Using Concept Cartoons to investigate future primary school teachers' pedagogical content knowledge on addition

Usando Cartoons concetuais para investigar o conhecimento pedagógico do conteúdo de futuros professores sobre adição

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Abstract. This paper introduces an exploratory empirical qualitative study that has been carried out with two diverse groups of future primary school teachers (before vs after the attendance of a course on didactics of mathematics). The study uses an educational tool called Concept Cartoons accompanied by a set of six indicative questions as a means of collecting data on pedagogical content knowledge (PCK) in mathematics. In particular, the study focuses on future teachers' written responses to virtual pupils' opinions in a virtual classroom situation related to the algorithm of written addition of natural numbers. The findings reveal ten different code categories of displays of PCK related to knowledge of pupils (three categories), knowledge of tasks (one category) and knowledge of instruction (six categories), some of them related to strong PCK, others to weak PCK. According to the findings, all the categories related to knowledge of pupils occurred in the post-didactic group only, all the categories that appeared only in the pre-didactic group are connected to weak PCK, and all the categories that occurred only in the post-didactic group are connected to strong PCK.
Keywords: Concept Cartoons; future teachers; mathematics education; pedagogical content knowledge; pupils' misconceptions; written addition algorithm.

Resumo. Este artigo apresenta um estudo qualitativo empírico de natureza exploratória conduzido com dois grupos diferentes de futuros professores do 1.º ciclo (antes vs. após a frequência de uma unidade curricular em didática da matemática). O estudo faz uso de uma ferramenta educacional chamada cartoons concetuais, acompanhada por um conjunto de seis questões orientadoras como meio de recolha de dados sobre o conhecimento pedagógico do conteúdo (PCK) em matemática. Em particular, o estudo foca-se nas respostas escritas dos futuros professores em face das opiniões de alunos virtuais, numa situação virtual de sala de aula relacionada com o algoritmo escrito da adição de números naturais. Os resultados revelam dez categorias diferentes de evidências do PCK relacionadas com o conhecimento dos alunos (três categorias), conhecimento das tarefas (uma categoria) e conhecimento instrucional (seis categorias), algumas relacionadas com um PCK forte,

outras com um fraco PCK. De acordo com os resultados, todas as categorias relacionadas com o conhecimento dos alunos emergem apenas no grupo pós-didática, todas as categorias que surgem apenas no grupo pré-didática estão relacionadas com um fraco PCK, e todas as categorias que surgem apenas no grupo pós-didática se relacionam com um forte PCK.
Palavras-chave: cartoons concetuais; futuros professores; educação matemática; conhecimento pedagógico do conteúdo; ideias erróneas dos alunos; algoritmo escrito da adição.

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Introduction

This paper presents an educational tool called Concept Cartoons and its possible use in future primary school teachers' professional preparation. Concept Cartoons are pictures presenting several children in a bubble dialog related to the subject matter that is on agenda in the classroom, where the opinions inside the bubbles might be correct as well as incorrect (Keogh & Naylor, 1993). When Concept Cartoons are used as an educational tool in a primary or secondary school classroom, the teacher usually shows one picture to pupils and asks various questions concerning the opinions inside the bubbles. Such an arrangement appeared to augment motivation of pupils and provoke classroom discussion (Naylor & Keogh, 2013). Since Concept Cartoons present virtual pupils' opinions in virtual classroom situations related to particular content-related topics, they might be also perceived as a special kind of representation of school practice (Buchbinder & Kuntze, 2018), namely as a component of decomposition of practice (Grossman et al., 2009) focusing on various content-related pupils' opinions that might appear in the classroom. Some researchers employ this perspective to use the tool in professional preparation of teachers: mostly for assessing future teachers' subject matter content knowledge (Keogh et al., 2001; Ormanci & Sasmaz-Oren, 2011; Temel & Sen, 2019), much rarely for promoting their pedagogical content knowledge (Depaepe et al., 2018).

In the present empirical study, we focus on Concept Cartoons in relation to pedagogical content knowledge. In particular, we investigate displays of pedagogical content knowledge that appear in responses provided by future primary school teachers when working with Concept Cartoons accompanied by a given set of questions. The participants of the study consist of two groups of future primary school teachers in different stages of their professional preparation: before vs after the attendance of a course on didactics of mathematics.

