résolution de problèmes? *Grand N, 69*, 31-54.

Lefeuvre, G., Garcia, A. & Namolovan, L. (2009). Les indicateurs de développement professionnel *Questions Vives [En ligne], 5*. Retrieved from http://questionsvives.revues.org/627 doi: 10.4000/questionsvives.

Leontiev, A. N. (1975). *Activité, conscience, personnalité*. Moscou: Edition du progrès.

Leplat, J. (1997). *Regards sur l'activité en situation de travail*. Paris: Presses Universitaires de France.

Lewis, C. & Hurd, J. (2011). *Lesson study, Step by step, How teacher learning communities improve instruction*. Portsmouth, Etats-Unis.

Mangiante, C. (2007). *Une étude de la genèse des pratiques de professeurs des écoles enseignant les mathematiques: Prédermination et développement.* (Doctoral thesis), Université Paris 7.

Mangiante, C. (2012). Une étude de la cohérence en germe dans les pratiques de professeurs des écoles en formation initiale puis débutants. RDM176, 32(3), 1-43.

Murata, A. (2011). Introduction: Conceptual overview of lesson study. In L. C. Hart, A. S. Alston & A. Murata (Eds.), *Lesson study research and practice in mathematics education* (pp. 1-12): Springer Netherlands.

Peltier-Barbier, M.-L., Butlen, D., Masselot, P., Ngono, B., Pezard, M., Robert, A. & Vergnès, D. (2004). *Dur d'enseigner en ZEP. Dur pour les élèves. Dur pour les enseignants. Analyse des pratiques de professeurs des écoles enseignant les mathématiques en réseaux d'éducation prioritaire*. Grenoble: La pensée sauvage.

Perry, R. & Lewis, C. (2011). *Improving the mathematical content base of lesson study summary of results*. http://www.lessonresearch.net/IESAbstract10.pdf.

Robert, A. (2001). Les recherches sur les pratiques des enseignants et les contraintes de l'exercice du métier d'enseignant. *Recherches en Didactique des Mathématiques, 21*(1-2), 57-80.

Robert, A. & Hache, C. (2013). Why and how to understand what is at stake in a mathematics class. In F. Vandebrouck (Ed.), *Mathematics classrooms. Students' activities and teachers' practices* (pp. 23-74). AW Rotterdam, Netherlands: Sense Publishers.

Robert, A. & Rogalski, J. (2002). Le système complexe et cohérent des pratiques des enseignants de mathématiques: Une double approche. *Revue canadienne de l'enseignement des sciences, des mathématiques et des technologies, 2(4)*, 505–528.

Robert, A. & Rogalski, J. (2005). A cross-analysis of the mathematics teacher's activity. An example in a french 10th-grade class. *Educational Studies in Mathematics, 59*, 269-298. doi: 10.1007/s10649-005-5890-6

Roditi, E. (2011). *Recherches sur les pratiques enseignantes en mathématiques: apports d'une intégration de diverses approches et perspectives.* (Habilitation à diriger des recherches), Université Paris Descartes, Sorbonne.

Roditi, E. (2013). Diversity, variability and commonalities among teaching practices. In F. Vandebrouck (Ed.), *Mathematics classrooms. Students' activities and teachers' practices* (pp. 75-90). AW Rotterdam, Netherlands: Sense Publishers.

Rogalski, J. (2003). Y a-t-il un pilote dans la classe? Une analyse de l'activité de l'enseignant comme gestion d'un environnement dynamique ouvert. *Recherche en didactique des mathématiques, 23*(3), 343-388.

Rogalski, J. (2013). Theory of activity and developmental frameworks for an analysis of teachers' practices and students' learning. In F. Vandebrouck (Ed.), *Mathematics classrooms: Students' activities and teachers' practices* (pp. 3-22). AW Rotterdam, Netherlands: Sense Publishers.

Warfield, V. M. (2014). *Invitation to Didactique*. New York: Springer.

Yoshida, M. & Jackson, W. C. (2011). Response to Part V: Ideas for developing mathematical pedagogical content knowledge through lesson study. In L. C. Hart, A. S. Alston & A. Murata (Eds.), *Lesson study research and practice in mathematics education* (pp. 279-288): Dordrecht, Netherlands: Springer.