Development of pre-service teachers' noticing competencies within a mathematical modelling context: a case study

O desenvolvimento das competências de *noticing* de futuros professores num contexto de modelação matemática: um estudo de caso

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Abstract. Mathematical modelling is challenging for students as well as for teachers. Thus, fostering teachers' competencies to teach mathematical modelling is an essential component of teacher education. Recent research on teachers' competencies pays increasing attention to the role of noticing conceptualised as situation-specific competencies necessary for actions within the classroom. These situation-specific noticing competencies are needed for the teacher to react to unanticipated approaches and unforeseen obstacles students may encounter when working on modelling problems. To assess these competencies, videos can be used to simulate real classroom situations and measure the skills necessary for a spontaneous and adequate diagnosis and action. In this paper, the development of pre-service teachers' noticing competencies within a mathematical modelling context is analysed by qualitatively comparing pre- and post-test data. A modelling seminar at the master's level served as an intervention. Three pre-service teachers were selected, and their competence development is analysed qualitatively in-depth regarding growth indicators. The pre-service teachers showed different trajectories of learning. These were manifested in different ways and differed concerning the topics of students' difficulties, use of metacognitive strategies and approaches used to solving the problem.
Keywords: mathematical modelling; teacher's competencies; teaching mathematical modelling; noticing; pre-service teachers.

Resumo. A modelação matemática é desafiante para os estudantes bem como para os professores. Assim, a promoção das competências dos professores para ensinar a modelação matemática é uma

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componente essencial da formação de professores. Estudos recentes sobre as competências dos professores prestam cada vez mais atenção ao papel da observação, entendido como as competências específicas de cada situação, que são necessárias para agir dentro da sala de aula. Estas competências de observação, específicas da situação, são necessárias para o professor poder reagir a abordagens não antecipadas e a obstáculos imprevistos que os alunos podem encontrar ao trabalharem em problemas de modelação. Para avaliar essas competências, podem ser usados vídeos que simulam situações reais de sala de aula e aferir as competências que são necessárias para realizar espontaneamente um diagnóstico e uma ação adequados. Neste artigo, o desenvolvimento das competências de *noticing* de futuros professores, num contexto de modelação matemática, é analisado, comparando qualitativamente os dados de um pré e um pós-teste. Um seminário de modelação num curso de mestrado serviu como intervenção. Foram selecionados três professores em formação inicial e o desenvolvimento das suas competências foi analisado, qualitativamente e em profundidade, no que diz respeito a indicadores de progressão. Eles revelaram diferentes trajetórias de aprendizagem, que se manifestaram de diferentes formas, diferenciando-se quanto aos tópicos das dificuldades dos alunos, da utilização de estratégias metacognitivas e das abordagens usadas para resolver o problema.
Palavras-chave: modelação matemática; competências do professor; ensino de modelação matemática; *noticing*; professores em formação inicial.

Introduction

Many curricula for mathematics education worldwide include mathematical modelling as an essential component. Working on mathematical modelling problems should support students in critically reflecting on aspects of their everyday life with the help of mathematics and prepare them to be responsible citizens (Kaiser, 2017). However, mathematical modelling is a difficult task not only for students but also for teachers (Blum, 2015). When teaching mathematical modelling it is essential for teachers to be able to perceive obstacles in students' working processes, interpret them based on theoretical concepts and to be able to make decisions on how to best support the students. These requirements form the concept of noticing.

To assess these competencies, videos can be used to simulate real classroom situations and measure the skills necessary for a spontaneous and adequate diagnosis and action (Kaiser et al., 2015). For this purpose, we developed an instrument using staged videos as *stimuli* to measure noticing competencies for teaching mathematical modelling. This instrument was used in a pre-post-test design before and after a modelling course at master's level. The development of the instrument as well as its validity is presented in Alwast and Vorhölter (2021): we ensured all relevant content is covered (content validity), showed that the three coded levels of interpretation matched participants underlying reasoning (elemental validity) and checked the assumed one-dimensional structure of noticing competencies in a mathematical modelling context (construct validity).

For analysing the data obtained with this instrument we used a coding scheme (described in Alwast and Vorhölter (2021)), which allowed to identify different levels of interpretation on an aggregated stage and make general statements about developments. However, it was not possible to identify how these developments are manifested in detail. To give a more detailed picture of pre-service teachers' competence development, the use of growth indicators as suggested by Jacobs et al. (2010) seems promising. Therefore, in this paper, we present a case study consisting of a subsample of pre-service teachers participating in the main study, in which the development of three pre-service teachers' competencies is analysed by using these growth indicators. Here we limit ourselves to the facet of interpretation as one of the three sub-facets of noticing.

Theoretical framework

Teachers' competencies

In mathematics education research, significant large-scale studies have examined teachers' competencies with a focus on cognitive and affective-motivational facets, for example TEDS-M (Kaiser et al., 2015) or COACTIV (Kunter et al., 2013), and validated their constructs, which distinguish among various cognitive facets according to Shulman (1987).

To overcome the gap between a holistic approach assessing behaviour in real life and an analytical approach focusing on the underlying dispositions of competence, Blömeke et al. (2015) developed a model of *competence as a continuum*. This model integrates situation-specific skills as components of competence, which are needed to act in specific situations based on one's dispositions leading to an observable behaviour. We are, in particular, interested in these competencies needed to notice students' actions in the moment.

