Changing mathematical beliefs and achievement: The synergies of mindset ideas and effective teaching

Mudar as crenças matemáticas e o desempenho: Sinergias entre questões de mentalidade e um ensino eficaz

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Abstract. This paper reviews research showing the importance of changing students' mindsets and mathematical beliefs, and the promising research that has shown the impact of infusing mindset ideas through the teaching of mathematics. Previous research has demonstrated increased motivation and brain activity in learners with a growth mindset when facing mathematics problems. This paper proposes a shift in focusing solely on positive mindset messages towards the reorganization of learning environments and teaching approaches, to what we call a Mathematical Mindset approach. It also presents the outcomes of a summer camp that was designed with a mathematical mindset approach and taught in 10 US districts and four countries.
Keywords: mathematics; mindsets; neuroscience; camp; math anxiety.

Resumo. Este artigo faz uma revisão da investigação que evidencia a importância de mudar a mentalidade e as crenças matemáticas dos alunos, incluindo a investigação promissora que tem mostrado o impacto da infusão de questões de mentalidade no decurso do ensino da matemática. Estudos anteriores demonstraram um aumento da motivação e da atividade cerebral em alunos com uma mentalidade construtiva (growth mindset) quando estes enfrentam problemas matemáticos. Este artigo propõe que o foco seja alterado das simples mensagens de mentalidade positiva com vista

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à reorganização dos ambientes de aprendizagem e às abordagens de ensino, para o que chamamos de uma abordagem de mentalidade matemática. Apresenta também os resultados de um acampamento de verão que foi concebido com uma abordagem de mentalidade matemática e implementado em 10 distritos dos EUA e em quatro outros países.
Palavras-chave: matemática; mentalidades; neurociências; acampamento; ansiedade matemática.

Introduction

Mathematics achievement is important for young people; it contributes to a pathway that leads students away from poverty and towards greater prosperity and health (OECD, 2019). But most countries report that mathematics achievement is low and negative feelings about math are widespread (Hecht et al., 2023). Mathematics education scholars have contributed a great deal towards understanding the source of students' negative feelings about mathematics and set out an approach to learning that is enjoyable and impactful (Driscoll, 1999; Blum, 2011; Schoenfeld & Sloane, 2016; Gutierrez, 2000; Boaler, 2022), but systems are slow and difficult to change, and the approach that has resulted in widespread disaffection – procedural teaching with frequent tests – continues to be widespread. In addition to the prevalence of procedural teaching, change is often restricted due to educators' lack of attention to students' feelings about mathematics and themselves (Cheng, 2022; Suri, 2023). Many people believe that student cognition and learning is unrelated to their beliefs and feelings, and that these exist in different parts of the mind and brain. But recent research is showing the inter-relations between ideas and knowledge, and the importance of paying attention to the ways young people think about mathematics and their own potential to learn (Chen et al., 2018). This paper will review some of this important research and unpack the complex relations between mathematical beliefs, mindset, achievement, and teaching.

Math anxiety

It has been known for some time that students with positive attitudes towards their learning, achieve at higher levels (Aiken & Dreger, 1961; Aiken, 1976). Positive attitudes reduce anxiety towards learning, enhance motivation, and boost students' persistence (Pajares & Miller, 1994; Singh et al., 2002). But sadly, negative feelings about mathematics are commonplace – across the achievement spectrum. For some individuals, negative feelings reach the level of what has been termed "math anxiety" – a debilitating condition that often starts at a young age (Ramirez et al., 2018) and snowballs as people go through their lives. Math anxiety is characterized by an intense emotional response that arises in math-related situations, leading to avoidance behaviours, reduced confidence, and ultimately, a decline in math performance (Suárez-Pellicioni, et al., 2016).

The impact of math anxiety is significant, with research suggesting that it can lead to a decrease in performance equivalent to almost a year of schooling (Suárez-Pellicioni, et al., 2016). This decline in performance is not limited to the classroom; high math-anxious individuals tend to perform worse on math tests, particularly when faced with complex problems that require higher-order thinking and problem-solving skills (Suárez-Pellicioni, et al., 2016). Math anxiety can also negatively impact professional development, employment, and salary prospects. The fear and avoidance of math-related tasks can limit career choices, particularly in fields that require a strong foundation in mathematics. This can lead to a cycle where math-anxious individuals avoid careers in math-intensive fields, further perpetuating the stereotype that math is difficult and inaccessible (Suárez-et al., 2016). Scientists have even found that when people with math anxiety are given math questions, the same fear centre lights up in their brains as the fear centre that lights up when we see snakes and spiders. As the fear centre of the brain becomes activated, activity in the problem-solving centres of the brain is diminished (Young, et al., 2012).

