Mathematical modelling in the teaching and learning of mathematics: Part 1

Modelação matemática no ensino e aprendizagem da matemática: Parte 1

Susana Carreira
Faculdade de Ciências e Tecnologia, Universidade do Algarve
UIDEF, Instituto de Educação, Universidade de Lisboa
Portugal
scarrei@ualg.pt

Werner Blum
Institute of Mathematics, University of Kassel
Germany
blum@mathematik.uni-kassel.de

Opening note

The editorial policy of the Quadrante journal includes the publication of one thematic issue in each annual volume. In the year 2021, with an ongoing pandemic crisis that will remain in the collective memory, the Quadrante journal will publish its thirtieth volume. The world-wide spread of the Sars-CoV-2 virus has brought, around the world, countless discussions, news, reports, and articles related to the modelling of this spread and to possible consequences that can be drawn from these models. Nonetheless, the decision to produce a special issue on *Mathematical Modelling in the Teaching and Learning of Mathematics* was taken before the pandemic began, which means that this decision is neither a reaction nor a lucky opportunity at a fitting moment. Rather, this coincidence sheds further light on the enormous societal importance of mathematical models and modelling. The relevance and necessity of disseminating research and development work in this field has become apparent from curricular rationales and recommendations across the globe, to long-term educational policies and international trends on the aims and contents for the education and training of the new generations of citizens (Bakker, Cai, & Zenger, 2021). Much research and development activities in this field over the last four decades can be seen, for instance, in the ICMI Study on Modelling and Applications in Mathematics Education (Blum,

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Galbraith, Henn, & Niss, 2007) and in the Proceedings of the biennial series of International Conferences on the Teaching of Mathematical Modelling and Applications (ICTMA, https://www.ictma.net/conferences.html). In 2019, at ICTMA-19 in Hong Kong (see Leung, Stillman, Kaiser, & Wong, 2021), Gabriele Kaiser, President of the International Executive Committee of the ICTMA Community, set the challenge of increasing the visibility of the research produced worldwide on modelling in mathematics education, by advocating for the publication of more high-quality articles in well-established scientific journals.

The current impetus given by the Quadrante journal to the work of a considerable number of researchers on mathematical modelling in mathematics education confirms the fertile activity in this field throughout the world. As guest editors, we were grateful for the positive response to our call from a prestigious group of authors across the world. The contributions that we received proved to be remarkable in quantity and quality. Due to this response, an editorial decision was made to handle two thematic issues instead of only one as originally planned. These two issues will make up the current volume No. 30 of Quadrante. This first issue contains the first section of the total collection of papers. It is composed of a set of 13 articles, which cover the following research foci: i) types and design of modelling tasks; ii) the implementation of mathematical modelling in the early years; iii) the processes involved in modelling and the models produced by students and teachers; iv) the use of technology in mathematical modelling; and v) curricular approaches that promote the integration of modelling in mathematics teaching and learning.

An overview of this thematic issue

The first article, by *Elfringhoff and Schukajlow*, provides a starting point for considering the nature and effects of modelling tasks on student learning. They conducted a qualitative study with 9th grade students aiming to know the sources of interest in a modelling task. The study intended to see if interest motivated by a modelling problem would change from the condition where the student is only invited to read the problem and consider its situational context to the condition where the student is invited to solve the problem. Some enlightening findings were achieved, namely that the real-world context is decisive for triggering interest and that emotions about the context together with self-efficacy beliefs play a predominant role. After solving the problem, the students seemed to retain interest in modelling tasks where the level of difficulty is compatible with their confidence in achieving a solution and where the problem is open enough to allow them to develop their own approaches to solve it.

The situational context of modelling problems is clearly pertinent in the case of estimation problems in real contexts, namely in the so-called Fermi problems. The article by *Segura, Ferrando and Albarracín* assumes that Fermi problems represent a special category of modelling problems, which can be used to initiate students into modelling, given that the mathematics involved tends to be reasonably accessible. In a study with future primary education teachers, the way in which the subjects incorporate complexity factors into their plans and later into their solution processes was investigated. Initially, the tasks were presented on paper with images of the real places and situations and later the subjects actually solved the problems in the real sites to which the problems referred. The results showed that the problems proved to be cognitively demanding. The study also revealed the importance of field work in the process of solving estimation tasks formulated in real contexts familiar to the students.