Theoretical framework

Teachers' knowledge

The study presented in this paper focuses on future primary school teachers' knowledge that would play a role when teaching mathematics in the classroom. Theoretically, it proceeds from Shulman's concept of *knowledge base of teaching* (1986, 1987), namely from the two content-related categories: *subject matter content knowledge* (SMK) and *pedagogical content knowledge* (PCK). In that sense, SMK in mathematics is understood as knowledge needed for one's own learning and performing mathematics (e.g. for learning new mathematical concepts, solving mathematical problems, reading mathematical texts), and PCK in mathematics is understood as knowledge needed for teaching mathematics to others. The relationship between SMK and PCK is close and might differ for different individuals (Ball, Lubienski, & Mewborn, 2001; Depaepe et al., 2015). However, generally, it can be stated that SMK and PCK are two unequal sets with a non-empty intersection.

There are various methods for assessing PCK of future teachers or practicing teachers and these methods use various tools for data collection: interviews with individual teachers, video recordings or direct observations of lessons taught by them, video recordings of discussions among several teachers, teachers' written narratives on critical moments in teaching, etc. (Depaepe, Verschaffel, & Kelchtermans, 2013). In mathematics, an extensive study on SMK and PCK was carried out within a COACTIV research project (Krauss, Baumert & Blum, 2008), followed by other studies (e.g. Kleickmann et al., 2013). The latter study, which focused on the impact that teacher education could have on SMK and PCK, used written tests as a tool for data collection, and distinguished three components of PCK emerging from quantitative data analysis: *knowledge of pupils* (of their strategies, strengths and possible difficulties, conceptions and misconceptions, sources of possible misunderstandings, etc.), *knowledge of tasks* (of various was of solving and their explanations, potential for teaching various topics, etc.) and *knowledge of instruction* (of different didactical models, modes of explanation, educational tools, etc.). Findings of the study show that teachers might gain their PCK from three main sources: from own experience in the role of a pupil, from teacher professional preparation courses, and from own experience in the role of a teacher.

As for the particular mathematical content in the focus of the present study, the topic of the written algorithm of addition of natural numbers belongs to earlier grades of primary school, but many future primary school teachers have weak PCK on this topic due to their struggling to understand and explain issues related to place value (Thanheiser, 2009). Similar weaknesses with the same source appear also with the other written algorithms of operations with natural numbers (Ma, 1999; McClain, 2003). Research has recommended that primary school teacher preparation programs stress the conceptual understanding of place value (Ma, 1999; McClain, 2003) and use numbers of three or more digits within the algorithms (Thanheiser, 2009).

Concept Cartoons

The educational tool called Concept Cartoons appeared in Great Britain in the 90s, as a means to augment motivation and provoke discussion in primary and secondary school science classrooms (Keogh & Naylor, 1993; Naylor & Keogh, 2000). Later on, the tool also permeated to other school subjects, e.g. English language (Turner, Smith, Keogh & Naylor, 2013) or mathematics (Dabell, Keogh, & Naylor, 2008). Each Concept Cartoon is an independent picture of several children in a bubble dialog that focuses on an issue related to the subject matter that is on agenda in the classroom. The issue in focus might be given in a formal school form (in mathematics: e.g. in a form of a presentation of a calculation or an algorithm that is needed to run the calculation, in a form of a statement about the validity of a certain mathematical property), or in an informal form that introduces a certain mathematics-related situation from everyday reality. The opinions given inside the bubbles might be correct as well as incorrect, but also with unclear or conditioned correctness. One of the bubbles is sometimes blank, with just a question mark inside – to boost the belief that there might be other opinions not displayed in the picture yet. No teacher or other adult is present in the picture. For a sample of a Concept Cartoon see Figure 1.