While math anxiety has been studied considerably, new evidence has moved the field by examining the neurological basis of mathematics beliefs, more generally. Chen et al. (2018) found that students' attitudes towards math – whether they liked or disliked it, correlated with their math achievement, (but not their reading achievement) even after controlling for IQ scores, age, working memory, and math anxiety. Significantly, the researchers also found that positive attitudes were related to activity in the hippocampus – activating the right and left hippocampus regions. This is important because many people – in and outside the scientific world – believe that people's attitudes are unrelated to their mathematical cognition, and that they exist in a different part of the brain. But the hippocampus is one of the most mathematical regions of the brain, it plays a vital role in learning, and in spatial navigation (Beilock, 2015). Chen et al. (2018) found that what people believe about mathematics changes the functioning of their hippocampus, underscoring the importance of cultivating positive beliefs and avoiding practices and messages that lead to mathematics dislike and mathematics anxiety.

Research from Neuroscience

A promising area of research for developing positive ideas about mathematics, comes from research on the brain and learning. A cycle that leads to mathematics anxiety and dislike comes from students struggling with mathematics, and believing, or being told, that they do not have a "math brain". This often causes students to give up and to question their potential not only as mathematics learners but as people. This makes the research on brain plasticity particularly important to share with learners. Neuroscientists have now shown that the widespread idea that mathematics achievement requires innate talent is a myth, with evidence that all brains have the potential to grow, develop and change (Merzenich, 2013; Doidge, 2007). The brain's receptiveness to change is influenced by factors such as attention, motivation, and engagement. When individuals are alert, focused, and motivated, the brain releases chemicals that facilitate plasticity and enable learning. Brain plasticity occurs through the strengthening of connections between neurons that are simultaneously engaged in specific activities (Merzenich, 2013).

As further evidence for the malleability of the brain, teaching interventions have been shown to induce neuroplasticity, leading to a boost in learner outcomes. Iuculano and colleagues (2015) gave 9-year-old students with moderate to severe learning disabilities (MLD) in mathematics a treatment of 8 weeks of one-to-one cognitive tutoring. The tutoring sessions consisted of games and activities that reinforced number knowledge, number strategies and systematic knowledge of number families. At the end of the 8-week intervention the students who had recorded learning disabilities achieved at the same levels as the students without learning disabilities and had the same brain functioning. Pre and post functional magnetic resonance imaging (fMRIs) showed brain activation patterns that were indistinguishable from traditional developing students. Pre and post measures of number fluency also showed no gap between students with disabilities and students without this diagnosis.

In addition to neuroscience, personal accounts of people are important in dispelling the myth of fixed brains. Letchford's (2018) record is one example, as she describes her parental journey with her own son who was diagnosed with significant special educational needs but went on to earn a doctorate in applied mathematics from Oxford University.

Mindset research

One of the most significant areas of research that has shown the importance of changing students' ideas about their own potential comes from research on mindset, pioneered by Stanford psychologist, Carol Dweck. Decades of research have shown that students vary in their beliefs about their own potential, having what she has termed "fixed" or "growth" mindsets. Individuals with a fixed mindset believe that their intelligence, talents, and abilities are fixed, unchangeable traits. People with a fixed mindset often focus on documenting their intelligence instead of developing it. In contrast, individuals with a growth mindset believe that their abilities can be developed through dedication and hard work. They view intelligence as a potential that can be realized through effort and learning. These people tend to embrace challenges, persist through setbacks, see effort as a path to mastery, learn from criticism, and find lessons and inspiration in the success of others (Dweck, 2006).

Dweck's research has shown that students with a growth mindset tend to achieve more than those with a fixed mindset because they worry less about looking smart and put more energy into learning. When students are helped to develop a growth mindset, they often show a boost in their grades and achievement levels (Blackwell et al., 2007). Dweck and colleagues have developed mindset interventions to help students develop a growth mindset – sharing information on brain growth, separated from any specific content messages. These interventions, that teach students that the brain is like a muscle that gets stronger with use, have shown positive results in improving students' motivation (Yeager et al., 2019).

Yeager et al. (2019) conducted a landmark study in the US with a national representative sample of 9th grade students with prior low achievement. The intervention was delivered completely online, lasting about an hour in total. Students in the same schools were randomly assigned to treatment and control groups, adding weight to their causal inferences. The results were overwhelmingly positive and showed that treatment students shifted to a growth mindset as measured by surveys, increased their overall GPA (Grade Point Average), including math and science courses, and took more advanced math courses in their following year of school. The study showed that a mindset intervention changed 9th grade students' academic mindsets and learning trajectories. A follow-up analysis showed that schools with classroom practices and norms that were aligned to a growth mindset tended to have more sustained outcomes such as increased math course-taking. This is important as it points to the need to build mindset ideas into classroom approaches, as later sections of this paper will share.

