A method for meta-reviews of research unfolding the unavoidable systemic nature of affect

Um método de meta-revisão da investigação que revela a inevitável natureza sistémica do afeto

Chiara Andrà
University of Eastern Piedmont, Vercelli
Italy
chiara.andra@uniupo.it

Andrea Amico
University of Eastern Piedmont, Vercelli
Italy
andrea.amico@bigpicturelearning.it

Cristina Scalvini
University of Eastern Piedmont, Vercelli
Italy
20043754@studenti.uniupo.it

Luca Doria
University of Eastern Piedmont, Vercelli
Italy
20043885@studenti.uniupo.it

Peter Liljedahl
Simon Fraser University
Canada
liljedahl@sfu.ca

Matteo Pezzutto
University of Eastern Piedmont, Vercelli
Italy
20044385@studenti.uniupo.it

Quadrante: Revista de Investigação em Educação Matemática 32(2) 6-24
https://doi.org/10.48489/quadrante.31565
Received: June 2023 / Accepted: November 2023

Abstract. This paper can be considered a methodological contribution for qualitative meta-reviews of research published in the field of affect in mathematics education. We apply this method to a Special Issue and to an ICMI Book, in order to test whether affect emerges in its systemic nature, or whether single affective variables are considered in each paper under consideration. Different types of cases emerged in our investigation.
Keywords: qualitative meta-review; affective variables; unidimensional; multidimensional; affect system.

Resumo. Este artigo pode ser visto como uma contribuição metodológica para a realização de meta-revisões qualitativas de trabalhos de investigação publicados na área do afeto em educação matemática. Aplicamos esse método a um número especial de uma revista científica e a um livro da série de estudos do ICMI, a fim de averiguar se em cada artigo em consideração o afeto emerge na sua natureza sistémica ou se, ao invés, as variáveis afetivas são consideradas isoladamente. Diferentes tipos de casos surgiram na nossa investigação.
Palavras-chave: meta-revisão qualitativa; variáveis afetivas; unidimensional; multidimensional; sistema de afetos.

Introduction and background

In the paper presented at the annual International Conference "Mathematical Views" (MAVI) in 2017, Peter Liljedahl starts with noting:

> Research in the affective domain has always been restricted to focused attention to a single affective variable. This is ironic given that we know that affective variables tend to cluster. Perhaps the reason for this is that we lack theories for thinking about affective clusters (Liljedahl, 2018, p. 21).

This paper can be considered as seminal in pursuing the aim of understanding affect in mathematics teaching and learning not as a collection of variables that should be studied separately, but rather as a system of variables. The metaphor of a system entails both an idea of relationships among variables and an idea of structure among these relations, a structure that is both tied to personal experiences and dynamic (Malmivuori, 2006). Building upon Green's (1971) idea of beliefs as a system, Liljedahl (2018) proposes to consider affect as a constellation of beliefs, attitudes, emotions, goals, and efficacy. Each person holds their own affective system, which can be represented using a connected graph.

In this paper, we aim at better understanding why affect-related research in mathematics education has mostly focused on a single variable at a time, and to propose a theoretical framework to examine the existing literature in terms of uni- versus multi-dimensional focuses with respect to affect-related phenomena. The theoretical background for this is represented by McLeod's paper written in 1992, and Di Martino and Zan's (2011) and Hannula's (2012) ones, which represent two moments, 20 years apart, in the history of affect-related research in Mathematics Education, when researchers felt the need to define what affect is and how it is structured. We recall the content of these contributions, starting with the 1992 paper.

Talking about affect in mathematics education, we can say that there is a "before McLeod" and an "after McLeod": indeed, he was the first who tried to provide a complete picture about the state of the art until the early 90s, reconceptualizing much of the research in the cognitive affective field. McLeod's (1992) framework identifies three major categories in the research of mathematics-related affect: beliefs, attitudes, and emotions. McLeod identifies these three characteristics as manifestations of affect, characterised by different degrees of stability, intensity, and cognitive involvement. Beliefs represent one of the extremes of the scale: for McLeod, in fact, beliefs are the most stable, the most emotionally modest and the most cognitively associated, in contrast to emotions, which are the most unstable and intense. Attitudes lay somehow in the middle between these two extremes and originate from repeated emotional reactions. The socio-cultural context of origin contributes to the formation of an individual's beliefs, which are then also shaped by personal experiences. Beliefs seem to play an important role in students' emotional reactions in mathematical situations. McLeod's first elaboration on affect in mathematics learning and teaching seems to be oriented towards separating and defining distinct affective dimensions and it has shaped the way researchers still today approach affective variables. In fact, McLeod's merit is to define and separate the variables so that they can be compared and their interactions can be studied.

While it is true that McLeod was a pioneer in research on affect in mathematical education, Hannula (2012) argues that some issues remain open after McLeod's systematisation. Terminological ambiguity, in McLeod's paper, represents a main problem and for Hannula the most problematic one is the definition of *attitudes*: McLeod, in fact, defines them as "affective responses that involve positive or negative feelings of moderate intensity and reasonable stability" (p. 581). While other researchers have provided a variety of different definitions of *attitude*, increasing ambiguity around the construct, Hannula leaves it aside and focuses on three features of affective variables, namely: *cognitive*, *motivational,* and *emotional*. *Cognition* concerns information, personal or determined by the environment, *motivation* guides behaviour in terms of goals and choices, and *emotions* act as a feedback system to the motivational and cognitive processes.

Moreover, in Hannula's work, *stability*, which McLeod considered as an indicator in the distinction between beliefs and emotions, takes on an independent dimension: in this way, the emotions involved become the *state* of the subject and the emotional stability works as a *trait* factor. The same holds for beliefs and motivation, both having a trait (stable) and a state (instable) dimension. Finally, Hannula further distinguishes among the biological and the social dimensions of affective variables, pointing both to the internal, physiological aspects of all affect-related phenomena in mathematics education, and to the socially shaped ones. Like McLeod's one, also Hannula's work has a systematic nature, hence it is aimed at classifying and distinguishing affective variables, to avoid overlaps and areas of intersection among them. Thus, we note that also this approach, yet relevant for defining and distinguishing, does not encourage researchers in searching for links among affective variables.