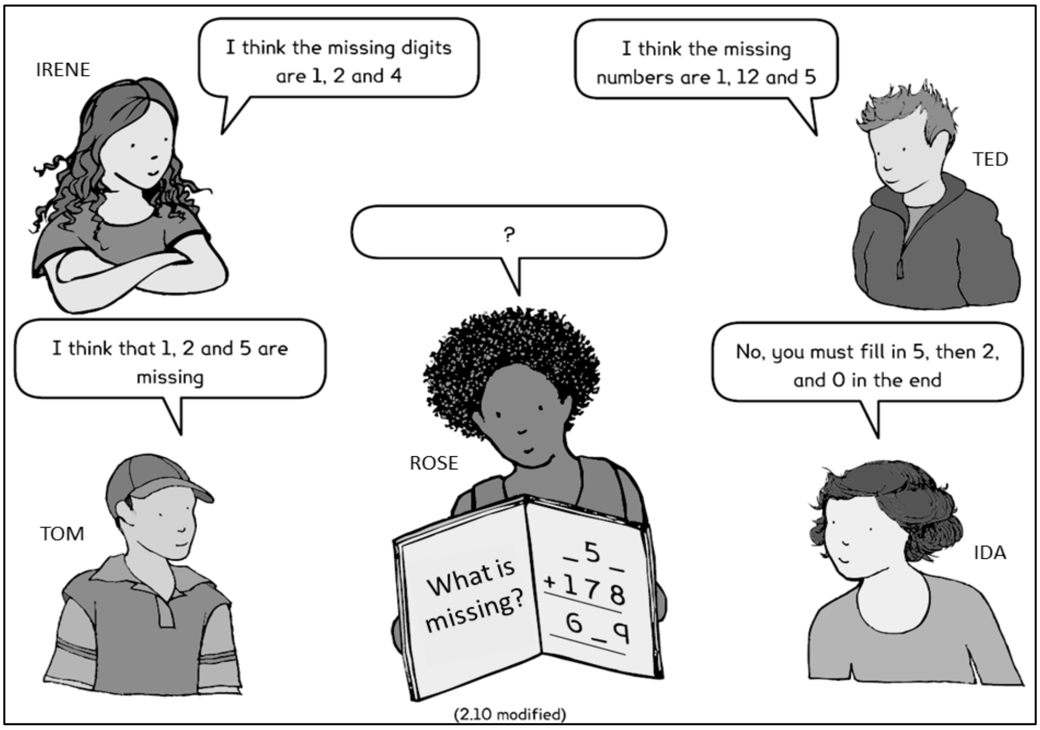

Figure 1. Concept Cartoon on missing digits in the written addition algorithm; template with children, empty book and empty bubbles taken from (Dabell et al., 2008: 2.10)

Summarized, each Concept Cartoon is a picture that presents virtual pupils' opinions in a virtual classroom situation related to a particular content-related topic. In that sense, Concept Cartoons might be perceived as a special kind of representation of school practice (Buchbinder & Kuntze, 2018), namely as a component of decomposition of practice (Grossman et al., 2009) concentrating on various content-related pupils' opinions that might appear in the classroom. For this purpose, the Concept Cartoons might be created towards chosen intentions, by filling the bubbles with particular pupils' conceptions and misconceptions. These opinions in bubbles might be based on own or shared teaching experience, on own or shared observational experience, on textbooks for future teachers and their educators (e.g. Anghileri, 2007; Ashlock, 2010; Hansen, 2013), and on educational research focusing on correct and incorrect solving strategies (e.g. McIntosh, 2002; Ryan & Williams, 2011; Selter, 2001; Thompson, 1994).

When Concept Cartoons are used in a primary or secondary school classroom, the teacher usually shows the picture to pupils, asks questions like "What do you think about it?", "Which of the children are right?", "Why?", and the pupils discuss the answers. The authors of Concept Cartoons ran several empirical studies that confirmed that such an arrangement can augment motivation of pupils and provoke their discussion (Naylor & Keogh, 2013). Thus, Concept Cartoons are able to involve even the pupils that are not used to taking part in classroom discussions, and that a promising design for these purposes consists in splitting the pupils for the discussion into several small groups (Naylor, Keogh & Downing, 2007). The authors of Concept Cartoons did not provide any research on the use of this tool in mathematics classrooms.

Most research on Concept Cartoons focuses on the tool in science classrooms: on the methodology of its use (e.g. Reyes-Roncancio, Romero-Osma, & Bustos-Velazco, 2019; van den Berg, 2013) and on its influence on pupils' knowledge of science concepts (Atasoy & Ergin, 2017; Chin & Teou, 2009; Pekel, 2019).