Depending on the respective focus, noticing is conceptualized differently. In general, it refers to the situation-specific cognitive processes needed when confronted with a complex situation such as in a classroom, which requires teachers to use their knowledge to support students in the moment. Thus, the overall goal is to improve the effectiveness of teaching and learning (Mason, 2011; Sherin et al., 2011a). Noticing is usually distinguished into (1) (selectively) perceiving noteworthy aspects in a complex teaching situation with an overwhelming amount of information, (2) attending to or interpreting these based on knowledge and former experiences, and (3) making a decision and thinking about alternative actions (Schack et al., 2017; Sherin et al., 2011b).

Competencies for teaching mathematical modelling

In general, what we know about teachers' competencies and quality also holds true for the teaching of mathematical modelling (Blum, 2015; Niss & Blum, 2020). More specifically for teaching mathematical modelling, Borromeo Ferri and Blum (2010) developed a theoretical model which consists of four dimensions of pedagogical content knowledge and skills: theoretical, task, instruction and diagnostic dimension. For example, the task dimension includes among others the ability to solve a modelling problem, while the diagnostic dimension contains the knowledge and ability to identify phases of the modelling process as well as difficulties. Furthermore, Eames et al. (2018) developed theoretical principles (six competencies as well as five beliefs) relevant for teaching mathematics, which partly overlap with the dimensions by Borromeo Ferri and Blum (2010). For example, they mention that teachers should "develop capability of designing instruction that is responsive to questions raised by learners themselves either implicitly or explicitly" (p. 263) and thus stress the importance of quickly reacting to a teaching situation in the moment.

Based on the model *competence as continuum* by Blömeke et al. (2015), competencies needed for teaching mathematical modelling include *dispositions*, which lead to a *performance* mediated by *situation-specific skills*. In addition to the dispositions covered in the models of Borromeo Ferri and Blum (2010) and Eames et al. (2018), one's own modelling competencies – as for example distinguished by Kaiser (2007), Maaß (2006) and Stillman (2011) – are a requirement for teaching mathematical modelling and therefore an essential part of dispositions. Another important aspect for teaching mathematical modelling is meta-metacognition (Stillman, 2011), which is characterized by monitoring and supporting students' metacognitive processes. This multitude of dispositions provide the basis on which classroom situations are perceived and interpreted, whereupon decisions about how to proceed must be made, i.e., noticing competencies.

Noticing competencies for teaching mathematical modelling

Galbraith (2015) identified three types of noticing that are important for the teaching and learning of mathematical modelling, with the type "noticing as a mentor" being especially relevant to the teaching of mathematical modelling; via modelling-specific noticing, the mentor needs to ensure that students are working productively toward the goal of the task while following their individual approach. However, Galbraith (2015) extends the concept of noticing to a wider context (e.g., mathematicians' noticing, students' noticing), while the framework of this article will follow the narrower conceptualizations described in literature from the discipline of noticing as discussed above.

For the presented study, we adapted the competence model developed by Blömeke et al. (2015) to conceptualise and specify teachers' situation-specific skills within a mathematical modelling context (see Figure 1; for more details see Alwast and Vorhölter (2021)). Regarding mathematical modelling, this model illustrates that teachers must be able to grasp modelling-specific situations based on cognition (i.e., modelling-specific pedagogical content and mathematics content knowledge) and affective-motivational aspects (e.g., modelling-specific beliefs) in order to be able to adequately behave and competently support students.

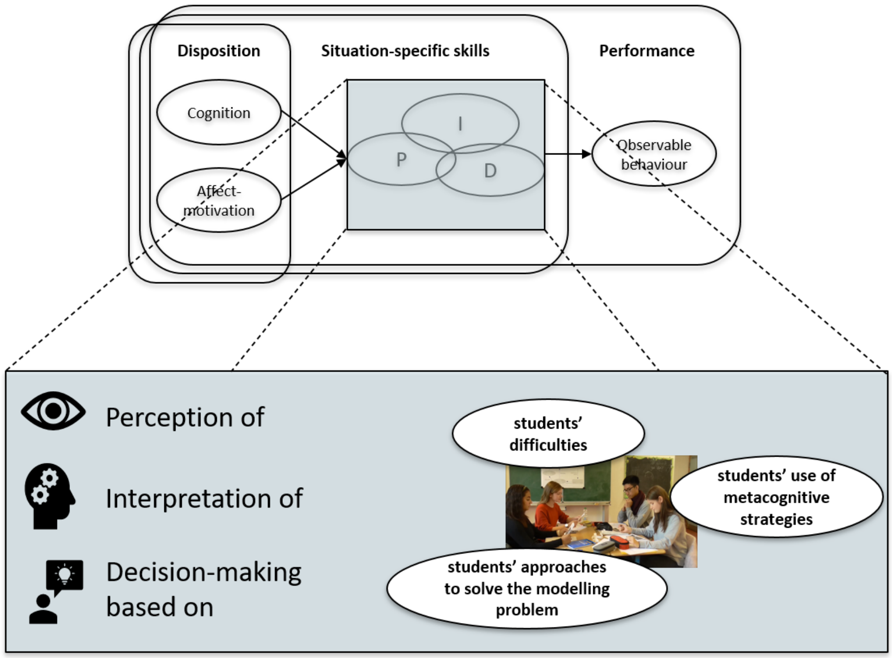

Figure 1. Conceptualisation of noticing within a mathematical modelling context (Alwast & Vorhölter (2021)) based on Blömeke et al. (2015)

Noticing is specific to each profession and can be understood as using a particular lens to view situations, which are specific for teaching. Therefore, noticing in a modelling context requires a modelling-specific lens and thus entails:

1. perceiving students' modelling-specific difficulties, perceiving the potential in diverse approaches to solving the problem, and perceiving the successful or missing use of metacognitive strategies as an organisational structure that is needed for autonomously working on complex problems;
2. interpreting the perceived aspects based on meta-knowledge about the characteristics of modelling problems (for example as conceptualised by Borromeo Ferri and Blum (2010), and Klock and Wess (2018)) and theories about students' working processes;
3. making a decision about an appropriate reaction by taking into account the interpretation and considering multiple ways to intervene. Students' approaches to solving a modelling problem cannot always be anticipated due to the openness of modelling problems, and a decision on how to respond should evaluate whether or not a new idea might lead to an appropriate model and an adequate result.