The approach of Di Martino and Zan (2011) highlights that many of the previous constructs related to affect do not have a systematic nature but depend largely on subjective experiences: "attraction emotions are influenced by subjects' tastes; affective reactions of being pleased and displeased are influenced by subjects' goals; affective reactions of approving or disapproving are influenced by subjects' beliefs and values" (Di Martino & Zan, 2011, p. 5). This questions the possibility of finding cause/effect laws that can define the interaction between affect and cognition; Di Martino and Zan's work shifts from a normative approach, aimed to explain causes of behaviour to predict it, and focus on an interpretative one, through the free narration of students' relationship with mathematics; in this way it is possible to highlight what aspects they consider relevant in their experience and how these interact with each other. The introduction of personal interpretation in the construct of attitude allows the researchers to characterise *attitudes* on the basis of three aspects: (i) emotional disposition towards mathematics, (ii) perception of being/not being able to accomplish the task (often called "perceived competence") and (iii) personal vision of mathematics. The first one is expressed through the appreciation towards the matter ('I like/I dislike mathematics'), while perceived competence relates to the students' ability ('I can do it/I can't do it'). Di Martino and Zan focus on the nature of the interplay between beliefs and emotions; their analysis finds different patterns of attitude, depending on the student's perceived competence and vision toward mathematics, which in turn can be associated with the same negative emotional disposition. This model of attitude does not only seem to create an explicit link between beliefs and emotions, but also underlines how they interplay. Di Martino and Zan's (2011) work, differently from McLeod's (1992) and Hannula's (2012) ones, tries to define theoretically a construct stemming from the relations among affective variables such as beliefs and emotions, hence trying a systematic approach to affect-related variables.

A few interesting things emerge from this review. The first one is that some affective variables, like attitudes, are understood as being multidimensional in their very nature, whilst other variables, like beliefs, are so complex that need to be considered on their own in order to be studied. This entails that some studies in affect-related research in mathematics education might have considered mostly one dimension of affect and focused on better defining it, whilst other studies might have considered theoretical (i.e., drawing on theories that consider affect as systemic in nature), or empirical (i.e., emerging from data) approaches that have led them to consider the interactions of different affective variables to explain some teaching/learning phenomena. The second interesting fact that emerges from our review is that there seems to be a quite reasonable agreement among the main dimensions in affect-related research, which are identified as: the cognitive (beliefs), the affective (emotions) and the conative ones (motivation), with a distinction among the self and the social that emerges somehow independently from the other three dimensions. The dimension of self acknowledges the explosion that identity research in mathematics education has seen in the past two decades (Graven & Heyd-Metzuyanim, 2019). According to the seminal work by Sfard and Prusak (2005), identity is a collection of reified, significant and endorsable narratives about a person. These narratives can be authored not just by the person about him/herself (First person narratives) but also by others to her (Second person) and about her (Third person). Many researchers have adopted Sfard & Prusak's definition, not just because of its socio-cultural roots, but also because of its operationality (Graven & Heyd-Metzuyanim, 2019). The social dimension acknowledges the so-called *social turn* (Lerman & Zevenbergen, 2004) in mathematics education, a turn that took place at the beginning of the 2000s.

With these five dimensions in mind, we propose to analyse a sample of papers written in the field of affect in mathematics education: 6 from the 2006 ESM Special Issue, which came 14 years later than McLeod's (1992) seminal work, and 6 from the 2015 ICMI book, which came about 4 years later than the turning point established by Hannula's (2012) and Di Martino and Zan's (2011) contributions. We, then look at how much these 12 papers address each one of these three dimensions, and how they link them (if they do so). In order to do so, we adopt a pentagon (Figure 1).

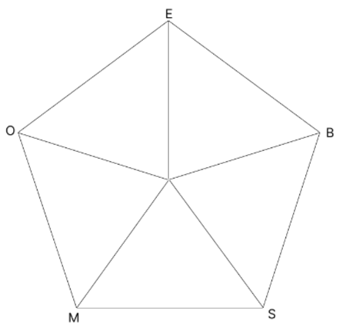

Figure 1. The five dimensions according to which we analyse papers in affect-related research. E stays for Emotions, B for Beliefs (and represents the cognitive dimension), S for self (and includes self-efficacy and identity), M for motivation (the conative dimension) and O for Social (the "others" and the environment).

Why a pentagon? Because it offers a model useful to represent the dimensions, on which a paper focuses, in a versatile way. With its five vertices, each representing a different aspect (emotions, beliefs, motivation, self, and social), the figure allows a holistic view of the multidimensional system of affect. Each vertex of the pentagon is linked to the centre of the figure and a point will be fixed on each one of the five segments that link the centre with each vertex: the closer the drawn point approaches the perimeter, the more it is considered decisive to define the affectivity researched in the considered paper. Given the wealth of studies on the subject and the divergence of approaches, the pentagon offers a good general eye on how much the main aspects have been considered, also in relation to others.

Methodology

Our research hypothesis stems from the consideration we drew in the previous section, namely that two main types of research can be conducted on affect-related phenomena in mathematics education.

Type 1 papers focus on a single variable and type 2 ones are multidimensional. Usually, the aim of type 1 papers is to define the role of the researched variable in a certain setting or situation and usually it is theory-driven. Namely, it (i) employs an established theory on a certain construct (e.g., anxiety), which usually comes with a questionnaire that allows to measure such a construct, then (ii) applies the theory and the questionnaire to the context under analysis and finally (iii) analyses the responses to the questionnaire and answers the research questions. The merit of type 1 papers is to contribute to better understanding each affective variable, to enrich our knowledge on how it changes in different contexts, to have new tools in our hands to investigate and focus on it.

Type 2 papers usually adopt a qualitative/interpretative methodology. They start from some theory, but it is the nature of their data that counts. Being it a series of interviews, or open-ended questionnaires, or commented drawings, or the like, it is as if from these data more than one affective variable emerges and needs to be considered in order to understand what is going on with the data. It is as if the data pushes the researcher to consider more than one theory, and thus more than one variable, in order to be able to analyse it. We introduce such a distinction between type 1 and type 2 papers at the beginning of methodology, because for us this is the core of our contribution. In fact, the tool of the pentagon allows us to see how many variables, and to which extent each one of them, are taken into consideration in an individual research paper. It is a visual representation of this, but our methodology stems from a fundamental distinction: the one between type 1 and type 2 papers, indeed.