Some researchers use Concept Cartoons in professional preparation of teachers, mostly for assessing future teachers' SMK on science concepts. Future teachers write their answers to questions like "Which of the children are right?", "Why?", and the answers are assessed either quantitatively by scoring (Keogh, Naylor, Boo, & Feasey, 2001; Ormanci & Sasmaz-Oren, 2011), or qualitatively by content analysis of the text (Temel & Sen, 2019). Research studies focusing on Concept Cartoons in relation to PCK of future teachers are rather scarce; Depaepe et al. (2018) used Concept Cartoons as one of the educational tools in a university preparation course on didactics of mathematics. After the course, the group of future teachers that regularly worked with video recordings and Concept Cartoons showed stronger SMK and PCK than the group that did not use them.

This paper presents a different methodological approach to Concept Cartoons in relation to future teachers and their PCK, an approach that focuses on qualitative assessment of future teachers' PCK and uses Concept Cartoons accompanied by a set of questions as a tool for collecting written data. This methodology has been derived on the basis of a qualitative exploratory study with more than 150 respondents – future teachers in various stages of university preparation (Samková, 2018). Within this approach, Concept Cartoons are submitted to future primary school teachers printed on the upper third of a sheet of A4 paper, and the future teachers are asked to answer a set of six indicative questions that are related to didactical aspects of the pictured situation (see Table 1). For this purpose, the individual children in the picture are labelled by names (similarly as in Figures 1, 2).

Table 1. The set of indicative questions

1)	Which child do you strongly agree with?
2)	Which child do you strongly disagree with?
3)	Decide which ideas in bubbles are right and which are wrong. Give reasons for your decision.
4)	Try to discover the cause of the mistakes.
5)	Advise the children who made the mistakes how to correct them.
6)	Propose two texts that could be filled in the blank bubble – one correct and the other incorrect.

The respondents work individually and write their answers on the sheet of paper, below the Concept Cartoon picture. The respondents are given enough time for work, usually about 20 to 30 minutes per picture. The joint discussion of the Concept Cartoons follows only after the respondents return the sheets where the individual answers to the questions are written down. Sometimes, the joint discussion is postponed to the next seminar so that there is enough time to analyse the answers and prepare additional questions or additional materials for the discussion (textbooks, records of pupils' solutions related to the task, classroom videos, etc.).

As shown in Samková (2018), such an arrangement allows to assess informal foundations of PCK of future primary school teachers that have not attended any course on didactics of mathematics yet, and distinguish the three components of PCK given by Kleickmann et al. (2013).

Methodology

The present exploratory qualitative empirical study focuses on displays of PCK that appear in responses that future primary school teachers provide when working with Concept Cartoons accompanied by a set of indicative questions. In particular, the study addresses the following research question: What displays of PCK related to the algorithm of written addition of natural numbers can be observed in written responses to Concept Cartoons provided by future primary school teachers in different stages of their teacher preparation (before vs after enrollment in a course on didactics of mathematics)?

Participants

The participants of the present study were two groups of future primary school teachers – students of primary school teacher professional preparation in a Faculty of Education in the Czech Republic. Their professional preparation lasts five years and covers all primary school curriculum subjects. The group called *pre-didactic* consisted of 29 second-year students that had not attended a course in didactics of mathematics yet, and the group called *post-didactic* consisted of 12 third-year students that had already attended the course in didactics of mathematics. The students in this latter group had started observing regularly instances of school practice, as a part of the training program, but they had not yet experienced any teaching practice. As for the content perspective, the participants from the pre-didactic group had recently completed the "Natural numbers" part of a compulsory course in mathematics, where they discussed the structure of natural numbers, the structure of the decimal system (the place value system), algebraic operations on natural numbers and their properties; data collection took part in a seminar that belonged to this course in mathematics. The participants from the post-didactic group had recently completed the "Natural numbers" part of a compulsory course in didactics of mathematics, where they discussed general didactical issues related to the primary school mathematics curriculum (various educational approaches, the content of the primary school mathematical curriculum, mathematical knowledge of pupils entering primary school, methods of preparation for own teaching practice in mathematics), as well as issues related particularly to the "Natural numbers" topic (typical learning tasks, various representations and modes of explanations, educational tools, some common pupils' strategies, etc.).; data collection took part in a seminar that belonged to this course in didactics of mathematics.