Mindset and stereotypes

One of the most promising applications of mindset research is in countering negative stereotypes about who can achieve highly in mathematics. Steele (2011) developed the idea of stereotype threat, referring to the experience of anxiety or concern that develops when a person is worried about confirming a negative stereotype about their social group – such as the stereotype that males are better at math. The negative impact of the idea that success in mathematics requires innate brilliance is compounded by stereotypical ideas of who has brilliance, frequently attributed to males and to white and Asian people. Cimpian and Leslie (2017) conducted groundbreaking research showing that the more professors in different academic fields believe in the idea of a gift, the lower is the participation of women and people of colour in their field (Cimpian & Leslie, 2017). Mathematics is one of the fields with the highest numbers of professors believing that students need an innate gift to succeed (Cimpian & Leslie, 2017; Chestnut et al., 2018).

In a study of nearly 600 students with a median age of 11 years, Bagès and colleagues (2016) studied the effects of providing booklets about different role models to students before giving them a difficult math test. With random assignment to treatment groups, students received booklets that a) explained how the role model worked hard to achieve a high math score (growth mindset), b) explained how the student was naturally good at the subject (fixed mindset), or c) did not provide any explanation. The role models in the booklets were of males and females. Results showed that girls scored as highly as boys when given role models that gave growth mindset messages, regardless of the gender of the role model. The girls performed lower than the boys when given role models that signalled an explanation of giftedness (fixed mindset) or when no explanation was given. The authors propose that giving girls messages of the malleability of intelligence may counter stereotypical ideas they have been given and make mathematics achievement more equitable.

Research by O'Rourke and colleagues (2014) demonstrated that educational games can also incorporate mindset interventions. By adjusting incentive structures to promote growth mindset ideas, educational games can foster the belief that intelligence is malleable. The researchers followed 15,000 elementary students, observing that the group receiving such incentive modifications, particularly the low-performing students, persisted longer in the game, employed a wider range of strategies, and maintained engagement despite challenges.

Mindset ideas are not only important for learning; researchers have shown that the core belief – that change is possible – impacts people's fitness and health (Zahrt & Crum, 2020), and can help students dial back on aggression (Yeager et al., 2013).

Despite the range of positive evidence supporting the importance of changing students' mindsets, and countering stereotypes, some researchers have found it challenging to replicate the effects of mindset interventions (Brez et al., 2020; Rienzo et al., 2015). Notably, for this paper and for the field, researchers find no or little impact when giving mindset interventions, when they only attempt to change students' ideas without changing the teaching practices that students are experiencing. Such interventions have led other scholars to critique the idea of mindset (Kohn, 2015), claiming that it places the responsibility for change upon students, rather than their teaching environments or broader systems in which they learn and work.

Mindsets and teaching

Fortunately, the field of mindset research is now expanding to include interventions that aim to create teaching environments that infuse mindset ideas through practices, rather than share isolated growth mindset ideas that are often contradicted by fixed teaching practices.

Dweck and Yeager (2019) describe the newest frontiers of mindset research as studies that consider the building of growth mindset cultures. In their work with teachers, they found that teachers who professed to hold a growth mindset often enacted practices that countered those messages unintentionally. This has been coined as a false growth mindset (Dweck, 2015; Gross-Loh, 2016). One version of false mindset practices is when teachers tell students that they can learn and they should try harder, but present mathematics questions that are short, procedural, and closed. When students do not see a way to learn and grow, mindset messages fall flat. Boaler and colleagues have defined a Mathematical Mindset approach as one that supports growth mindset ideas by providing more open tasks that allow students to see their learning and growth, along with assessment systems that give feedback to students and opportunities to revise their work (Boaler, 2022).

Daly, Bourgaize, and Vernitski (2019) studied the impact of a mathematical mindset approach on students learning mathematics in college. Researchers in mathematics, psychology and neuroscience investigated the difference for students when working on narrow or more open mathematics tasks, studying students' brains using electroencephalogram's (EEGs), looking for stimulation of the brain areas that are associated with motivation. This interdisciplinary collaboration found that students who were given standard math problems in tests reported less interest in continuing the test as they answered more questions. By contrast, the students answering the more open mathematical problems became more motivated as they worked. Additionally, the EEG testing found stronger patterns of activity associated with motivation and engagement – shifting activity to the left side of the prefrontal cortex – in the brains of students who were working through more open mathematical problems. In prior studies, this pattern of "motivation-related" brain activity had been shown to decline as students worked through challenging problems, but it increased when students worked on mathematics problems that were more open ended. The researchers concluded that questions that encourage multiple ways of solving them, with visuals, create positive learning experiences for students.

Boaler and colleagues (2018) studied the impact of a Mathematical Mindset intervention delivered online, as a six session, student online course. The study was a randomized controlled experiment involving middle school teachers who taught two classes. One of the classes for each teacher – the experimental condition – were given the online class at the start of the school year. Both sets of students were then taught by the same teachers. During the online class students were taught mindset ideas and they were introduced to mathematics as an open, visual subject. The students who took the online class at the start of the school year, achieved significantly higher scores on state tests at the end of the year, and were 68% more engaged in their mathematics classes during the year.