The data of our research consist in a selection of papers that we do not consider exhaustive but serve some purposes we account for in what follows. First, we selected some papers from the 2006 Educational Studies in Mathematics (ESM) Special Issue on affect and some chapters from the 2015 ICMI Book, which in our view represent collections of papers from distinguished scholars in the field. Secondly, by applying the model of the pentagon to analyse the affective variable(s) under focus for each contribution, we can see whether a paper is uni- or multi-dimensional in its own focus, and the weight of each variable. Then, comparing the pentagons, we can compare the papers, but also examine possible evolutions over time in the field. Finally, we mentioned the weights of each variable in the pentagon, thus we now clarify how we introduced a scale from 0 to 5 to determine how much a variable of the pentagon is taken into consideration. The weight of 0 means that the respective variable is not even mentioned, and on the opposite the weight of 5 means that the variable is very central to the paper: its name can appear in the title, it is pervasive of many sections and, in particular, it is present both in the theoretical framework and in the data analysis/results. The intermediate values have been assigned in intermediate cases. For example, 1 is assigned if a variable is mentioned in one section of the paper, whilst 4 is assigned if a variable is central for, say, the theoretical framework, but it is not used in the data analysis, and it can be mentioned again in the conclusion or the discussion.

Data analysis

In analysing our data, we first present a summary of each selected paper, then we draw the pentagon.

In the 2006 ESM Special Issue on affect, DeBellis and Goldin (2006) introduce the constructs of mathematical intimacy (tied to emotions and engagement) and mathematical integrity (related to persistence), and they consider affect as an internal representational system, exchanging information with cognitive systems. The authors conceptualise the existence of affective structures that incorporate: values, beliefs, attitudes, feelings. Affective structures interact with cognitive configurations.

Emotions are tied to intimacy and to engagement and are very central in DeBellis and Goldin's (2006) work. They are rated 5 over 5 in our scale. Another important dimension is motivation, which is related to perseverance and integrity in Debellis and Goldin's (2006) work, who however dedicate not much space in order to elaborate on them, if compared to emotions. Thus, a score of 4 over 5 is assigned to motivation in our scale. Also, the social dimension has a certain relevance in this paper, as an individual interacts with other individuals. Beliefs are just mentioned (and a score of 1 is assigned), and the self lies at the background, and it can be considered marginal (score: 0). From these considerations, the pentagon in Figure 2a is, in our view, a summary of the variables considered in this paper.

Malmivuori (2006) focuses on students' self-perception systems and self-regulation processes, emphasising the importance of self-evaluations and ongoing self-regulation. The framework integrates affect, cognition, and behaviour, going beyond traditional static concepts and highlighting the dynamic nature of these factors. The article suggests that students with high self-awareness, positive self-evaluations, and effective self-regulation are able to improve their mathematics learning and problem solving. By consciously dealing with negative affective responses, students can optimise their learning processes. The study shows that self-confidence and affective responses significantly influence self-regulation, with high self-efficacy and positive affective responses, favouring persistence and preference for mathematical challenges. Conversely, low self-esteem and maths anxiety hinder self-regulation and performance. The results highlight the interdependencies between students' self-evaluation, affective responses, and self-regulation, thus assigning to emotions a predominant role (enjoyment, fear and anxiety are central in the study), followed by the self-related aspects (both self-efficacy and self-esteem), which are strongly related to emotions in this research, in turn followed by the social ones. To note, socio-cultural theories are central for the theoretical background of Malmivuori's study, but they are not applied in the data analysis. Thus, a score of 4 in our scale is assigned. Beliefs are mentioned in the data analysis, being integration of new knowledge and risk-taking considered as beliefs; similar considerations for motivation, for which persistence is mentioned in data analysis. Since beliefs and motivation are just mentioned, a score of 1 is given to each of them. All in all, Figure 2b emerges as the outcome of our analysis of Malmivuori's paper.

We now summarise Hannula (2006), whose analysis is sketched in Figure 2c. Frank's case, as presented in the opening of ESM Special Issue, explores the relationship between motivation, goals and needs in the context of self-regulated learning. Frank's beliefs about mathematics and his mathematical ability influence his motivation. Frank sees mathematics as an ever-evolving field with multiple approaches to problem solving and recognizes its relevance to daily life. Frank has both general and specific maths-related goals, including doing well, solving the assigned problem, and achieving mastery. His confidence in his mathematical proficiency is high, but he is less confident about the specific task assigned in the episode under analysis. Frank demonstrates a desire for fluency and a preference for solving problems without relying on a calculator. Beyond Frank's case, Hannula argues that motivation regulation is central to understanding students' behaviour in the classroom. Motivation, structured by needs and goals, has the potential to direct behaviour through emotional control mechanisms. Frank's case illustrates that students can have multiple simultaneous goals, and choices between these goals are influenced by the learning environment and cues from teachers. Negative emotions, such as worry and frustration, can affect problem-solving performance, while positive affective states such as curiosity and interest can facilitate it. This view of motivation aligns with some aspects of other theoretical frameworks discussed in the field. Motivation is seen as a representation of goals and needs, closely related to the concept of identity and desired identity, conceptualised as mathematical self. Creating a supportive learning environment involves meeting students' needs for autonomy, competence and social belonging. In analysing Hannula's (2006) paper with the pentagon tool, we can notice that at theoretical level the paper establishes links between affective variables: motivations are the very heart of this paper (and they are mentioned also in its title), emotions and beliefs are deemed as important, while social needs and mathematical self are mentioned but less central if compared to the other affective variables.