Measurement of noticing competencies

Measurement of noticing competencies in mathematics education

Regarding the concept of noticing, several instruments were developed to assess this situation-specific competence and its development, also in the field of mathematics education, for example, van Es and Sherin (2002), Kersting (2008) or large-scale studies such as TEDS-FU (Kaiser et al., 2015). Moreover, a few frameworks classify learning trajectories of noticing competencies, for example, the learning to notice framework (van Es, 2011). Many of these instruments use videos for simulating a classroom situation. In comparison to a real teaching situation, where any kind of assessment can cause a disruption of the teaching process, instruments using video vignettes offer a standardization. Santagata et al. (2021) did a meta-study on studies of video-based programs; this meta-study distinguished different perspectives of noticing and showed that different foci frame the study design and results.

For analysing the data of open items in the area of noticing, different coding schemes exist. For example, Kersting (2008) distinguished three levels of interpretation, which range from purely descriptive comments to remarks, which include some analytic chunks, to a coherent analysis. By using different quantified levels, she only looked at aggregated data.

For a more differentiated analysis of the development of noticing competencies, the results of Jacobs et al. (2010) are of particular interest. They assessed the noticing competencies of pre- and in-service teachers with different levels of experience and thereby focused on children's mathematical thinking. As prompts they used classroom video clips and a set of written student work. Thus, compared to Kersting (2008), they not only looked at aggregated data but analysed the development content-wise. As a result, Jacobs et al. (2010, p. 196) formulated a set of six growth indicators that illustrate shifts in teachers' professional noticing of children's mathematical thinking:

1. a shift from general strategy descriptions to descriptions that include the mathematically important details;
2. a shift from general comments about teaching and learning to comments specifically addressing the children's understandings;
3. a shift from overgeneralizing children's understandings to carefully linking interpretations to specific details of the situation;
4. a shift from considering children only as a group to considering individual children, both in terms of their understandings and what follow-up problems will extend those understandings;
5. a shift from reasoning about next steps in the abstract (e.g., considering what might come next in the curriculum) to reasoning that includes consideration of children's existing understandings and anticipation of their future strategies; and

6. a shift from providing suggestions for next problems that are general (e.g., practice problems or harder problems) to specific problems with careful attention to number selection.

It should be stressed that the growth indicators were not used as a coding scheme but were a result of their analysis. Although they are not classified regarding perception, interpretation, and decision-making, the first growth indicator mainly describes the sub-facet perception, while the second to fourth focus more on interpretation and the fifth and the sixth deal primarily with decision-making. As these indicators seemed promising for analysing data regarding the development of noticing competencies, they are of particular interest for this study. However, they cannot simply be applied to the assessment of the development of teaching competencies for mathematical modelling in general, because on the one hand mathematical modelling comes with specific challenges and, on the other hand, the growth indicators need to be adapted to the instrument used.

Measurement of noticing competencies in a mathematical modelling context

Situation-specific skills as included by Blömeke et al. (2015) are not yet assessed comprehensively in the area of mathematical modelling. As an approach to assess the perception, interpretation and decision-making of a situation, the instrument developed by Klock and Wess (2018) used text vignettes with closed items. Compared to video vignettes, text vignettes require the participant to imagine the situation, offer less sensory data and allow the participant to experience the situation multiple times and are therefore more distant from real classroom settings than video vignettes (Lindmeier et al., 2013). Thus, for assessing pre-service teachers' noticing competencies within a mathematical modelling context, we developed and validated a video-based instrument (see Alwast & Vorhölter, 2021). The instrument contains two staged videos that simulate real classroom situations and last three minutes each. Both staged videos display a group of four students in ninth grade who are working on the modelling problem "Uwe Seeler's Foot", which requires students to verify a newspaper statement and, based on a picture, compare the volume of a statue showing Uwe Seeler's foot with the volume of his real foot (Vorhölter et al., 2019). The first video vignette shows them at the very beginning of the working process, while in the second video the students are working mathematically and starting to interpret their solution. To develop staged videos, which on the one hand represent typical, simultaneously occurring difficulties in modelling processes, but at the same time are short with a high information density and focus on substantial issues, we first theoretically identified noteworthy aspects as well as used recorded videos to choose relevant situations and generate authentic scripts including several noteworthy incidents. After several revisions we created two staged videos with student actors. Two versions of each video were made: although both versions had the exact same content, different student actors played a role.

Thus, we were able to use the second version of the video in the post-test to minimize memory effects. In total, 14 intended incidents related to students' modelling-specific difficulties (e.g., unsuitable use of routines to solve a problem without considering the context), to students' use of metacognitive strategies (e.g., planning the procedure) and to students' diverse approaches to solving the problem (e.g., calculating the volume of both the sculpture and of a foot with shoe size 42 to verify or falsify the ratio presented by a newspaper article) were implemented in the videos and thus could be perceived and interpreted by pre-service teachers. These were included in the staged videos, because they display a variety of common student behaviour during the work on a mathematical modelling problem as known from literature (for example, Blum, 2015; Maaß, 2006; Stillman, 2011; for an overview, see Niss & Blum, 2020).

