

Towards the construction of a mathematical identity through a reflective role-play in problem solving

Em direção à construção de uma identidade matemática através da interpretação de papéis reflexiva na resolução de problemas

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Abstract. In this paper we present the design of a role-play in problem-solving, aimed at promoting the construction of students' mathematical identity. On one hand, there is a role-play on the cognitive functions identified as pivotal of the mathematical identity. On the other hand, there is a role-play concerning the group engagement: actors, who solve the problem; onlookers, who observe and take notes on how the cognitive roles are played by the actors. The double-level structure of actors and onlookers, implemented in a digital environment, allows the onlookers both to view in real-time the storytelling created by the actors do and to discuss it focusing on one cognitive role. All the students are required to fill-in a logbook, concerning their remarks on the cognitive role observed. We discuss the findings of a pilot, which involved 14-year-old students, and the mathematical identity emerged from the students' logbooks, by analysing the cognitive roles as experienced and perceived by the students.
Keywords: role-playing; mathematical identity; narratives; cognitive roles; problem-solving.

Resumo. Neste artigo, apresentamos o desenho de uma atividade de interpretação de papéis (role-play) durante a resolução de problemas, que visa promover a construção da identidade matemática dos alunos. Por um lado, há uma interpretação de papéis relacionados com as funções cognitivas identificadas como centrais na identidade matemática. Por outro lado, há uma interpretação de papéis que tem a ver com o envolvimento do grupo: os atores, que resolvem o problema; os espectadores, que observam e tomam notas sobre o modo como os papéis cognitivos são desempenhados pelos atores. A estrutura de dois níveis, composta por atores e espectadores, implementada num ambiente digital, permite que os espectadores vejam em tempo real a narrativa criada pelos atores e que, simultaneamente, a discutam, tendo como foco um papel cognitivo. Todos os alunos terão de preencher um diário de registos, contendo as suas observações sobre o papel cognitivo observado. Discutimos aqui os resultados obtidos numa experiência piloto, que envolveu alunos de 14 anos, e a identidade matemática que emergiu dos diários de registo dos alunos, analisando os papéis cognitivos que por eles foram vivenciados e percebidos.

Palavras-chave: interpretação de papéis; identidade matemática; narrativas; papéis cognitivos; resolução de problemas.

Introduction

This work is part of a larger research, developed in the project “Digital Interactive Storytelling in Mathematics: a competence-based social approach”¹, funded by the Italian Ministry of University and Research. The project aimed at developing a teaching/learning methodology, called DIST-M, which explores technology for integrating the digital storytelling approach (Porter, 2004), a competence-based approach to mathematics learning (Niss, 2003) and the Vygotskian social perspective of learning (Vygotsky, 1986). The DIST-M methodology should support the development of the mathematical literacy required to the students at the end of compulsory education (at the age of 15). On the teachers’ side, it provides guidelines for designing competence-based activities, exploring the storytelling metaphor in a digital environment. On the students’ side it provides an immersive approach to mathematics learning.

In this paper, we focus on a specific strand of the project, which concerns the design of a role-play in problem-solving, aimed at promoting students’ construction of their mathematical identity, which constitutes a key issue for learning (Radovic et al., 2018). On one hand, the students are engaged in a role-play on the cognitive functions that compose the mathematical identity (Albano et al., 2021). On the other hand, the students, split into groups, are involved in a role-play concerning the group engagement: actors, who solve the problem; onlookers, who observe and take notes on how the cognitive roles are played by the actors. The digital environment allows the actors to contribute to create the storytelling and the onlookers both to view in real-time the storytelling created by the actors and to discuss it, by focusing on one cognitive role. The role-play activity occurs in the digital environment (e.g., e-learning platform such as Moodle), in class or outside the class, depending on the teacher’s choice. All the students are required to fill-in a logbook, concerning their remarks on the cognitive role observed.

Our research questions concern two aspects:

(RQ1) What is the mathematical identity that emerged from the students’ experience in the digital storytelling designed setting?

(RQ2) How does the double-structure of actors and onlookers contribute to the construction of students’ mathematical identity?

We discuss the mathematical identity that emerged from the role-playing, by analysing the cognitive roles as experienced and perceived by the students through their logbooks.