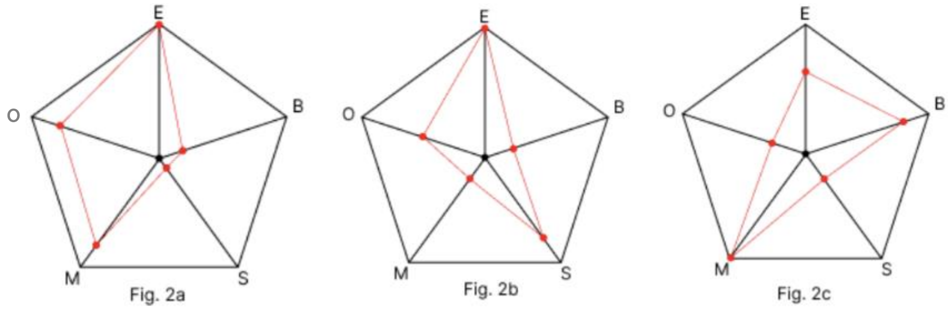

Figure 2. The pentagons that summarise (a) DeBellis and Goldin's (2006), (b) Malmivuori's (2006), and (c) Hannula's (2006) papers in *Affect in Mathematics Education: Exploring Theoretical Frameworks: A PME Special Issue*, published in Educational Studies in Mathematics

Brown and Reid (2006) explore Damasio's somatic marker hypothesis and its implications for understanding decision making in different contexts. The hypothesis suggests that many decisions are unaware and are influenced by emotional cues known as somatic markers. These markers serve as an emotional basis for unconscious decision making, occurring before conscious action and reflection. The authors argue that in areas such as teaching and mathematical reasoning, most decisions are unconscious and becoming aware of them could interrupt ongoing activities. They emphasise the importance of identifying decision points and detecting inherently unobservable aspects of decision making. Using the somatic marker hypothesis, researchers can uncover these hidden processes. Furthermore, the authors highlight the fundamental role of emotions in cognition and decision making. Emotional orientation and purpose are considered significant factors in the development of teachers and mathematical reasoning. The authors suggest that the incorporation of somatic markers into research and teaching methodologies could enhance learning experiences and contribute to a transition from simplistic and dichotomous decision making to more complex and relational decision making in teachers' practices. Somatic markers have a social and an internal nature: the former remains in the background of Brown and Reid's study, while the latter is emphasised particularly because somatic markers offer insight into emotions. Somatic markers guide actions, which in turn are supported by purpose, which is tied to motivation. Beliefs and the self are not mentioned in this study and the pentagon that we draw is shown in Figure 3a.

Op't Eynde et al. (2006) explore the students' emotional experiences when solving mathematical problems in the classroom. They find that there is a unique and individual flow of emotion for each student, influenced by her interpretations and evaluations of problem-solving events. Different students may interpret and evaluate the same events differently based on individual factors and the context of the assignment. Negative emotions, particularly frustration and anger, are often experienced by students when confronted with difficult problems or when an immediate method to the solution is not apparent. The study suggests that teaching students how to deal effectively with these negative emotions is important for teaching problem solving in mathematics. Allowing space for negative emotions can be cognitively and motivationally beneficial, as it indicates that students are interested in finding solutions and are motivated to succeed. Understanding emotions in the mathematics classroom requires acknowledging their situatedness and complexity, and researchers need methods that capture students' real-time interpretation and evaluation processes. In drawing the pentagon represented in Figure 3b, we considered the centrality of emotions in Op't Eynde et al.'s work, emotions that are social and situated, and this feature allows us to consider the social dimension as important for the researchers. Beliefs and motivation are mentioned in the theoretical background, while the self emerges from data. In fact, the authors argue in the theoretical framework that the students refer to their underlying beliefs when they report on their emotions, and in the data analysis the researchers mention a student's self-perception of being able to solve the problem.

Evans, Morgan, and Tsatsaroni (2006) explore how the ideas, emotions, and actions of participants in a mathematics classroom are shaped by their interactions and the discursive practices at play. The relationship between positions in discourse and positions assumed in practice is highlighted, showing how hierarchical positions are reproduced and the role of emotions in adopting, modifying, submitting or claiming a position. The study reveals that students' enthusiasm and anxiety are associated with their positions within different discursive practices. An interplay between mathematics discourses and everyday discourses is observed, where anxiety is linked to the competition and value conflicts between official pedagogical discourse and local practice. Pleasure and enjoyment play a significant role in forming students' positions, with pleasure deriving from youth culture and progressive pedagogical discourse influencing submission and acceptance of subordinate positions. Evaluation, both of oneself and of others, is crucial in establishing individual positions and identity. The nature of mathematics and the pedagogical discourse, especially the evaluation criteria, interact with other discursive resources and personal stories, creating specific positions and generating emotions. Emotion is seen as an integral part of the social organisation of the practice. The study recognizes that the focus on group work in the classroom may limit the expression of observed emotions, as school mathematical discourses often offer little opportunity for their expression. However, it is suggested that explicit evaluation criteria may allow less powerful students to evaluate their own work and that of others. We notice, overall, that emotions are central also in Evans et al.'s paper, being them both socially organised and implicated in constructing identity. The social aspect is related also to positioning and the self to identity. Furthermore, this study highlights that the social, the self and the emotion are strongly related and figure 3c tries to capture this rather unique feature that emerges.

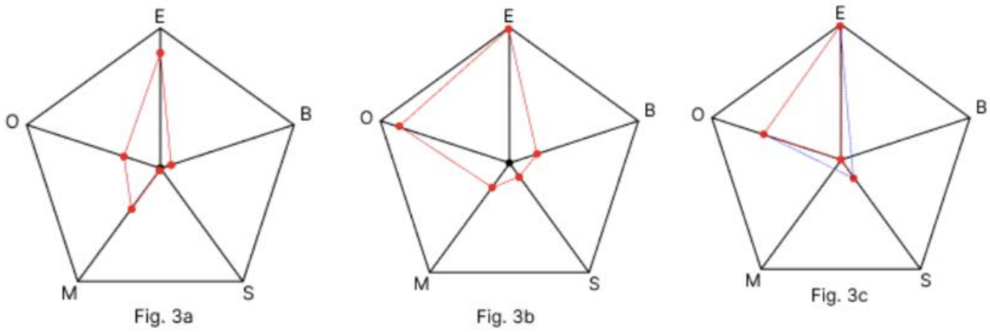

Figure 3. The pentagons that summarise (a) Brown and Reid's (2006), (b) Op't Eynde et al.'s (2006) and (c) Evans et al.'s (2006) papers in *Affect in Mathematics Education: Exploring Theoretical Frameworks: A PME Special Issue*, published in Educational Studies in Mathematics