In a separate experimental study of a Mathematical Mindset approach, 5th grade teachers were given professional development that introduced them to open, rich tasks and mindset ideas. The professional development was a "blended" approach of an online class as well as in person training from county office staff. Observations were conducted in the teachers' classrooms, with instructional coaches trained to look for mindset teaching practices defined in a Math Mindset Teaching Guide. The Guide identifies 15 teaching practices across 5 focal areas, with each practice featuring 3 levels of implementation: beginning, developing, and expanding. Professional development allowed teachers to progress along this scale, cultivating a growth mindset and implementing growth mindset teaching strategies in mathematics. The study showed significant positive changes in student beliefs, teacher's instructional practices, and student scores on standardized state mathematics tests. The mathematical mindset approach was particularly beneficial for girls, English learners, and economically disadvantaged students.

Hecht and colleagues (2023) conducted a synthesis of mindset studies directed at students, including interventions that focused on the mindset culture of classrooms, including teacher instructional practices. For example, teachers who hold a growth mindset tend to allow students to revise and submit their work. In their work, they provide several powerful case studies to show examples of growth mindset teaching practices, and details of teachers adopting these practices, and evidence of the ways these practices can be sustained over time. At a global level, the authors point out that while growth mindsets are associated with positive learning outcomes in 72 of 74 counties, there is still much to be learned about how growth mindset practices are enacted and differ across the globe.

Boyd and Ash (2018) add more texture to the field's understanding of changing teacher practice and beliefs toward a growth mindset approach in mathematics. They studied 7 early grade teacher-researchers who worked across 2 years to implement the Singapore math curriculum in England. Starting lessons with exploratory anchor tasks, teachers experimented and reflected together on affordances and barriers. Two major shifts were found: project teachers came to see mathematics as a creative and collaborative endeavour, and teachers moved away from the common practice of in-class grouping by ability as they came to see the benefits of heterogeneous classrooms. Despite these important shifts, the researchers noted the persistence of labelling students as "low" and of the teachers worrying that school auditors would expect them to be preparing students for high stakes assessments.

Wang and colleagues (2019) found that the infusion of growth mindset messages through an intervention on fractions significantly improved student performance. Their study implemented a structured 13-week intervention focusing on fraction magnitudes with two groups of low-performing 3rd grade students. The first group received the intervention program alone, while the second received the same intervention complemented with growth mindset, goal setting, and self-regulation support. This resulted in considerable learning increases among the students who received the intervention with mindset messages.

Bonne and Johnston (2016) studied 7–9-year-olds in four New Zealand primary schools, two of which formed an intervention group ($n = 41$) and the remaining two, the comparison group ($n = 50$). In the intervention group teaching practices were consistent with growth mindset principles (giving students feedback, providing encouragement, and helping students learn from errors). These resulted in significant gains in growth mindset beliefs, self-efficacy, and mathematics achievement.

Designing teaching with a Mathematical Mindset approach

Several studies reviewed have shown the potential of mindset interventions, particularly when mindset ideas are infused through teaching. In the final case of mindset change that we review, researchers planned a four-week teaching approach that was built with mindset principles and taught by the authors. The approach was given as a mathematical mindset summer camp and was piloted at Stanford University in 2015 (Boaler, 2019). The 18-day intervention targeted 83 middle school students from a low-resourced local school district, who reported negative perceptions of their math learning abilities. After four weeks of mathematics lessons that shared mindset ideas but also enacted mindset teaching practices – such as the use of open tasks, celebration of struggle and mistakes, and the provision of feedback that students could use to improve their learning – the students significantly improved their achievement on state tests, a change that was equivalent to 2.8 years of schooling.

Following the successful pilot, the summer camp was offered to students in ten US school districts. In 2021, multiple camps across the US replicated the mindset-math summer camps using the same curriculum, teaching strategies, and growth mindset messages. The study of the impact of the camps included 536 students enrolled in grades 5-7. A mixed-method approach with a matched comparison group showed that students attending the camps developed more growth mindset beliefs, scored higher on pre/post math assessments, and achieved significantly higher math GPA's in the following semester (Boaler et al., 2021). Notably, teachers who taught in the camps also incorporated more growth mindset teaching practices when they returned to their teaching in their regular academic school year (Leshin et al., in press). The mindset camps have now been taught in the US, Scotland, Italy and Brazil, and the details of the camps, including the tasks used, are shared in English and Portuguese on youcubed.org.