In both cases, the study involved, as participants, all the future teachers that were present at the particular compulsory seminar where data collection took place. None of the participants had been previously acquainted with the format of Concept Cartoons.

Data collection

The data collection tool in this study took the form of a Concept Cartoon picture from Figure 2, accompanied by the six indicative questions from Table 1. For each of the groups of participants, data collection took place at one of the compulsory seminars. The participants were asked to work individually and answer the questions in written form. The Concept Cartoon from Figure 2 shows a completed algorithm of a written addition of two three-digit numbers, with a hidden mistake and an instruction of "try again" that is directed towards the author of the mistake. The children in the picture provide various comments on the issue. The fact that the addends are both three-digit numbers is in agreement with the research recommendations to use numbers of three or more digits in teaching written addition algorithm to primary school pupils as well as future primary school teachers

(Thanheiser, 2009). The mistake hidden in the algorithm is common among pupils, especially in early stages of the learning process (Ashlock, 2010; Hansen, 2013). It relates to a failure in understanding that the position of a digit determines its value, i.e. a place-value error. In particular, the mistake consists of a singular erroneous handling of an intermediate result in the tens and hundreds columns; however, a similar situation in the units and tens columns (that preceded the erroneous one) was handled properly.

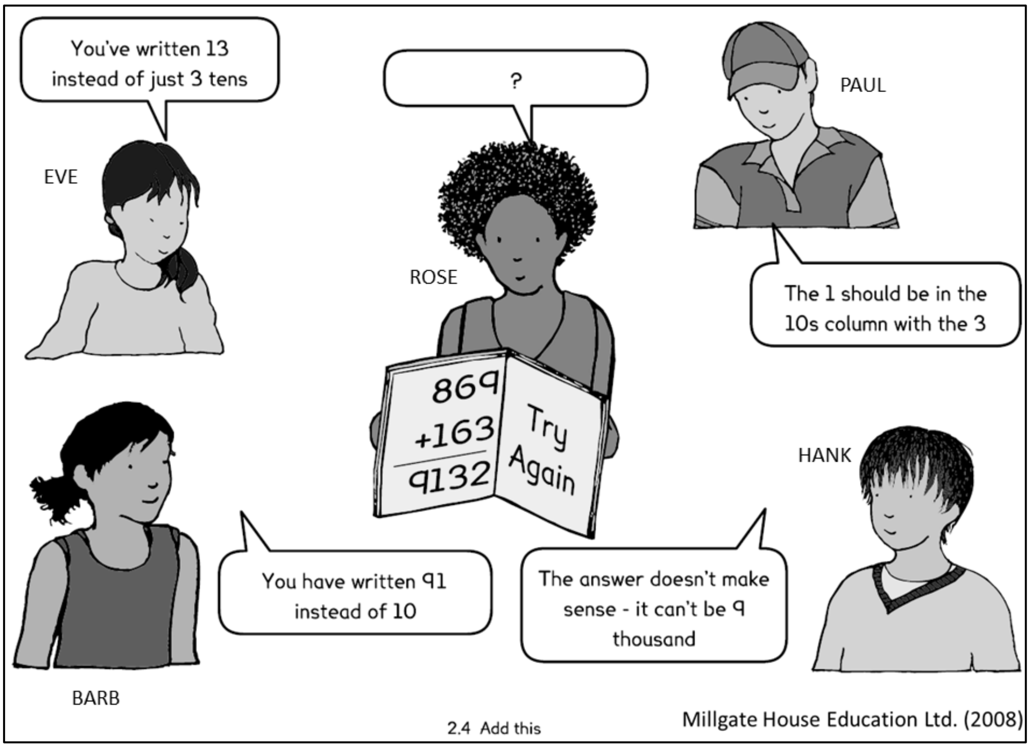

Figure 2. An original Concept Cartoon on the written addition algorithm; picture taken from (Dabell et al., 2008: 2.4), names added

The opinions offered inside the bubbles in Figure 2 are of various nature. Eve, Paul and Barb each comment just a particular intermediate step of the algorithm and its result that is placed in a particular column or columns, while Hank does not address any intermediate steps of the algorithm but comments the result of the operation in general. Hank points rightly out the nonsensicality of the result; however, the source of his opinion is not revealed and his opinion is not justified, i.e. further questioning from the teacher would be helpful to reveal the thinking process. Eve, Paul and Barb, in their comments to intermediate steps, more or less correctly deal with place values. From the perspective of the learning process, each of them might reflect a semi-finished outcome of learning of the algorithm that might be considered by a teacher as a proper constituent of the learning process. But none of the three bubbles provides a complete justification of the correct relation to the meaning of the algorithm, and further questioning from the teacher is necessary to ascertain whether the opinion in the bubble is really a reflection of a proper understanding of place value or not.