Open questions were used, which could be read by the pre-service teachers before watching the video. This information was provided beforehand because practising teachers know what they want to focus on in their class. Then, the pre-service teachers had to watch each staged video once without stopping to approximate the simultaneity and transience of a real classroom situation. Afterwards they individually had to answer nine open questions in written form. In this way, the pre-service teachers could show to what extent they were able to perceive and interpret incidents. Furthermore, a decision on how to solve a problematic situation was required at the end of the video. However, although decision-making is also an important facet, the focus of the instrument is on perception and interpretation. A detailed description of the instrument and the validation process can be found in Alwast and Vorhölter (2019; 2021).

Figure 2 displays one of the nine open questions included in the instrument. One of the intended incidents related to this question is students' unsuitable use of routines to solve the problem without considering the context; more specifically, the students in this video do not know how to deal with an underdetermined task and use the given numbers without understanding their meaning because they lack meta-knowledge on modelling problems.

Question 1: Analyze the students' difficulties in relation to the modelling cycle and reason about it. Use relevant didactic concepts and terminology as far as known.

Figure 2. Example question from the instrument regarding students' difficulties

As prerequisite for being able to answer the open questions, participants need to be reminded of the modelling cycle and other important concepts, which are referred to in the test, beforehand. This procedure was chosen, because the test should not assess mere knowledge or a lack of it but the application of knowledge through noticing. Furthermore, it needs to be ensured that the modelling problem and different ways to solve it are already known.

In the main study, it proved feasible to perceive all incidents and achieve the highest level of interpretation for all incidents. However, only a small group of participants was able to achieve the maximum, which is in line with the assumption that pre-service teachers are not experts yet and cannot be expected to notice on such a level. Yet, there was a significant difference between the pre- and the post-test in some areas as presumed. However, due to the evaluation method, it could only be determined that participants' interpretations were more theory-based in the course of the study, but not how exactly these interpretations differed at the beginning and at the end of the study. Therefore, using the growth indicators developed by Jacobs et al. (2010) as a coding scheme seems to be a way of describing these developments more precisely.

Related to the developed instrument, the items used and the modelling context in mind, the first growth indicator seems to be less suitable, as working mathematically is part of the modelling cycle but not specific for mathematical modelling. The second, third and fourth growth indicator focus on students' understanding in a specific situation, which can be adapted to the context of modelling. Therefore, these growth indicators match the situations and incidents shown in the developed videos and therefore seem beneficial to capture more precisely the development of the pre-service teachers in terms of their interpretation skills. The fifth and sixth growth indicator describe decisions, which are focused on the trajectory of learning in the long run. As this was not required within the test, these two growth indicators are also not appropriate for our study. Therefore, in this study we limit our further analysis to the second, third and fourth growth indicator.

Research questions

As described before, teaching mathematical modelling is challenging and it is important that teachers are able to react in the moment and make informed decisions when confronted with unforeseen obstacles and complex teaching situations. For the evaluation of interventions aimed at improving these competencies, it is therefore important to have an instrument that can trace these developments.

The case study presented in this paper aims at adapting growth indicators (Jacobs et al., 2010) to analyse developments of pre-service teachers' noticing competencies within a mathematical modelling context based on a developed video-based instrument. The growth indicators as described by Jacobs et al. (2010) provide a tendency of development independent of the content. Using the selected growth indicators for mathematical modelling, we then analyse the development of three pre-service teachers in depth during a modelling seminar (described in the next section) and compare the characteristics of these three cases.

Thus, the following research questions are examined in this paper:

1. How does pre-service teachers' focus of interpretation as a facet of noticing competencies develop during a modelling seminar regarding the adapted growth indicators?
2. Which insights about pre-service teachers' development does the in-depth analysis using the growth indicators as coding scheme provide?

Methodology

To answer the research questions, we use data collected in the main study and specifically examine three cases. A seminar at the master's level that focused on mathematical modelling served as an intervention and is described in the following section. At the beginning and at the end of the seminar a video-based test (as described above) was used to assess pre-service teachers' noticing competencies regarding mathematical modelling. Hereby, insights about pre-service teachers' noticing competencies within a mathematical modelling context were gained on an aggregated level. For the in-depth analysis, we use a different method for data analysis than in the main study, namely the concept of growth indicators by Jacobs et al. (2010).

Modelling seminar

The seminar lasted one semester and included 13 sessions of 2.5 hours each. Pre-service teachers in their master's program had the choice between taking the modelling seminar, which was taught by the second author, or two other seminars in mathematics education. During the study (3 years), about 15 to 30 pre-service teachers participated in the seminar per term. The modelling seminar contained theoretical input covering aspects relevant for teaching mathematical modelling and thus included most aspects of the four dimensions by Borromeo Ferri and Blum (2010). Concerning the *theoretical dimension*, background knowledge on modelling competencies, modelling cycles, metacognitive strategies, or digital tools was conveyed and applied to practical examples. The *task dimension* includes working on and analysing modelling tasks as well as potential barriers. During the seminar, pre-service teachers worked on modelling problems of varying complexity. In particular, two complex modelling problems were worked on in depth, for example, the Spidercam problem (Klöckner et al., 2016) or a suggestion on how to air the classroom. In addition, many smaller but still authentic problems in a didactical context were worked on and analysed. Regarding the *instructional dimension*, participants specified the working process during the Hamburg Modelling Days in relation to the specific modelling problem. To prepare the participants for this, lesson planning and possible interventions were discussed. Practice-oriented activities such as analysing staged and recorded videos as well as student artifacts (for example, written solutions) were used to evaluate possible interventions. Videos and student artifacts were also used for fostering the *diagnostic dimension* by analysing in different ways the behaviour and the solution process of students and discussing typical difficulties together. Thus, a major focus was the transfer from theory to practice – for more details see Alwast and Vorhölter (2019), Vorhölter and Freiwald (in press).