The Math Mindset summer camp is an intervention designed to engage students in mathematics that is open, visual, and creative, while fostering a growth mindset. During the camp, students view and discuss short videos to learn about growth mindset and neuroplasticity, and these messages are reinforced throughout the camp. Students participate in math routines such as "number talks and dot talks" to experience the range of approaches and perspectives even with math problems that have a single solution. The majority of the time is spent on “low floor, high ceiling” tasks, developed at youcubed. Such tasks are accessible to everyone (low floor) but extend to high levels (high ceiling), allowing for a range of approaches from intuitive and informal to more formal mathematical approaches. The tasks selected for the camp focused especially on developing number sense, as well as algebraic reasoning and generalization, through the exploration of mathematical patterns. Two sample tasks are provided. In Figure 1, the Squares to Stairs task asks students to think on their own about how they see the shape growing. After students have an opportunity to draw and describe how they see the shape changing they are ready to engage in group work and further study. Figure 2 shows a task called Number Visuals, where students colour-code collections of circles as they look for patterns, leading to explorations in types of numbers, including even, odd, composite, and prime numbers.

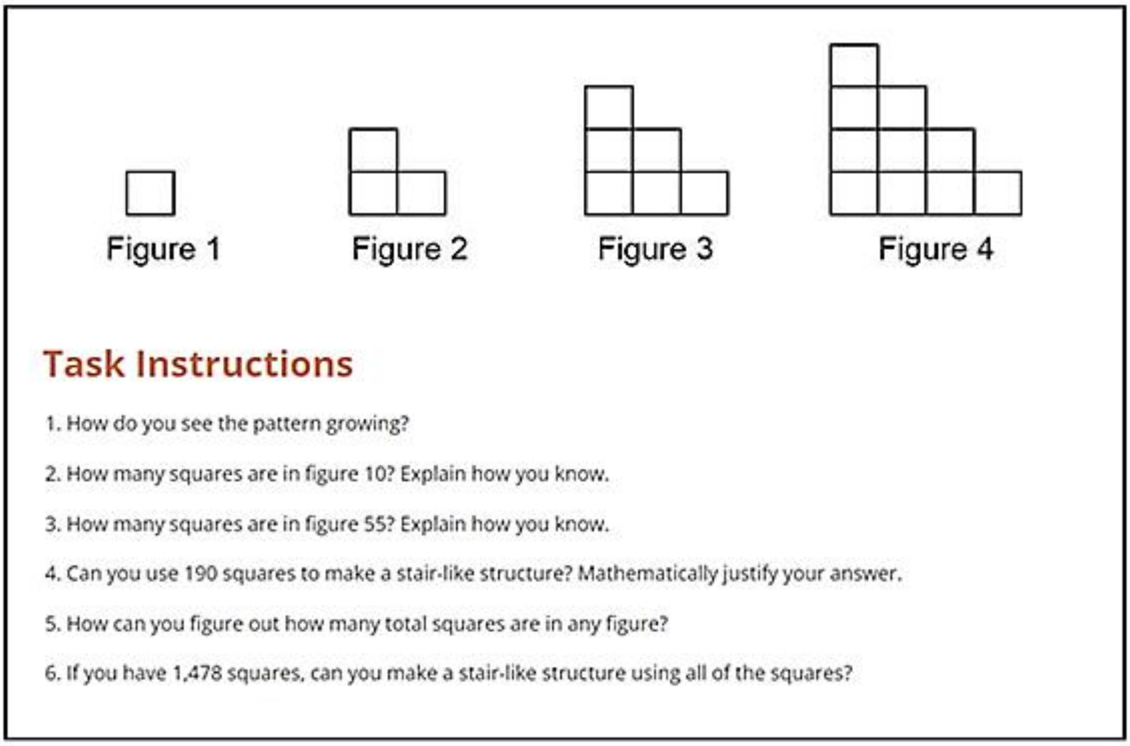

Figure 1. Squares to Stairs Task

Students work collaboratively on the tasks, sharing and comparing strategies, representing their ideas in small groups and in whole class discussions. The cumulative effect of students experiencing mathematics in this way leads to a deeper engagement with mathematics, a positive mathematical identity, and an increase in mathematical performance (Boaler, 2019; Boaler et al., 2021).

Different studies, that have been reviewed, show the impact of mindset interventions in changing students’ beliefs and increasing their achievement. More recent and promising research suggests that impacts are magnified and sustained when mindset messages are consistent with teaching practices that emphasize growth and learning, through tasks and assessment practices used.