Summarized, all of the opinions provided by the children in the picture are open for further questioning from the teacher, leave room for doubts on the part of the teacher regarding the correctness of a particular children's thinking, are not fully connected to the meaning of the algorithm nor precise or complete from the justification point of view. Thus, the opinions create a lot of opportunities for wide discussion with future teachers, as well as for investigating their PCK about the algorithm, place value, alternative strategies (e.g. estimation strategy that might led to Hank's opinion), etc.

Data analysis

The present study is of a qualitative exploratory design, with the process of data analysis based on open coding and constant comparison (Miles, Huberman, & Saldaña, 2014). During data analysis, all the written data materials were open coded focusing on aspects related to PCK, the codes were sorted to categories, and then the process of constant comparison was executed. At the end of the process of constant comparison, each of the categories was assigned to one of the PCK components by Kleickmann et al. (2013) and denoted by a plus or minus sign referring respectively to strong or weak PCK. As a result, we have got ten relevant PCK-related categories arranged into the structure shown in Table 2.

Table 2. The PCK-related code categories, their labels and signs

Component of PCK	Label	Category	Sign
knowledge of pupils	c1	considering possible unclear phrasing	plus
	c2	pointing out possibly problematic reasoning	plus
	c3	pointing out improbable sources of mistakes	plus
knowledge of tasks	c4	content-related errors in explanations	minus
knowledge of instruction	c5	insisting on one particular procedure	minus
	c6	rejecting an unknown opinion	minus
	c7	providing procedural explanations only	minus
	c8	providing approval or objection without explanations	minus
	c9	providing an inappropriately chosen example in an explanation	minus
	c10	providing a constructive explanation	plus

Findings

General findings

In this subsection, we present general findings accompanied in brackets by references to particular responses that will appear in transcripts in the two following subsections of the Findings section. The references to responses of participants from the pre-didactic group begin with a code letter “B” (in the sense of “before the course in didactics of mathematics”), the references to responses of participants from the post-didactic group begin with a code letter “A” (in the sense of “after the course in didactics of mathematics”). Each of the references contains also the name of the pictured children whose statement is addressed in the particular response.

The above stated list of relevant PCK-related categories has already shown that the participants’ responses spread across all components of PCK, with the largest number of relevant categories belonging to the component of knowledge of instruction. Some of the categories occurred in both the pre-didactic and post-didactic groups (categories c7, c8), some categories occurred only in the pre-didactic group (c4, c5, c6, c9, c10) or only in the post-didactic group (c1, c2, c3). Four of the five categories that occurred only in the pre-didactic group are minus-categories; all three categories that occurred only in the post-didactic group are plus-categories. The categories belonging to knowledge of pupils occurred only in the post-didactic group.

In particular, in both groups, there were some participants that provided only procedural explanations (e.g., B23/Paul, B14/Rose, B20/all, A6/Eve) or provided apprehension or objection without explanations (e.g., B27/Paul, B27/Eve, B32/Paul, A10/Hank). In the pre-didactic group, there were participants that displayed content-related errors in their explanations (e.g., B11/Paul), insisted on one specific procedure (e.g., B14/Hank, A5/Hank), rejected an unknown procedure (e.g., B39/Eve), provided explanations with an inappropriately chosen example (e.g., B27/Hank) or provided a constructive explanation (e.g., B28/Paul, B13/Rose). In the post-didactic group, there were participants that considered possible unclear phrasing (e.g., A2/Paul, A10/Paul), pointed out possibly problematic reasoning (e.g., A11/Barb, A5/Barb) or pointed out improbable sources of mistakes (e.g., A6/Barb).