Sample

In total, more than 60 pre-service teachers participated in the main study. For the case study presented in this paper, we selected three cases from the main study as examples: Sarah, Dorothea and Alex[1]. These were chosen because they vary in gender and course of study, and had chosen different second subjects. They range in age from 22 to 31. While Sarah and Dorothea study mathematics education for teaching at primary school and secondary level, Alex will become a teacher for a higher track school. They are all in their first semester of their master's degree and the modelling seminar was their first course on mathematical modelling except for one session of the introductory lecture on mathematics education during their bachelor's degree. Furthermore, they did not participate in any mathematics courses that dealt with mathematical modelling.

Moreover, according to the aggregated results (Alwast & Vorhölter, 2021) they differed in their analysis in the pre-test, but also showed quite contrasting developments. Therefore, we were interested to analyse the developments of these three cases in depth and achieve more detailed results.

Data analysis

For analysing the data, qualitative content analysis (Kuckartz 2014) was used. As outlined above, three of the six growth indicators developed by Jacobs et al. (2010) were selected for this study and serve as deductively developed codes. For adapting the growth indicators for the specific instrument and content, we inductively expanded the coding scheme to fit the characteristics of mathematical modelling (Table 1).

We used consensual coding (Kuckartz, 2014) to discuss and agree upon the coded parts and extended the standardized coding scheme in the process.

Table 1. Coding scheme

	Description	**Example**
A shift from general comments about teaching and learning to comments specifically addressing the children's understandings		
general comments	Descriptions of students' actions and working process as seen and detached from the specific situation. These include general knowledge about teaching and learning without linkage to students' thinking.	"The students exchanged ideas about the goal of the task. After they had clarified this, they discussed their approach. The planning took place independently. "
specific comments	Comments specifically addressing the students' understanding; indicators are the usage of verbs such as "assume, realize, think, mistake"; drawing conclusions from students' actions and linking it to cognitive processes.	"The students have difficulties checking if their solution gives an answer to the modelling problem. They only realize this through the teachers' intervention."
A shift from overgeneralizing children's understandings to carefully linking interpretations to specific details of the situation		
overgeneralizing children's understandings	Overgeneralized statements are characterized by an abstractness that makes them true for any random situation.	"Through communication they try to explain their suggestions and try to confirm or disprove the ideas of the others."
carefully linking interpretations to specific details of the situation	Statements include reference to a specific situation by using modelling-specific knowledge or knowledge about the modelling problem.	"First, the group wanted to divide the number given by the newspaper by the shoe size. This approach is not expedient, because lengths and solid figures are mixed. The ratio cannot be determined this way because they combine different dimensions."
A shift from considering children only as a group to considering individual children, both in terms of their understandings and what follow-up problems will extend those understandings		
considering children only as a group	The statements refer to the whole group and no distinction is made between the individual students, even if the difficulties were only experienced by one person or the strategies were only used by one person.	"To plan their approach the students asked questions such as 'What should we do now?' They collected different ideas and reflected on them."
considering individual children	Specific reference is made to individual students, and their behaviour is interpreted (possibly in relation to the behaviour of other group members).	"The teacher controls the results when the passive students told him they were done. The other two students might have found a solution without the help of the teacher."

Results

As a starting point for our analysis, we considered the aggregated results obtained in the main study for the three selected cases (see Alwast & Vorhölter, 2021). Just looking at the breadth of perception, there is striking variance concerning the development of the three cases.