Looking at figures 2 and 3, we can notice that emotions are the most investigated dimension in almost all the papers considered in the 2006 ESM Special Issue on affect. Also, the social is an important dimension in 3 over 6 papers, followed by motivation (2 over 6). The self and beliefs are only considered as central in one paper each. A reader might ask what these diagrams show more, with respect to what is summarised in the paragraphs above, and this is a good question. First of all, and naively, we can say that the diagrams provide a holistic representation of a quite lengthy analysis made in words. In doing so, it becomes easier to pinpoint the dimensions that are more relevant in each study. Secondly, in building the pentagons, we also have to rate how important is each dimension for each paper, thus introducing a scale that can be used for comparison. Thirdly, and consequently, we can look at the areas identified by the irregular polygons and immediately identify the papers that address more and more than one dimension (being the area in red larger), in contrast to those that address mostly one dimension. Figure 3a is an example of the latter, whilst Figures 3b and 3c consider emotions and the social, showing a larger area compared with Figure 3a, and Figure 2a shows an even larger area considering three dimensions as quite important.

We conduct a similar analysis on a selection of chapters from the 2015 ICMI Study Springer Book on Affect, to further test this methodology. In particular, we consider the first three chapters of the section dedicated to interest, motivation and values, and the first three in the section dedicated to interest and flow, for sake of space.

In their chapter, Achmetli and Schukajlow's (2015) investigate whether constructing multiple solutions while solving real-world problems affects the students' experience of competence and their interest in mathematics, and how constructing multiple solutions works together with experience of competence in improving the students' interest. The researchers draw on theoretical considerations and empirical results, showing that constructing more than one solution for one problem provides the students with feedback about their competence in the learning situation and positively affects their understanding of mathematics. In Achmetli and Schukajlow's (2015) theoretical framework, interest is linked to attention, goals, levels of learning, thus it is related to strategy use, self-regulation, performance goals and achievement. The (need for) competence is tied to motivation, self-regulation, ability, beliefs and goals. Despite this model, the students in Achmetli and Schukajlow's (2015) study reported a similar level of interest at post-test after a four-lesson teaching unit, in both the multiple solution and in the one solution conditions. A possible explanation for this incongruence between theory and data is given by the authors: i.e., in the current study, students in the multiple solution condition had to use specific mathematical procedures, which were presented at the beginning of the teaching unit, and were not given the opportunity to choose their own individual routes to the solution during the teaching units.

The relevance of students' prior interest to their experiences of competence as well as their interest at post-test was another investigated point, however, the direction of this connection was not completely clear. The results of this study show that students' prior interest positively influences their experiences of competence, while a higher experience of competence during the teaching unit did not lead to a higher interest at post-test.

In drawing the pentagon (Figure 4a), we first consider that emotions and beliefs are not considered in this study. The self is, in Achmetli and Schukajlow's (2015) words, the most important dimension, especially self-efficacy, which is tied to the need for competence. Motivation (interest) is also very important for their study, whilst the importance of the social aspect emerges from data, as the students had to use a specific mathematical procedure.

Dobie's (2015) research takes on a sociocultural approach to the study of perceived utility value in mathematics. In particular, it examines students' values and considers the role that one's values might play in ideas about the usefulness of mathematics. Dobie (2015) maintains that there is a multitude of direct and indirect influences on one's achievement-related choices and academic performance. Some indirect influences are personal beliefs, such as one's own self-concept or one's perceptions of others' expectations, while others are features of one's environment, such as family demographics and existing cultural stereotypes. All of these beliefs and features of the environment influence, in Dobie's (2015) view, two factors that directly affect achievement-related choices and performance, namely: expectations of success and beliefs about the value of a task. The findings from Dobie's research highlight three main themes. First, students emphasised the importance of mathematics being useful. Second, students in this study exhibited strong interdependent values, related to both family and collaboration with others. Third, some students made connections between their interdependency values and the usefulness of mathematics, highlighting ways they could use mathematics to help others. This third theme emerges from data and allows us to conclude that the two dimensions of the affective system are connected, somehow a posteriori, by Dobie, after having analysed the data. In drawing the pentagon (Figure 4b), we consider that Dobie stems from the definition of values as beliefs, as well as we tie interdependence with the social dimension of our model. Emotions and motivations are not mentioned, whilst the models of the self are mentioned and a socio-cultural approach to the self is taken. However, we also notice that the self takes on a rather marginal role in this study. We draw a blue line to represent the established connection between interdependence and values (third theme).

Middleton, Mangu, and Lee (2015) investigate the motivations of high school students, self-declaring their intentions (or not) to attend a STEM academic path after high school, and the volatility of such intentions between grade 9 and grade 11 in the US. The authors understand interest and utility as sub-dimensions of motivation, which in turn positively influence self-efficacy, which is meant as another sub-dimension of motivation as well. According to some of the theories of motivation employed by other papers analysed in this study (e.g., Achmetli and Schukajlow, 2015), interest is a sub-dimension of motivation, but usually utility belongs to the beliefs area, while self-efficacy belongs to the one of the self. Hence, in drawing the pentagon for this paper (Figure 4c), we opted for using the red line to analyse the dimension that emerges, which is motivation, but we employed the blue one to somehow unpack and keep track of the actual dimensions considered.