From the perspective of the six indicative questions, three of the questions produced a large amount of relevant PCK-related data (questions 3, 4, 5), while the other three questions produced no relevant PCK-related data (questions 1, 2, 6). In the case of question 6, the lack of relevant data was probably caused by the fact that the participants considered Rose as the author of the text in the workbook, responded to Rose in the same manner as to the other children within questions 3 to 5, and so left the sixth question unanswered.

Pre-didactic group

Decisions about wrong and right statements

Among the 29 respondents of the pre-didactic group, 14 assessed as correct both Barb and Hank, and nine assessed as correct Barb, Eve and Hank together. The remaining six offered various combinations of children assessed as correct (Eve and Hank, Barb and Eve, all, just

Hank, just Barb). On the other hand, 16 of the 29 respondents assessed as incorrect Paul, six assessed as incorrect both Eve and Paul, and four assessed none as incorrect. The remaining three offered various combinations of children assessed as incorrect (Hank and Paul, Barb and Hank, Eve and Hank and Paul).

Ten of the 29 respondents were not able to decide between correctness and incorrectness for some of the children: five of them did not have a decision for Eve, three for both Eve and Paul, one for Barb, and one for both Barb and Eve.

Since the correctness of all of the statements is unclear due to missing justifications and all of the statements require further questioning from the teacher to ascertain the (in)correctness, the diversity in decisions given by participants shows how individual views and preferences of teachers might impact the assessment of pupils.

Comments, justifications and advices

The comments, justifications and advices provided by the pre-didactic group were of varied nature. Some of them objected appositely against imprecision or incompleteness of the statements in the bubbles. But in their objections, they did not provide any constructive advices nor explained possible sources of the mistakes:

B27	Paul:	He cannot write more digits at once.
	Eve:	Basically, she is right, but you cannot write a higher number than 9 inside one column.
B32	Paul:	You cannot inscribe a two-digit number into one column.

Other respondents presented opinions that included procedural advices:

B23	Paul:	Basically, he is right, the result of the addition is 13, but we do not write the 1, we add it in the next column instead.
B14	Rose:	You should have added the 1 to the 9, not to write it besides.
B20		It might be helpful for children to keep the 1 to transfer on their fingers (to remind themselves about the transfer) or to write the transferring digit directly inside the calculation notation.

There were also responses rather inappropriate or odd. For instance, one of the respondents did not agree with Eve just because she had not understood her statement:

B39	Eve:	Nonsense! I do not understand what she means.

Another one placed Hank among those who were not correct, and thought that she should push him to think numerically:

B14	Hank:	His opinion is not correct. Although, he says the true, but he does not think directly in numbers, which he should. Hank, try to justify your opinion in numbers.

The oddly unbreakable connection to numbers appeared also among responses that considered Hank as correct; this particular response is based on an inappropriately chosen example:

B27	Hank:	Hank is correct. He logically thinks that the result cannot be bigger than 1998 when adding hundreds: 999 + 999 = 1998.

Some of the participants provided indirect constructive advices as they tried to show the incorrectness of children's views by bringing them into absurdity:

B28	Paul:	If the 1 were in the column with the 3, there would be a 4 there.
B13	Rose:	869 163 91212

Also terminological mistakes appeared in data: some of the participants confused the terms "number" and "digit" in their responses:

B11	Paul:	He does not know that he can write only one number in each column.

Post-didactic group

Decisions about wrong and right statements

Among the 12 respondents of the second group, eight assessed as correct Eve, Barb and Hank together, two assessed as correct just Eve and Hank, and two assessed as correct just Hank. All of the 12 respondents decided between correct and incorrect on all of the children. In comparison with the pre-didactic group, the diversity in decisions given by post-didactic-group participants is much smaller. Data excerpts below will show that the reason is mainly hidden in a common approach that the post-didactic-group participants applied to all of the statements. This common approach takes into account possible unclear phrasing of pupils, and thus eliminates the impact of individual teacher's views and preferences.

Comments, justifications and advices

Generally, the participants from the post-didactic group showed greater insight. Also, they were not so strict in their judgements. For instance, some of the participants were able to realize that children may sometimes be unclear in their phrasing, that they may think of something but say otherwise:

A2	Paul:	He probably meant it well, but what he said is nonsense.
A10	Paul:	The wrong opinion of Paul could also arise in such a way that he had the result written correctly but expressed himself poorly.