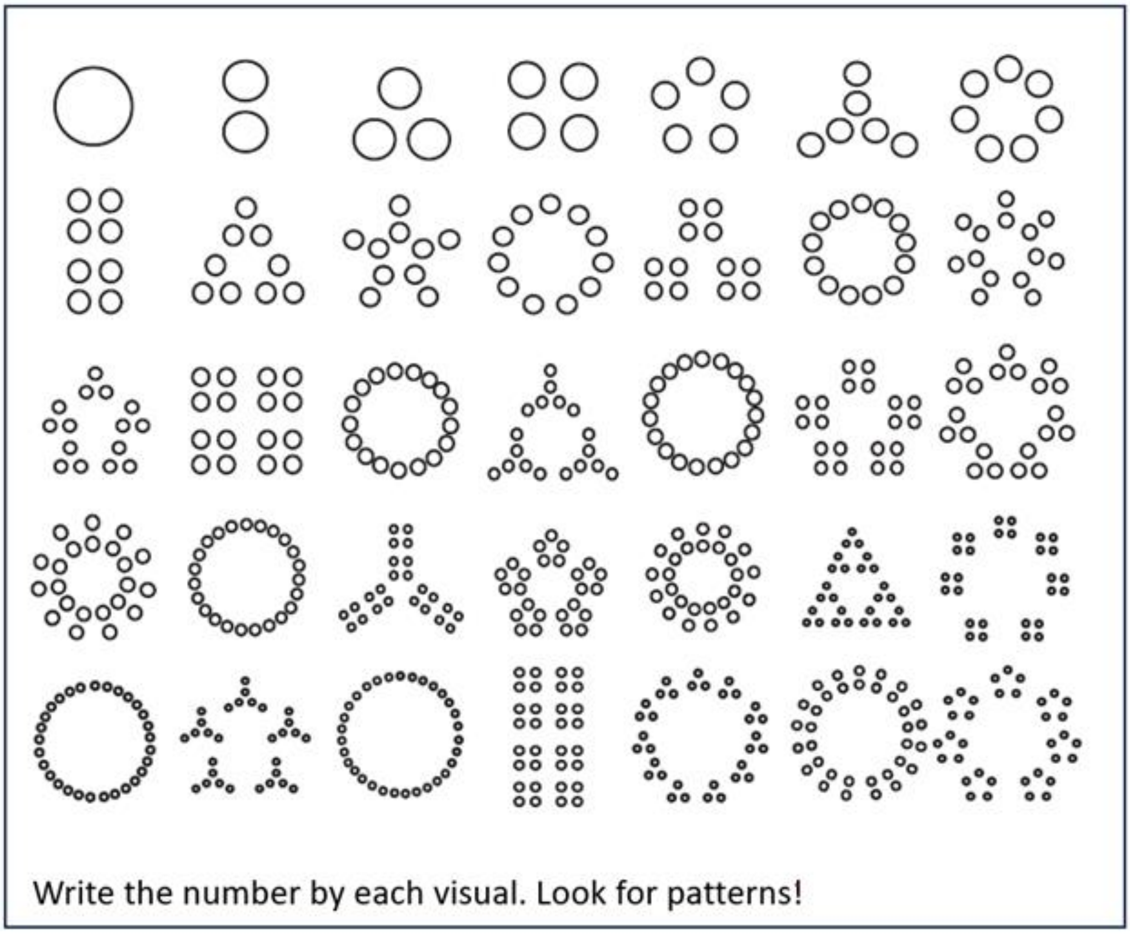

Figure 2. Number Visuals Task

Conclusion

Mathematics achievement nationwide is low, and the global covid pandemic served to reduce mathematics achievement even more (Suri, 2023). This phenomenon has many sources, including ineffective teaching methods that emphasize procedures over concepts and ideas. What this paper has aimed to show, is that what you believe about yourself and what you believe about mathematics matters. So much funding and attention in education is given to changing students' knowledge and understanding when huge boosts in achievement (and more) come about when students' attitudes and ideas about their own potential change.

Despite the potential of mindset research and practice, there are barriers to the take up and spread of mindset interventions and teaching approaches. One barrier comes from researchers and others claiming that mindset ideas have little impact, dissuading others to take them up. What this paper has hoped to show is that while isolated mindset ideas may have little or short impact when they are contradicted by teaching practices (Rienzo et al., 2015), mindset ideas that are infused through mathematics teaching are highly impactful.

Another goal of this paper was to share the latest neuroscience showing that students' beliefs about mathematics are intricately connected to the cognitive areas of their brains. This evidence alone should halt the use of anxiety inducing practices in mathematics teaching and learning and encourage an approach that infuses mindset and positivity into the content.

There have been many initiatives and interventions focused upon changing mindset ideas. There have also been many initiatives to teach mathematics as a multidimensional subject of investigations and connections. New research is suggesting that when these two initiatives come together, and mathematics is taught in a broad and multidimensional way, with mindset messages, the impact of both approaches is amplified. This happens because there is a synergy between a mathematics approach that is conceptual and open, and the idea that students can learn and grow. Just as there is a synergy between assessment practices that encourage feedback and revision and the idea that learning is about improvement over time. As we move forward with initiatives to improve students' experiences of mathematics, arguably one of the most important proposals for current times, it seems wise to investigate more fully the connections between mindset ideas and the ways students encounter mathematics in classrooms and in their lives.