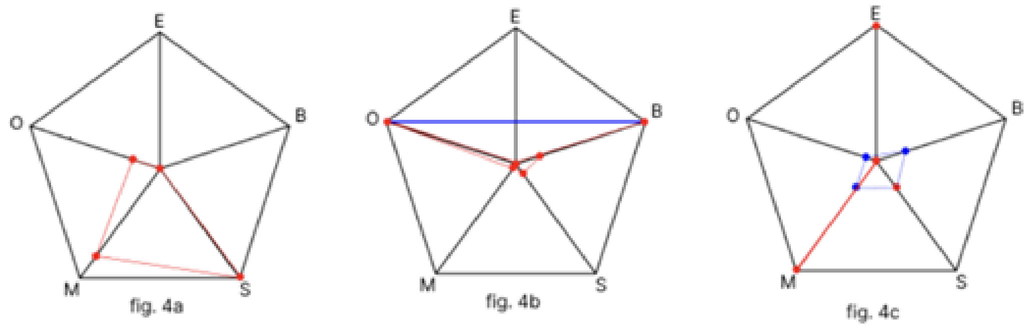

Figure 4. The pentagons that summarise (a) Achmetli and Schukajlow's (2015), (b) Dobie's (2015) and (c) Middleton et al.'s (2015) chapters in the book entitled *Affect and mathematics education: Fresh perspectives on motivation, engagement, and identity*, published by Springer

The first three chapters analysed come from the section of the book dedicated to interest, motivation, and values. This focus on motivation is best mirrored in Figure 4c, being the only dimension investigated and making the paper an almost perfect example of type 1 papers. Also Figure 4a reflects the focus on motivation, but more attention is given to the dimension of self in the paper considered. In Figure 4b, indeed, motivation is close to zero and beliefs (values) and the social play a predominant role.

Finally, our data analysis considers three chapters coming from the section on engagement and flow.

De Simone's (2015) research objective is to study rationality in teachers' decision-making processes, using the theoretical framework of the philosophical theory of rationality by Habermas (1998), adapted to mathematics education (Boero et al., 2010; Boero & Planas, 2014). Habermas speaks of the discursive rationality of a human being who "is able to account for his orientation towards validity claims" (Habermas, 1998, p. 310). He further explains that discursive rationality has three different roots: knowledge, action and language (knowing), acting and speaking. Each root of discursive rationality is related to a specific type of rationality: (i) knowledge is related to epistemic rationality (i.e., an individual's ability to evaluate available information, to make evidence-based judgments, and to reach rational and well-founded conclusions); (ii) action is connected to teleological rationality (i.e., the link between an individual's actions and the results that are intended to be achieved through those actions); (iii) language is linked to communicative rationality (in fact, the communicative use of linguistic expressions not only gives "life" to the subject's intentions, but also represents the communicative state actually in progress and also establishes interpersonal relationships with other individuals).

Habermas is criticised by other authors (Rienstra & Hook, 2006), who claim that Habermas did not take into account the human being and all his sensorial experiences, of despair, humiliation and adventure. De Simone (2015), therefore, hypothesises that the rational factors of decisions are closely intertwined with the affective ones, leading her research to a study concerning the interweaving of these factors. The article analyses the actions of a mathematics teacher within her class, using a theoretical framework that tries to integrate the cognitive and affective dimensions. To conduct this analysis, Habermas' theory of rationality and the concept of emotional orientation developed by Brown and Reid (2006) in mathematics-related affective research are combined.

Through an interview with a teacher, her expectations about the lesson and the lesson itself are analysed focusing on emotional indicators (gestures, facial expressions, and the like), in order to identify the interconnection between the rationality of the teacher and her emotions. The article concludes that it is possible to outline the three different natures of the interweaving between rationality and emotionality of the teacher. Epistemic emotionality consisted of both solving a quadratic equation (rational key) and expecting students to feel the need for justification (emotional key). Furthermore, the teacher's teleological emotionality manifested itself in a meta-level teleological component that went beyond simply clarifying what the students were doing. In general, her discourse, being emotionally involved, could not be neutral and for this reason there were two different aspects to her discourse: on the one hand, there was what she was saying, but on the other, there were all the aspects different (e.g., gestures, prosody) which showed her emotional involvement in the discussion. Therefore, the teacher also had a communicative emotionality. Given that rationality is a cognitive dimension, the only affective dimension is emotion and the resulting pentagon is the one drawn in Figure 5a.

The work of Khalil, Lake, and Johnson (2015) has roots in the theory of engagement structures (Goldin, 2014) and takes into account the professional development of teachers through trajectories and teaching practices (Borko et al., 2014). Through the engagement structures theory, researchers intend to take into account the whole affective sphere (i.e. behavioural, affective and social constellation) that positions and describes the state "in the moment" that the student experiences. In the same way, through the professional development of teachers, meant as the need to improve their teaching practice, Khalil et al. (2015) also took into account the 'teaching' components, whose practice is mediated by their history and dispositions as learners. Teaching behaviours have three dimensions: affective, which guides the feelings; motivational, useful for the determination of needs and goals; and cognitive, considered the least important. The investigation was carried out through the observation of two teachers in action: an expert and a novice. What has emerged is the importance of taking into account one's affective domain, to encourage a change in attitudes and beliefs. Teachers who develop these characteristics while teaching mathematics, can create experiences of insight, a valuable resource in fostering mathematical performance. The novice teacher, in fact, immediately brings these intuitions into practice: during her second rehearsal, she benefits from her positive beliefs and teaching skills built during the first experience. Considering the engagement structures theory, there is a recognition of the driving need to decrease the disparity between the current perception of teachers' teaching ability and their motivation to become what they consider an ideal teacher. The most effective professional development is seen after experiencing tensions caused by gaps; this seems to stimulate not only the pedagogical development of teachers, but also their adaptability to classroom situations. In drawing the pentagon (Figure 5b), we firstly dwell on the high importance given to emotions in this chapter, as emotions affect a teachers' practice. Also, motivation, understood as tied to "what we need", influences it and is quite important in this study. Beliefs and the social contests emerge in the data, especially when the turning points for teachers are taken into account. The self lays in the very background.

The goal of Montoro and Gil (2015) is to explore the characteristics of a mathematical task in order to help students experience a state of flow, useful to create the appropriate educational environment. The term flow was introduced by Csikszentmihalyi (1975) and represents a state of deep concentration in which people feel more motivated and achieve better performance; it is characterised by the isolation of the subject from what is not the activity itself and a feeling of reward from challenge accomplished. This state of extreme concentration requires clear goals, balanced tasks, and immediate feedback: the odds go up when the ongoing activity is a challenge neither too easy nor too difficult. According to

Liljedahl (2016), the mentioned state can be a tool of analysis for teacher's professional practice. From the data analysis, Montoro and Gil discovered how the number of students experiencing the state of flow increases during group work, when the mathematical task is perceived with a balanced difficulty. Thanks to the discussion that emerged within the group, the students improved the resolution procedures starting from their initial misconceptions. The more the task is perceived as easy, the more students are able to trigger the flow state. The only task considered as problem solving showed the lowest percentage of flow: in this case groups of students with heterogeneous abilities hinder the flow experience in students with lower mathematical skills (they do not have time to reflect on the task). In conclusion, to facilitate the flow when working in a group, it is necessary the presence of these three factors: "pre-service primary school teachers must be clear about task requirements, receive immediate feedback on their performance, and feel able to face challenges and overcome difficulties. Moreover, they must notice that they significantly contribute to the group" (p. 305).