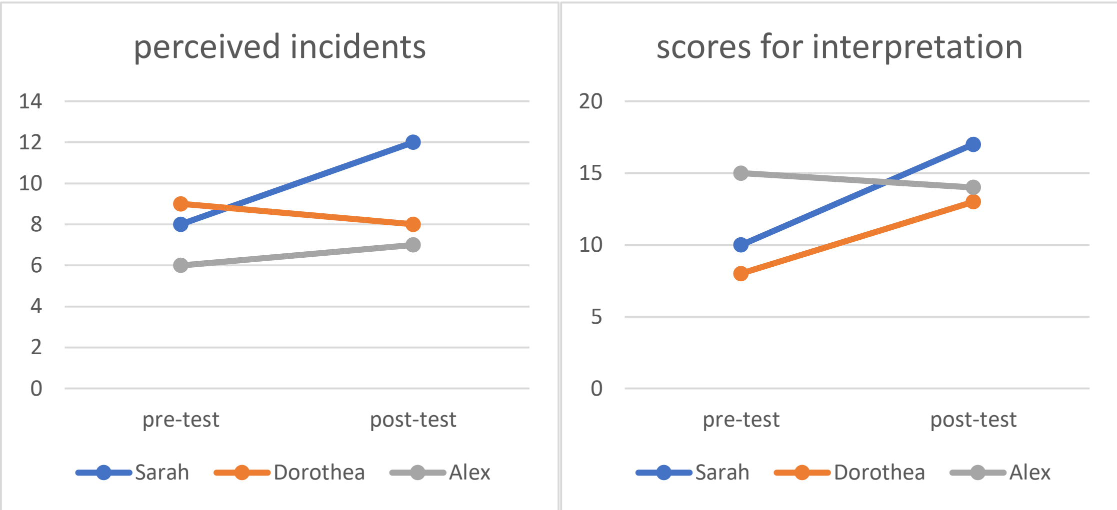

Figure 3. Aggregated results

Sarah already perceives 8 out of 14 intended incidents before the intervention. After the intervention she shows a great range of perception, naming 12 out of 14 incidents. This strong growth can be found concerning all topics (students' difficulties, students' metacognitive strategies, students' approaches to solving the problem). Similar to Sarah, Alex perceives 9 incidents in the pre-test. In the post-test he only mentions 8 out of 14 incidents. Thus, there is no growth visible in this regard, rather a slight decrease. Dorothea perceives 6 out of 14 incidents in the pre-test and showed a slight increase with 7 perceived incidents after the intervention. All in all, while Alex and Dorothea change only slightly in the number of incidents they perceive and mention, Sarah shows a great increase.

For interpreting, Sarah and Dorothea showed a great increase with scores of 10 (pre-test) and 17 (post-test) for Sarah and 8 (pre-test) and 13 (post-test) for Dorothea. In comparison, Alex' score concerning his level of interpretation did hardly change (15 and 14) but was already high at both times of measurement.

Pre-service teachers' development regarding the growth indicators

In the following, the adapted growth indicators are used for analysing the development of the three pre-service teachers in depth. For this purpose, the three aspects of students' difficulties, approaches to solving the problem, and use of metacognitive strategies are considered separately.

A shift from general comments about teaching and learning to comments specifically addressing the children's understanding

This shift deals with comments that are generally true about teaching and learning and describe students' actions in contrast to statements that specifically interpret students' thinking and learning process though their actions.

In the pre-test, Sarah addresses the students' thinking regarding their difficulties with a few remarks. She mainly uses descriptive comments that focus on the knowledge that students lack, that is, she mentions that students forgot a step in their calculation and later realize their mistake. After the intervention, Sarah interprets students' actions and utterances in order to draw conclusions about students' thinking and expresses them in a way that shows she reflected and interpreted what she perceived regarding students' difficulties and understanding. She uses evaluative comments about students' understanding that are based on her knowledge about characteristics of modelling task and draws conclusions about reasons for students' understanding, such as students' meta-knowledge concerning modelling problems:

> Apparently, it is difficult for the students that there is a task that does not automatically provide all the relevant information.[2]

There she points out a characteristic of modelling problems that led to students' confusion and unstructured approach at first. She remains in giving general comments about teaching and learning regarding the aspects of approaches and metacognitive strategies. This development can only be found concerning students' difficulties, concerning the other two areas she keeps using general comments.

In comparison, Alex does not use a lot of explicit references to students' thinking regarding metacognition and approaches to solving the problem. It is not possible to identify differences between the pre- and the post-test. However, similar to Sarah's statement, he recognizes students' mistakes regarding a lack of knowledge about modelling problems. All in all, a development cannot be found.

Looking at the third case, it can be noted that Dorothea refers to students' thinking a couple of times in the pre-test. Compared to Sarah, these comments are a bit more distant from the situation and the students. She uses terms such as "there is a problem with", "the first approach was" and "they made a mistake when calculating because". After the intervention, Dorothea still uses a lot of these terms and there are some direct references to students' thinking:

> They were not able to remember where their numbers came from and did not understand their own solution.

Dorothea describes different approaches and evaluates students' possibilities regarding the respective approaches and discusses the following questions: What do students already know/understand? What do they still need to do/understand in order to successfully solve the problem with this approach? Therefore, in Dorothea's case, a development was only found regarding students' approaches to solving the problem.

Comparing all three cases, this growth indicator manifests quite differently regarding the three aspects (see Table 2):

- regarding difficulties, Alex already addressed students' thinking in the pre-test, while Dorothea stuck to general comments about teaching and learning. However, for Sarah a shift from general comments about teaching and learning to specific comments addressing students' thinking can be analysed.
- in terms of approaches, only a change in Dorothea's statements could be detected, while both Sarah and Alex maintain general comments about teaching and learning.
- regarding the use of metacognitive strategies, all three pre-service teachers used general comments about teaching and learning in the pre- as well as in the post-test.

Table 2. Development regarding the second growth indicator (↗: increased; $\overline{\rightarrow}$: stable on a low *level*; $\underline{\rightarrow}$: *stable on a high level*)

	Sarah	Alex	Dorothea
Difficulties	↗	$\underline{\rightarrow}$	$\overline{\rightarrow}$
Approaches	$\overline{\rightarrow}$	$\overline{\rightarrow}$	↗
Metacognitive strategies	$\overline{\rightarrow}$	$\overline{\rightarrow}$	$\overline{\rightarrow}$

To sum up, based on this growth indicator, we can assume that Sarah's and Dorothea's level of expertise increased in one area each, while Alex shows no development in this regard, although already specifically addressing students' understanding regarding students' difficulties in both pre-and post-test and thus showing a high level of expertise in this regard.

A shift from overgeneralizing children's understandings to carefully linking interpretations to specific details of the situation

This shift deals with general comments that could be true about any other situation to statements that are linked to the specific situation and refer to details that are relevant for the interpretation.

Regarding Sarah, this indicator can be reconstructed concerning students' difficulties and students' approaches to solving the problem. The specific situation is only referred to a couple of times before the intervention. For example, she explicitly mentions a problematic situation where students assume that shoe size units are equal to centimetres. After the intervention, Sarah repeatedly refers to the specific details of a situation and uses indirect speech. In addition to repeating the references already made in the pre-test, she reasons about students' difficulties and approaches to solving the problem by explicitly remarking on the relevant incidents. Sarah also refers to the characteristics of the modelling problem to argue the adequacy of the students' approaches to solving the problem, such as finding out the scale of the photo.