References

Aiken L. R. (1976). Update on attitudes and other affective variables in learning mathematics. *Review of Educational Research*, *46*(2), 293–311. https://doi.org/10.3102/00346543046002293

Aiken, L. R., & Dreger, R. M. (1961). The effect of attitudes on performance in mathematics. *Journal of Educational Psychology*, *52*(1), 19–24. https://doi.org/10.1037/h0041309

Bagès, C., Verniers, C., & Martinot, D. (2016). Virtues of a hardworking role model to improve girls' mathematics performance. *Psychology of Women Quarterly*, *40*(1), 55–64. https://doi.org/10.1177/0361684315608842

Beilock, S. (2015). *How the body knows its mind: The surprising power of the physical environment to influence how you think and feel*. Simon and Schuster.

Blackwell, L. S., Trzesniewski, K. H., & Dweck, C. S. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. *Child Development*, *78*(1), 246–263. https://doi.org/10.1111/j.1467-8624.2007.00995.x

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in Teaching and Learning of Mathematical Modelling. (International Perspectives on the Teaching and Learning of Mathematical Modelling)* (pp. 15–30). Springer. https://doi.org/10.1007/978-94-007-0910-2_3

Boaler, J. (2019). Prove it to me!. *Mathematics Teaching in the Middle School*, *24*(7), 422–428. https://www.youcubed.org/wp-content/uploads/2019/05/prove-it-to-me-JB.pdf

Boaler, J. (2022). *Mathematical mindsets: Unleashing students' potential through creative mathematics, inspiring messages, and innovative teaching* (2nd ed.). Jossey-Bass.

Boaler, J., Dieckmann, J. A., LaMar, T., Leshin, M., Selbach-Allen, M., & Pérez-Núñez, G. (2021). The transformative impact of a mathematical mindset experience taught at scale. *Frontiers in Education*, *6*. https://doi.org/10.3389/feduc.2021.784393

Boaler, J., Dieckmann, J. A., Pérez-Núñez, G., Sun, K. L., & Williams, C. (2018). Changing students' minds and achievement in mathematics: The impact of a free online student course. *Frontiers in Education*, *3*. https://doi.org/10.3389/feduc.2018.00026

Bonne, L., & Johnston, M. (2016). Students' beliefs about themselves as mathematics learners. *Thinking Skills and Creativity*, *20*, 17–28. https://doi.org/10.1016/j.tsc.2016.02.001

Boyd, P., & Ash, A. (2018). Mastery mathematics: Changing teacher beliefs around in-class grouping and mindset. *Teaching and Teacher Education*, *75*, 214–223. https://doi.org/10.1016/j.tate.2018.06.016

Brez, C., Hampton, E. M., Behrendt, L., Brown, L., & Powers, J. (2020). Failure to replicate: Testing a growth mindset intervention for college student success. *Basic and Applied Social Psychology*, *42*(6), 460–468. https://doi.org/10.1080/01973533.2020.1806845

Chen, L., Bae, S. R., Battista, C., Qin, S., Chen, T., Evans, T. M., & Menon, V. (2018). Positive attitude toward math supports early academic success: Behavioral evidence and neurocognitive mechanisms. *Psychological Science*, *29*(3), 390–402. https://doi.org/10.1177/0956797617735528

Cheng, E. (2023, May 29). What if nobody is bad at maths? *The Guardian*. https://www.theguardian.com/books/2023/may/29/what-if-nobody-is-bad-at-maths

Chestnut, E. K., Lei, R. F., Leslie, S. J., & Cimpian, A. (2018). The myth that only brilliant people are good at math and its implications for diversity. *Education Sciences*, *8*(2), 65. https://doi.org/10.3390/educsci8020065

Cimpian, A., & Leslie, S. J. (2017). The brilliance trap: How a misplaced emphasis on genius subtly discourages women and African-Americans from certain academic fields. *Scientific American*, *317*, 60–65. https://www.jstor.org/stable/27109296

Daly, I., Bourgaize, J., & Vernitski, A. (2019). Mathematical mindsets increase student motivation: Evidence from the EEG. *Trends in Neuroscience and Education*, *15*, 18–28. https://doi.org/10.1016/j.tine.2019.02.005

Doidge, N. (2007). *The brain that changes itself: Stories of personal triumph from the frontiers of brain science*. Penguin.

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers, grades 6-10*. Heinemann.

Dweck, C. S. (2006). *Mindset: The new psychology of success*. Random House.

Dweck, C. S. (2015). Carol Dweck revisits the growth mindset. *Education Week*, *35*(5), 20–24.