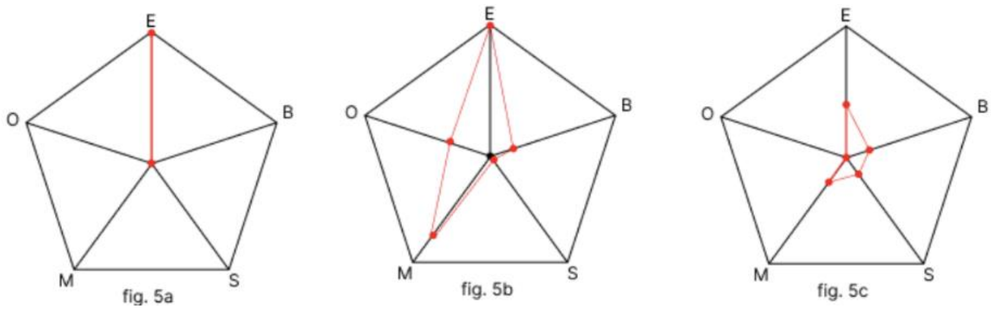

Figure 5. The pentagons that summarise (a) De Simone's (2015), (b) Khalil et al. (2015) and (c) Montoro & Gil's (2015) chapters in the book entitled Affect and mathematics education: Fresh perspectives on motivation, engagement, and identity, published by Springer

Being the three chapters in the section of the book dedicated to engagement and flow, it is not surprising that emotions play a major role, being the unique dimension investigated for Figure 5a, and the most investigated in Figures 5b (even reaching the maximum) and 5c. As well, it is not surprising that motivation is the second most considered dimension in these chapters. Beliefs are mentioned in two over three papers, while the social and the self in one paper each. In general, the areas identified by the red polygons are quite small, especially compared with the ones in Figure 2.

Discussion

This paper tries to quantify the extent to which each affective dimension, identified in the works of McLeod (1992), Di Martino and Zan (2011) and Hannula (2012), is taken into consideration in a research paper. Hence, the representation of the pentagon and a scale has been introduced.

In the sample of papers considered, two of them emerge as being "purely" unidimensional, namely De Simone (2015), which shows only the emotion dimension as having a weight different from zero (Figure 5a), and Middleton et al. (2015), for which we can see that only motivation is non-null (Figure 4c). While the former case can be claimed to be purely unidimensional because the author seeks for a theoretical model that intertwines the Habermas cognitive theory of rationality, the latter can be interpreted as being multidimensional, as motivation is split into sub-dimensions that belong to beliefs, self, and motivation dimensions.

If we consider the dimension of emotion as a case, we have already noticed that a majority of papers in the 2006 ESM Special Issue on affect have considered it as the most important. In fact, in 4 out of 6 papers its weight is set to the maximum. Of course, it is a sort of relative maximum, in the sense that it is the maximum relative to the single paper and to the extent to which this dimension emerges as relevant and central both in the theoretical framework and in the data analysis.

Dwelling further on emotions, we can also claim that, among the four papers that consider them prevalent in the 2006 ESM Special Issue, Malmivuori's (2006) and Op't Eynde et al.'s (2006) ones can be considered as rather unidimensional papers, given that there is one dimension (the Self in Figure 2b and the Social in Figure 3b, respectively) that emerges as secondarily important after emotions. In these cases, we acknowledge the importance for research of defining and focusing on a single variable (emotions), but we also claim that in this case the connection with the other variables is lost. Therefore, we are prone to conclude that, while it has been important for the history of affect-related research to define individual variables, we hope that in the future there will be more papers investigating the interconnections among them.

Figures 3a (Brown & Reid, 2006) and 5c (Montoro & Gil, 2015) show that, all in all, the affective dimensions have been investigated a few, and non-affective ones take on a major role. In other cases, more than one affective dimension is considered, and they are all relevant and important.

All these considerations allow us to conclude that, from our results, it emerges that a multidimensional approach to affect emerges: (i) in the theoretical framework, or (ii) from the data (in many cases, data add a dimension to an already multidimensional theoretical approach), or (iii) it emerges as a system and this feature of affect is purposefully investigated (but in the sample under consideration this is quite rare: only Dobie's (2015) paper analyses the structure and the nature of the relation between the Social and the Beliefs, in our pentagon.

Conclusions

The aim of our research was to investigate whether affect-related research in mathematics education tends to focus only on a single dimension of affect (what we called type 1 papers), or if the trend is to consider more than one (type 2 papers). In order to do so, we introduce the tool of the pentagon in our methodology, to capture the differences across different papers. Among the 12 papers analysed, we can say that two of them are unidimensional, whilst the other ten, at least to a limited extent, address more than one affective dimension. In saying this, we maintain that our investigation highlights the necessity, for researchers, to consider the relations among at least two affective variables in explaining didactical phenomena in mathematical classes. However, as we claimed in the introduction, a system is not only a collection of variables, it is not only a collection of relations among variables, but it also has a structure. The capability of the pentagon to capture the connections among the variables, as such connections emerge from a paper, seems to be less questionable than its capability to highlight the structure of the system. However, our study represents a step forward in the understanding of how the field of affect-related research is addressing the tendency of affective variables to cluster.

We also claim that another merit of this study is to attempt a meta-review that has a qualitative nature. Differently from the main trend of meta-reviews in these years, our lens of analysis allows an in-depth examination of the papers selected and the criteria to select the sample can be adapted to the aim of the research. Of course, this method brings with it all the limitations of a qualitative study, as well as all the potentialities.

References

Achmetli, K., & Schukajlow, S. (2015). Multiple solutions, the experience of competence and interest. In M. S. Hannula, G. C. Leder, F. Morselli, M. Vollstedt, & Q. Zhang (Eds.), *Affect and mathematics education: Fresh perspectives on motivation, engagement, and identity* (pp. 39–66). Springer.