In comparison, Alex almost always refers to the specific situation. However, in the pre-test, these references are sometimes irrelevant for reasoning about a certain aspect and do not support his chain of argument. In the post-test, Alex better identifies the important information and specific details that are relevant to interpret and reason about a situation and provides reasons for his statements, which are connected to the relevant situation at hand:

> After presenting 17.5 m3 as a solution, the girl on the left asks: 'Are you sure that this is the solution?' In this way, checking the results is encouraged.

Dorothea uses a lot of unspecific and general comments before the intervention, which were sometimes even irrelevant for the interpretation:

> Through communication they try to explain their suggestions and try to confirm or disprove the ideas of the others.

This quote is made with reference to students' use of metacognitive strategy monitoring and is too general to really indicate their monitoring process. In contrast, comments related to approaches to solving the problem are more specific and contain relevant details of the mathematical ideas mentioned as well as students' quote. In both pre- and post-test, all three approaches are described and interpreted regarding their feasibility and appropriateness as well as weighed against each other, while specifying ideas students mention and considering what is still missing. In the post-test, specific references are also made regularly concerning the other aspects.

Taken together, the nature of the development regarding this growth indicator differs from case to case and aspect (see Table 3):

- regarding difficulties, Alex linked his statements to specific situations in both, pre- and post-test. Sarah and Dorothea, in contrast, overgeneralized students' understanding in the pre-test, but developed, as they linked their statements to specific situations in the post-test.
- in terms of approaches, only a change in Sarah's statements could be identified, as both Alex and Dorothea already linked their statements to specific situations in the pre-test.
- regarding the use of metacognitive strategies, only for Dorothea a development can be found, the other two remain constant in their way interpreting: Sarah still overgeneralizes students' understanding in the post-test, Alex links his statements to specific situations at both times of measurement.

Table 3. Development regarding the third growth indicator (↗: *increased*; $\overline{\rightarrow}$: *stable on a low level*; $\underline{\rightarrow}$: *stable on a high level*)

	Sarah	Alex	Dorothea
Difficulties	↗	$\underline{\rightarrow}$	↗
Approaches	↗	$\underline{\rightarrow}$	$\underline{\rightarrow}$
Metacognitive strategies	$\overline{\rightarrow}$	$\underline{\rightarrow}$	↗

All in all, regarding this growth indicator, Sarah and Dorothea clearly demonstrate an increase in expertise regarding two aspects each, while Alex constantly refers to the specific situation in all aspects and at both points of measurement.

A shift from considering children only as a group to considering individual children

This shift deals with a focus on students as a group in contrast to parts of the group or individual students and their characteristic thinking and learning, which influences the other group members.

Even before the intervention, Sarah focuses on individual students regarding their metacognitive strategies. However, some of her observations are not correct, which might be due to an unfamiliarity with the concept of metacognition. After the intervention, Sarah still mainly considers individual students when it comes to their use of metacognitive strategies – now, however, her description of the individual students' use of metacognitive strategies is in line with the definition of the concept. Regarding students' difficulties and students' approaches to solving the problem, Sarah only considers the students as a group at all times.

Regarding Alex, a clear shift can be found from only considering the whole group of students to analysing individual students. Whereas before the intervention the "bored students" as part of the group are mentioned once, later on references to single students are made more often and more explicitly. These references are in most cases regarding students' use of metacognitive strategies and students' difficulties, and are followed by direct quotes from these students. In contrast, before the intervention, direct quotes are introduced by passive constructs such as "it was said: (...)".

In comparison, Dorothea refers to the students only as a group before the intervention. There is no reference at all that distinguishes between individual students or parts of the group. In the post-test, a few remarks concerning individual students can be found. Like Alex and Sarah, who mainly – but not only – described the students' actions in a more differentiated way according to their use of metacognitive strategies, Dorothea distinguishes between group members only regarding this topic.

> There were quite big differences concerning monitoring and regulation within the group. Two of the group members were always satisfied with the solution and

suggested to leave it there. The other two students asked whether the solution could be right and considered how to proceed.

Apart from this specific statement, Dorothea's remarks concerning individual students are quite vague most of the time.

All in all, this growth indicator could be mainly identified regarding students' use of metacognitive strategies (see Table 4):

- regarding students' difficulties, a shift from considering the whole group to considering individual students can only be identified for Alex. Sarah as well as Dorothea keep on mentioning the whole group.
- in terms of approaches, no development can be seen regarding the three pre-service teachers.
- regarding the use of metacognitive strategies, however, shifts can be stated for Alex as well as for Dorothea, although Dorothea's distinctions between group members are still vague after the intervention and not as differentiated as Alex's references. In contrast, Sarah already considered individual students in the pre-test. However, Sarah's interpretation of individual students' behaviour is not correct before the intervention but is accurate in the post-test.

Table 4. Development regarding the fourth growth indicator (↗: *increased*; $\overline{\rightarrow}$: *stable on a low level*; $\underline{\rightarrow}$: *stable on a high level*)

	Sarah	**Alex**	**Dorothea**
Difficulties	$\overline{\rightarrow}$	↗	$\overline{\rightarrow}$
Approaches	$\overline{\rightarrow}$	$\overline{\rightarrow}$	$\overline{\rightarrow}$
Metacognitive strategies	$\underline{\rightarrow}$	↗	↗

Summarizing, a clear development of expertise regarding students' metacognitive strategies and difficulties can be found in Alex's case, while Dorothea shows also small changes regarding students' metacognitive strategies. In the case of Sarah, no development could be determined since Sarah already focused on individual students at the beginning of the study regarding students' metacognitive strategies.