Dweck, C. S., & Yeager, D. S. (2019). Mindsets: A view from two eras. *Perspectives on Psychological Science*, *14*(3), 481–496. https://doi.org/10.1177/1745691618804166

Gross-Loh, C. (2016, December 16). How praise became a consolation prize. *The Atlantic*. https://www.theatlantic.com/education/archive/2016/12/how-praise-became-a-consolation-prize/510845/

Gutierrez, R. (2000). Advancing African-American, urban youth in mathematics: Unpacking the success of one math department. *American Journal of Education*, *109*(1), 63–111. https://doi.org/10.1086/444259

Hecht, C. A., Murphy, M. C., Dweck, C. S., Bryan, C. J., Trzesniewski, K. H., Medrano, F., Gianni, M. M., Mhatre, P., & Yeager, D. S. (2023). *Shifting the mindset culture to address global educational disparities*. *Science of Learning*, *8*. https://doi.org/10.1038/s41539-023-00181-y

Iuculano, T., Rosenberg-Lee, M., Richardson, J., Tenison, C., Fuchs, L., Supekar, K., & Menon, V. (2015). Cognitive tutoring induces widespread neuroplasticity and remediates brain function in children with mathematical learning disabilities. *Nature Communications*, *6*(1), 8453. https://doi.org/10.1038/ncomms9453

Kohn, A. (2015). The "Mindset" mindset: What we miss by focusing on kids' attitudes. https://www.alfiekohn.org/article/mindset/

Leshin, M., LaMar, T., & Boaler, J. (in press). "When you let kids have a little bit of freedom, they'll surprise you": The role of reasoning in raising teacher expectations of student potential.

Letchford, L. (2018). *Reversed: A memoir*. Acorn Publishing.

Merzenich, M. (2013). *Soft-wired: How the new science of brain plasticity can change your life*. Parnassus.

OECD. (2019). *Skills matter: Additional results from the survey of adult skills*. OECD Publishing. https://doi.org/10.1787/9789264258051-en

O'Rourke, E., Haimovitz, K., Ballweber, C., Dweck, C., & Popović, Z. (2014, April). Brain points: A growth mindset incentive structure boosts persistence in an educational game. In *CHI 14: Proceedings of the SIGCHI Conference on Human Factors in Computing Systems* (pp. 3339–3348). https://doi.org/10.1145/2556288.2557157

Pajares, F., & Miller, M. D. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: A path analysis. *Journal of Educational Psychology*, *86*, 193–203. https://doi.org/10.1037/0022-0663.86.2.193

Ramirez, G., Shaw, S. T., & Maloney, E. A. (2018). Math anxiety: Past research, promising interventions, and a new interpretation framework. *Educational Psychologist*, *53*(3), 145–164. https://doi.org/10.1080/00461520.2018.1447384

Rienzo, C., Rolfe, H., & Wilkinson, D. (2015). *Changing mindsets: Evaluation report and executive summary*. Education Endowment Foundation. https://eric.ed.gov/?id=ED581132

Schoenfeld, A. H., & Sloane, A. H. (Eds.). (2016). *Mathematical thinking and problem solving*. Routledge.

Singh, K., Granville, M., & Dika, S. (2002). Mathematics and science achievement: Effects of motivation, interest, and academic engagement. *Journal of Educational Research*, *95*(6), 323–332. https://doi.org/10.1080/00220670209596607

Steele, C. M. (2011). *Whistling Vivaldi: How stereotypes affect us and what we can do*. W. W. Norton & Company.

Suárez-Pellicioni, M., Núñez-Peña, M. I., & Colomé, À. (2016). Math anxiety: A review of its cognitive consequences, psychophysiological correlates, and brain bases. *Cognitive, Affective, & Behavioral Neuroscience*, *16*, 3–22. https://doi.org/10.3758/s13415-015-0370-7

Suri, M. (2023). *The Big Bang of Numbers.* W. W. Norton & Company.

Wang, A. Y., Fuchs, L. S., Fuchs, D., Gilbert, J. K., Krowka, S., & Abramson, R. (2019). Embedding self-regulation instruction within fractions intervention for third graders with mathematics difficulties. *Journal of Learning Disabilities*, *52*(4), 337–348. https://doi.org/10.1177/0022219419851750

Yeager, D. S., Hanselman, P., Walton, G. M., Murray, J. S., Crosnoe, R., Muller, C., … & Dweck, C. S. (2019). A national experiment reveals where a growth mindset improves achievement. *Nature*, *573*, 364–369. https://doi.org/10.1038/s41586-019-1466-y

Yeager, D. S., Trzesniewski, K. H., & Dweck, C. S. (2013). An implicit theories of personality intervention reduces adolescent aggression in response to victimization and exclusion. *Child Development*, *84*(3), 970–988. https://doi.org/10.1111/cdev.12003

Young, C. B., Wu, S. S., & Menon, V. (2012). The neurodevelopmental basis of math anxiety. *Psychological Science*, *23*(5), 492–501. https://doi.org/10.1177/0956797611429134

Zahrt, O. H., & Crum, A. J. (2020). Effects of physical activity recommendations on mindset, behavior and perceived health. *Preventive Medicine Reports*, *17*, 101027. https://doi.org/10.1016/j.pmedr.2019.101027