Boero, P., Douek, N., Morselli, F., & Pedemonte, B. (2010). Argumentation and proof: A contribution to theoretical perspectives and their classroom implementation. In M. M. F. Pinto & T. F. Kawasaki (Eds.), *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education*, (vol. 1, pp. 179–204). PME.

Boero, P., & Planas, N. (2014). Habermas' construct of rational behavior in mathematics education: New advances and research questions. In P. Liljedahl, C. Nicol, S. Oesterle, & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36* (Vol. 1, pp. 205–235). PME.

Borko, H., Koellner, K., & Jacobs, J. (2014). Examining novice teacher leaders' facilitation of mathematics professional development. *Journal of Mathematical Behavior, 33*, 149–167. https://doi.org/10.1016/j.jmathb.2013.11.003

Brown, L., & Reid, D. A. (2006). Embodied Cognition: Somatic markers, purposes and emotional orientations. *Educational Studies in Mathematics, 63*(2), 179–192. https://doi.org/10.1007/s10649-006-9027-3

Csikszentmihalyi, M. (1975). *Beyond boredom and anxiety*. Jossey Bass.

De Simone, M. (2015). Intertwining of rationality and emotions in mathematics teaching: a case study. In M. S. Hannula, G. C. Leder, F. Morselli, M. Vollstedt, & Q. Zhang (Eds.), *Affect and mathematics education: Fresh perspectives on motivation, engagement, and identity* (pp. 233–254). Springer.

DeBellis, V. A., & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: A representational perspective. *Educational Studies in Mathematics, 63*(2), 131–147. https://doi.org/10.1007/s10649-006-9026-4

Di Martino, P., & Zan, R. (2011). Attitude towards mathematics: A bridge between beliefs and emotions. *ZDM Mathematics Education, 43*, 471–482. https://doi.org/10.1007/s11858-011-0309-6

Dobie, T. E. (2015). A sociocultural examination of utility value in mathematics: The role of interdependence in middle school students' perceptions of usefulness. In M. S. Hannula, G. C.

Leder, F. Morselli, M. Vollstedt, & Q. Zhang (Eds.), *Affect and mathematics education: Fresh perspectives on motivation, engagement, and identity* (pp. 67–88). Springer.

Evans, J., Morgan, C., & Tsatsaroni, A. (2006). Discursive positioning and emotion in school mathematics practices. *Educational Studies in Mathematics, 63*(2), 209–226. https://doi.org/10.1007/s10649-006-9029-1

Goldin, G. A. (2014). Perspectives on emotion in mathematical engagement, learning, and problem solving. In R. Pekrun & L. Linnenbrink-Garcia (Eds.), *Handbook of emotions in education* (pp. 391–414). Taylor & Francis.

Graven, M., & Heyd-Metzuyanim, E. (2019). Mathematics identity research: The state of the art and future directions. *ZDM Mathematics Education, 51*, 361–377. https://doi.org/10.1007/s11858-019-01050-y

Green, T. F. (1971). *The activities of teaching*. McGraw-Hill.

Habermas, J. (1998). *The inclusion of the other: Studies in political theory*. MIT Press.

Hannula, M. S. (2006). Motivation in mathematics: Goals reflected in emotions. *Educational Studies in Mathematics, 63*(2), 165–178. https://doi.org/10.1007/s10649-005-9019-8

Hannula, M. S. (2012). Exploring new dimensions of mathematics related affect: Embodied and social theories. *Research in Mathematics Education, 14*(2), 137–161. https://doi.org/10.1080/14794802.2012.694281

Khalil, D., Lake, E., & Johnson, A. (2015). Teachers' classroom engagement structures: A comparative study of a novice US and an experienced UK mathematics teacher. In M. S. Hannula, G. C. Leder, F. Morselli, M. Vollstedt, & Q. Zhang (Eds.), *Affect and mathematics education: Fresh perspectives on motivation, engagement, and identity* (pp. 255–282). Springer.

Lerman, S., & Zevenbergen, R. (2004). The socio-political context of the mathematics classroom: Using Bernstein's theoretical framework to understand classroom communications. In P. Valero, & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 27–42). Springer. https://doi.org/10.1007/1-4020-7914-1_4

Liljedahl, P. (2016). Flow: A framework for discussing teaching. In C. Csíkos, A. Rausch, & J. Szitányi (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 203–210). PME.

Liljedahl, P. (2018). Affect as a system: The case of Sara. In B. Rott et al. (Eds), *Views and beliefs in mathematics education* (pp. 21–32), Springer. https://doi.org/10.1007/978-3-030-01273-1_3

Malmivuori, M.-L. (2006). Affect and self-regulation. *Educational Studies in Mathematics, 63*(2), 149–164. https://doi.org/10.1007/s10649-006-9022-8

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Learning and Teaching* (pp. 575–596). MacMillan.

Middleton, J. A., Mangu, D., & Lee, A. (2015). A longitudinal study of mathematics and science motivation: Patterns for STEM-intending high schoolers in the US. In M. S. Hannula, G. C. Leder, F. Morselli, M. Vollstedt, & Q. Zhang (Eds.), *Affect and mathematics education: Fresh perspectives on motivation, engagement, and identity* (pp. 89–106). Springer.

Montoro, A. B., & Gil, F. (2015). Exploring flow in pre-service primary teachers doing measurement tasks. In M. S. Hannula, G. C. Leder, F. Morselli, M. Vollstedt, & Q. Zhang (Eds.), *Affect and mathematics education: Fresh perspectives on motivation, engagement, and identity* (pp. 283–308). Springer.

Op't Eynde, P., de Corte, E., & Verschaffel, L. (2006). "Accepting emotional complexity": A socio-constructivist perspective on the role of emotions in the mathematics classroom. *Educational Studies in Mathematics, 63*(2), 193–207. http://www.jstor.org/stable/25472122

Rienstra, B., & Hook, D. (2006). Weakening Habermas: The undoing of communicative rationality. *Politikon*, *33*(3), 313–339. https://doi.org/10.1080/02589340601122950

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 14–22. https://doi.org/10.3102/0013189X034004014