Insights into pre-service teachers' development

As shown previously, Alex was characterized by relatively high aggregated scores in the facet of interpreting at the beginning of the study (15), which dropped minimally towards the end of the study (14). Dorothea, on the other hand, achieved the lowest aggregated scores at the beginning of the study (8), but these increased as the study progressed (13). Similarly, Sarah's scores were slightly above Sarah's at the beginning of the study (10), but showed a higher increase and were greater than Alex's at the end of the study (17).

With regard to the growth indicators, it can first be stated that no decline was observed, but rather all pre-service teachers remained at their level or increased. This is a different to the aggregated results in Alex's case. Furthermore, it can be noted that, unlike to the aggregated data, Dorothea showed a stronger development regarding the growth indicators.

Moreover, it can be observed that the three pre-service teachers did not develop identically with regard to the three aspects and the different growth indicators. For example, Sarah developed in relation to the first growth indicator in terms of students' difficulties, but not with regard to the other two aspects. In contrast, Dorothea, for example, developed with respect to the third growth indicator in relation to metacognitive strategies, but not regarding the other aspects.

Similarly, it can be noted that the individuals developed differently in each aspect across the growth indicators: with respect to difficulties, Sarah changed concerning the first and second growth indicator but kept commenting with respect to the entire group (third growth indicator). Concerning metacognitive strategies, Alex, as another example, sticks to generalizing comments regarding teaching and learning (first growth indicator), but constantly names specific situations (second growth indicator) and uses increased references to individual students (third growth indicator).

All in all, it could be determined that the greatest development occurred with regard to the second growth indicator, since both Sarah and Dorothea referred more precisely to details of the specific situation in the post-test. Alex had already done this at the beginning of the study; for him the greatest (and only) development can be found in terms of considering individual students as opposed to the whole group (third growth indicator).

Therefore, the differentiation with the help of the growth indicators enables to understand more precisely in which aspects exactly the development takes place.

Summary, limitations, and outlook

As it is important for teachers to react spontaneously to sometimes unforeseen situations when teaching mathematical modelling, we examined how pre-service teachers' noticing competencies develop. For this purpose, we adapted three of the six growth indicators by Jacobs et al. (2010) so that they were applicable to the instrument used (Alwast & Vorhölter, 2021). The goal was to get more detailed and in-depth results about the development. Three pre-service teachers who took part in a modelling course were analysed.

The first chosen growth indicator deals with a shift towards reasoning about students' thinking that requires meta-knowledge on mathematical modelling. The second chosen indicator focuses on interpreting with reference to the relevant modelling-specific situation, which should support the chain of argument. The last chosen indicator covers considering students as individuals and looking at their specific influence on the group that works on the modelling problem, for example, through monitoring the group work.

The analysis showed that with the help of the growth indicators, we have indeed been able to make more differentiated statements about the development of pre-service teachers' noticing competencies, specifically about the sub-facet interpreting. Regarding the three growth indicators, the following developments were additionally identified:

- a development concerning only one area (e.g., metacognition). For example, individual students were mainly considered when discussing students' use of metacognitive strategies. This is surprising, as the intervention covered all content areas equally and all pre-service teachers participated in the same intervention. It seems the intervention was successful for all cases but in different aspects.
- a development from offering situation-specific indicators that do not support the chain of arguments to providing details of a specific situation that are relevant for the interpretation: knowledge about certain concepts might not have been consolidated before the intervention (e.g., metacognition), which explains why an interpretation cannot draw on those concepts in a specific situation. During the intervention, videos and other student artifacts were analysed regularly with a specific focus on identifying relevant situations as a basis for an interpretation and therefore this development meets our expectations.
- a development towards explaining students' understanding with reference to modelling-specific knowledge and evaluating students' understanding based on knowledge about the characteristics of modelling problems: this finding can be explained in terms of literature by the competence model developed by Blömeke et al. (2015), which includes one's dispositions as basis for a perception and interpretation and thus indicates a qualitative validation of this construct. Theories concerning mathematical modelling were also conveyed in the intervention.

The presented analysis was based on selected growth indicators and led to differentiated insights into the development of three pre-service teachers concerning their noticing competencies in the context of mathematical modelling. Although the presented case study contributes to understanding the development of noticing competencies within a mathematical modelling context, the limitations should also be discussed. First, we chose three cases, and our results depend on this selection. An in-depth analysis of pre-teachers' ability to interpret was possible and complements the approach described in Alwast and Vorhölter (2021). Implementing these indicators with a greater sample might be suitable and might provide further insights. Second, we based our analysis on three out of six growth indicators and therefore decision-making was not evaluated in this analysis. As the growth indicators regarding future actions did not fit into the design of the instrument and the focus of the open questions, they could not be examined in this context; for example, we did not ask participants for a suggestion for further modelling problems, which is the focus of one of the excluded growth indicators. However, it would be desirable to analyse pre-service teachers' decision-making for a comprehensive analysis of noticing competencies – perhaps regarding Zech's (2002) taxonomy of support.

To gain more insight into the development of pre-service teachers' noticing competencies in a modelling context, the impact of knowledge and beliefs should be considered as well. In the course of the main study, pre-service teachers' modelling-specific knowledge and beliefs was also assessed, and it will be intriguing to investigate any emerging connections.

Notes

[1] All names are fictional to guarantee anonymity.
[2] Quotes are smoothened due to the translation and to allow a better readability.

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