In the work of *Greefrath and Frenken*, we find another approach to Fermi problems, grounded on the German educational scenario in which such types of problems have regularly been used in comparative diagnostic tests in grade eight (14-year-olds). This study addressed the design of modelling tasks for standardized assessment tests that are expected to be both credible and familiar to the students and to have an adequate degree of difficulty. The authors set the goal of classifying and comparing a set of tasks given in those diagnostic tests. They found considerable homogeneity in the tasks, in terms of openness, relation to reality, and authenticity. The authors concluded that Fermi problems have validity and are promising as tasks to be included in assessment tests and thus also in lesson development for the middle grades.

Although mathematical modelling is often associated with advanced levels of education, there is growing evidence that opportunities for modelling can be created in mathematics education from the early years. Two articles address this issue. The first article is the study by *Alsina, Toalongo-Guamba, Trelles-Zambrano, and Salgado*. Based on a theory-driven rubric, they offer a detailed analysis of young children's modelling processes when the children compared the temperatures of the water from the rain, from the refrigerator, and when heated in a microwave oven. In their study, the same situation was presented to children when they were 3 years old and 5 years old. In both cases, the results showed that, from an early age, children can carry out various processes associated with mathematical modelling. These authors also found that the older children showed a higher ability to create a specific model based on their mathematical knowledge.

The second article focusing on elementary school students by *González Galaviz, Lima, and Alvarado Monroy* developed a design-based research study concerning the early introduction of functional thinking. The research adopts the Models and Modelling Perspective (MMP; see Lesh & Doerr, 2003) and consists of two iterative cycles for the design and refining of a model eliciting activity. The first cycle was implemented with eight teachers, and the second with 21 elementary-school students (11 years old). The task involved a cryptography situation in which security codes are generated with a simple rotor machine. The analysis of the design process was based on the MMP pedagogical principles, the students' emerging models, and the teachers' feedback. The authors concluded that the cryptography context was effective for the students' construction and development of the fundamental understanding of function based on their own cognitive resources and experiences with the actual context.

Several articles address the students' modelling processes and their mathematical models in various real-world situations. Starting this set of papers, we introduce the study by *Sala Sebastià, Font, and Ledezma* that seeks to identify the relationships between the modelling process and the inquiry process. This research work was developed in the context of a co-disciplinary teaching, linking mathematics and history. In embracing an interdisciplinary approach, the teachers challenged their students to find out the type of Roman building that could be related to the archaeological remains found in their city. The analysis of the students' processes showed that when inquiry and mathematical modelling coexist, there are subprocesses of both cycles taking place, which complement each other in obtaining a solution to the real-world problem.

In *Buchholtz*'s article, attention is also directed to students' modelling processes in contexts that involve interaction with reality outside the school. As part of the project "Math & The City", Norwegian students' modelling processes were examined during a math trail. Throughout the trail, questions involving real objects and physical features of the surrounding space were posed; the questions related to calculating circle measures in these contexts. The students' interactions with the objects connected to different tasks on the trail were observed, as well as their mathematisation, interpretation, and validation strategies in estimating and taking measurements. One of the important findings refers to the fact that data collection plays a key role because it constitutes the core of the extended contextualisation of the real objects. When the relevant quantities have been determined, students must be able to put the data into a suitable mathematical context. Thus, one of the potentialities found in math trails is that the transition to the mathematical model via mathematisation takes place in a coherent and sustained way.

Ärlebäck, Frejd, and Doerr report on a teaching intervention, also drawing on the Models and Modelling Perspective, where 25 pre-service teachers solved a task in a context where they had to deal with nominal variables and devise a sampling model. The participants were asked to develop a sampling model that could be used to make predictions about the number and the distribution of different coloured beads in any size population. The pre-service teachers had to perform experiments using concrete materials carefully created to support the emergence of sampling models. The results showed that the future teachers, despite having struggled with the complexity of the problem, produced different sampling models. The models differed on the type of rules established for sampling, namely the sample size and the number of samples collected. Finally, the authors discuss the features of the task and the materials used and make new suggestions for task design that may improve the current knowledge on modelling processes involving inferential reasoning with categorical variables.

This set of articles is closed by the contribution of *Vorhölter and Krüger*, who examined the students' metacognitive strategies while solving modelling problems. The study compares the results of different research methods to measure metacognitive strategies. The authors point out that, in addition to the modelling sub-competencies necessary to successfully solve a modelling problem, there are general competencies that play an important role, such as metacognition. The MeMo project has shown that different methods (named as online and offline methods) may be used for measuring students' use of metacognitive modelling strategies. The article discusses the results of the application of different methods with two students, where their participation in a particular group was considered. The results support the authors' recommendations on the precautions to be taken with the use of different instruments, such as a questionnaire or an interview. The advantages and limitations of each method for measuring metacognitive strategies in modelling problems are highlighted.