They also drew attention to cases when the cause of the mistake was not addressed properly in the bubbles:

A11	Barb:	Yes, the difference is clear. However, she does not deal with its causes. Rose did not write 91 but 2, 13 and 9.
A5	Barb:	I agree; she corrects the result so that it comes out right. But she probably does not know how the mistake came into being.

And they pointed out that the causes given inside the bubbles were not very likely:

A6	Barb:	Yes, it is true. But the kind of mistake that the girl describes was not probably made by the solver.

Most of the post-didactic group was satisfied with Hank's opinion. However, some of them still were not:

A5	Hank:	We can see that Hank thinks logically. But his argument does not match the assignment "Try again".

As in the pre-didactic group, some of the post-didactic participants provided only procedural explanations:

A6	Eve:	Children sometimes do similar mistakes that they should remember a one and add it later but instead they write it down and do not add it.

Some also provided responses exhibiting apprehension without an explanation:

A10	Hank:	Hank’s opinion is logical. These numbers cannot yield such a high number.

Conclusions

In the present study, we have used an educational tool called Concept Cartoons to investigate future primary school teachers’ PCK related to the algorithm of written addition of natural numbers. By observing two different groups of future teachers in diverse stages of their teacher preparation (pre-didactic and post-didactic groups), we provide illustrations to the fact that future primary school teachers might obtain some informal foundations of PCK in mathematics as early as before the course of didactics of mathematics and own teaching practice (Kleickmann et al., 2013). But, for our particular pre-didactic participants and for the particular content related to the written algorithm, the foundations were rather weak and consisted mainly of procedural explanations or objections without explanations. On the other hand, the participants from the post-didactic group, who had not yet enrolled in teaching practice, provided rather strong knowledge of pupils (of the limits of their phrasing, problematic reasoning, probable sources of mistakes).

From the perspective of Concept Cartoons, we introduced the tool as a component of representation of practice (Buchbinder & Kuntze, 2018; Grossman et al., 2009) that focuses on content-related pupils' opinions and how the teacher responds to them. We employed the method for collection and analysis of PCK-related data provided by Concept Cartoons that had been derived in Samková's study (2018). In our particular case, the Concept Cartoon consisted of pupils' opinions that were not clearly expressed and required further questioning initiated by the teacher to reveal the thinking process and be able to assess it. The opinions were composed in such a way that they might or might not indicate some common pupils' mistakes (Ashlock, 2010; Hansen, 2013) or alternative strategies (estimation). The method proved its relevance in teacher preparation, since it helped to reveal responses to virtual pupils' opinions that displayed weak PCK, and thus created the space and opportunity to address such weaknesses properly during teacher professional preparation, before actual teaching practice.

From the perspective of the written algorithm, collected data confirm the difficulty of the topic for future primary school teachers (as already reported, e.g., by Thanheiser, 2009). Despite the fact that the structure of the decimal system with the place value system had been introduced to all of the participants within the course in mathematics, practically none of them used the structure or proper terminology in their objections and explanations, and none of them provided proper conceptual explanation. Such findings can be found also in research on other operations on natural numbers that require proper knowledge of the place value system (Ma, 1999; McClain, 2003). Moreover, the issue of proper terminology missed or misused in collected data draws attention to the fact that terminological and linguistic challenges need to be addressed during professional teacher preparation in order to construct mathematical knowledge properly (Schleppegrell, 2007). The addends in the written algorithm were three-digit numbers and such an arrangement proved its suitability and effectivity for diagnostic purposes related to the place value topic (as applied, e.g., in Selter, 2001; empirically justified and recommended in Thanheiser, 2009): some of our respondents were not able to orientate themselves in the algorithm and link the opinions inside the bubbles to particular locations within the algorithm.

Summarized, the present study showed how a Concept Cartoon consisting of bubbles with statements based on common pupils' misconceptions on a particular mathematical topic can be used in professional preparation of future primary school teachers to investigate their PCK related to the topic addressed in the statements.

As is usual for qualitative exploratory empirical studies, the limitation of the study consists in the size of the sample, in subjectivity of the process of data analysis and in an impossibility to generalize its results. However, the study offers an opportunity to observe in detail how Concept Cartoons can be implemented into teacher professional preparation.

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