Another series of articles address the role of technology in mathematical modelling. *Galbraith and Fisher* present a comprehensive picture of the various affordances provided by technology in solving modelling problems, while offering caution about the occasional misuse of technological devices in addressing modelling situations. In particular, the authors emphasize that technology has a potentially pervasive role in modelling practices as long as it is significantly used to improve the modelling process and its results. This is the key motivation for their analysis of the simulation process, in which technology becomes indispensable to modelling. Examples involving non-linearity and simultaneity among model relationships vividly illustrate that simulation is required for solution purposes and reveal the powerful role of dynamic software (such as Stella) in modelling real-world situations. Moreover, the examples discussed show how technology can enable the development of models that are inaccessible when having only hand methods of solution. The authors make it clear in their work that technology cannot be ignored and that its use should be well thought out within modelling in mathematics education.

Based on a teaching experiment in a linear algebra course at a Costa Rican university, *Ramírez-Montes, Carreira, and Henriques* studied the modelling routes of undergraduate students in a modelling task related to the construction of passwords for banking access. The task presupposed the use of concepts and procedures associated with vector spaces and required the construction of a model for the creation of a set of passwords using a spreadsheet. The authors analysed the modelling routes of two groups of students, one of which developed an incomplete modelling route and the other a more complete route. The results showed that the way in which students build and run a computer model drives the modelling process in very different ways, suggesting that students' competencies in using technological tools are relevant to their construction of effective models and to the creation of concrete solutions to a real problem. In the analysed modelling routes, the transitions between computer results, mathematical results, and real results were shown to play a significant role on the different groups' modelling processes. The possibility of simulating the real situation in the spreadsheet, together with the ability to articulate the mathematical and the computer model, were key factors in shaping the students' different modelling routes.

The next article, also addressing the role of technology in mathematical modelling environments, is contributed by *Irigoyen Carrillo, Alvarado Monroy, and González Astudillo*. The authors note that optimisation is a curricular theme in secondary education in Mexico and that students often learn differential calculus methods to solve optimisation problems, although they often use them in an algorithmic way without assigning meaning to the quantities involved. Based on the Models and Modelling Perspective, the researchers developed the instructional design of a model eliciting activity in an optimisation situation that required finding the dimensions of a fixed volume container to optimize its surface. During the second design cycle, the problem of optimizing the surface of a container was proposed to master students who were free to use technological tools to solve it. All groups created useful models to solve the planned optimisation situation. A diversity of models was observed, and the use of different mathematical representations was stimulated by the integration of different digital tools, such as spreadsheets, GeoGebra, and text editors. The solutions involved approximation techniques for exploring different particular cases and the use of more sophisticated techniques, such as differential calculus methods to find critical points of a function with the help of the derivative.

This special issue ends with an article by *Ikeda and Stephens*, who offer a theoretical reflection on curricular approaches to mathematical modelling. The paper aims to characterise what may be described as an integrated approach to mathematical modelling, in combining the views of modelling as content and of modelling as vehicle. An integrated approach to teaching mathematical modelling holds the longstanding idea that mathematical modelling and applications must be integrated into and contribute to elementary and secondary students' overall mathematical education. Drawing on a review of selected papers from the ICTMA Series books, the authors raise important questions, especially concerning the aims of mathematical modelling and the challenges that curricular design faces in promoting the integration of mathematical modelling in school and tertiary education. From their literature review, the authors draw the conclusion that there are two dominant conceptions of an integrated modelling approach. In the first, there is a predominance of mathematical activity, which means that mathematisation and the construction of mathematical knowledge acquire a central place in the modelling activity. The outcomes of mathematical modelling are seen as mathematical knowledge that can be used both in the real world and in the mathematical world. The second conception of an integrated modelling approach emphasizes mathematical modelling as an interaction between a real world and a mathematical world, giving the modelling activity a more multi-directional character. Rather than focusing on the outcome of modelling, this perspective takes the relationship between mathematics and reality as a central point. As such, the meaning and development of mathematical knowledge is seen to arise from the action of connecting the two worlds.

In closing this thematic issue, we believe that substantial evidence has been gathered that can fuel today's national and international debates on the integration of modelling in mathematics curricula and on the relevance of modelling in mathematics teaching and learning, inside and outside the school doors. This should be emphasized at the present time, especially, as is the case in Portugal, when a thoughtful curriculum revision in mathematics education is underway. As asserted in Niss and Blum (2020), despite the progress made in research and development, the manifest inclusion of mathematical modelling in the curriculum and practices of mathematics education still demands particular attention from most countries in the world.

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