Conceptual background

Cognitive functions in problem solving

In reviewing problem-solving activities, Albano et al. (2021) identified five key cognitive functions activated by a mathematician when solving a problem, recalled in the following:

1. **Prospecting:** the path to solving a problem always includes an exploration phase that allows the first steps to be taken. This is certainly true at the beginning of the resolution process, but it is repeated each time a piece of resolution is completed that reopens the question of how to continue.
2. **Planning:** when solving a problem, it is important to keep the thread of what we do as we go along by coordinating it with the starting point, the partial results obtained and the goal we want to achieve.
3. **Validating:** the path leading to the resolution of a problem is a path that is not known in advance and therefore subject to continuous checks on what is being done and the partial results that are obtained in order to validate them.
4. **Publishing:** solving a problem and presenting a solution to a problem are two different but related things. The former is a private process of the solver, while the latter serves to share the result of that process with others. This requires an editing activity to produce a text in a literate register (Ferrari, 2004; Ferrari, 2020) shared by the target community (Mariotti, 2006) so that it can be understood even by those who did not participate in the problem-solving process.
5. **Exploiting knowledge and wisdom:** when we face a problem, we first try to bring our own knowledge into play. Sometimes, however, this may not be enough, and in such case, we can turn outwards (e.g. ask a more experienced colleague or go to the library).

It is worthwhile to note that being able to solve a problem does not only bring cognitive functions into play, but also affective aspects: being able to cope with possible frustrations, distractions, contradictory stimuli, tiredness, etc. that may not only disrupt the activity, but even prevent the solver from being able to complete the task at hand.

The above cognitive functions allow to identify the following cognitive roles, which also incorporate some affective features:

1. **Promoter (prospecting):** he/she is in charge of exploring how to move on, he/she is in charge of exploring the way forward, trying out possible ways forward, offering cues for thinking about directions to take, proposing different possible paths, looking for insights.
2. **Boss (planning):** he/she is in charge of focusing the group's attention on the task, keeping track of what they did in terms of actions and strategies; he/she is also responsible of some affective aspects such as engaging all the group members and taking care of negative emotions that can hinder the solving process.
3. **Blogger (publishing):** he/she is in charge of producing a literate text, outlining the solution of the problem in accordance with the standards of the mathematical community that can be read and understood by an expert.

4. Pest (validating): he/she is in charge of questioning the processes activated by the group as well as the partial results obtained, and the arguments proposed in order to validate all the whole solving process.
5. Guru (exploiting knowledge and wisdom): he/she represents the external resources that can be called upon to retrieve useful knowledge for the solution process.

All the above cognitive roles should not be considered as strictly separated, but they coexist and operate within a person, although a closer relationship can be devised between the Promoter and the Guru. Indeed, while we communicate with ourselves, the 'various parts of us' (i.e. the various cognitive functions) that come into play in problem solving are actually communicating with each other.

Identity and narratives

As the above cognitive roles coexist within a problem solver, they contribute to construct students' mathematical identity. The definition of identity is a hard and controverse issue, as shown in literature reviews on this topic along with empirical and theoretical research studies undertaking many different perspectives (Darragh, 2016; Radovic et al., 2018). Often the definitions overlap and here we will blend ideas according to the social and narrative approaches of the research project, namely: participative identity, based on the notion of 'communities of practice' (Wenger, 1998), according to which identity comes from participation and engagement in a social community; and narrative identity, based on stories people tell about themselves or others (Sfard & Prusak, 2005). In fact, the authors just referred have themselves blended their communicational approach with the notion of community of practice, in conceiving identity-making as a communicational practice. Whereas in the participatory view, identity is constructed through an experience of engagement, in the narrative view it is emphasised that identity is not shaped by experience itself but by our view of our own experience or the experience of others. It should be noted that stories may vary depending on who constructs them and when they are constructed, so they may contrast and evolve. Sfard and Prusak distinguish narratives about identity in two categories: "*actual identity*, consisting of stories about the actual state of affairs, and *designated identity*, consisting of narratives presenting a state of affairs which, for one reason or another, is expected to be the case, if not now, then in the future" (2005, p. 18). From the discourse analysis point of view, they can be individuated by the tense of the verbs: the present tense characterizes the actual identity, whilst the future tense or expressions related to expectations/obligations characterize the designated identity. Co-authors of our identity create stories about us told by others as well as stories about others since they provide us possible elements to be incorporated into our designated identity.

Expansive learning

Expansive learning considers learners “involved in constructing and implementing a radically new, wider, and more complex object and concept for their activity” (Engeström & Sannino, 2010, p. 2). With his theory of expansive learning, Engeström and Sannino move to a multidimensional model that, among other things, understands learning as a process leading to the formation of theoretical knowledge and concepts rather than acquisition and creation of empirical knowledge and concepts.

The theory of expansive learning is based neither on the metaphor of acquisition (Skinner, 1953) nor on the metaphor of participation (Sfard, 1998), but on a metaphor of its own, which is that of expansion. Students learn something that is not yet there: they construct a new object and concept for their collective activity, and they implement this new object and concept in practice. The metaphor of expansion derives from the consideration that traditional modes of learning deal with tasks in which the content to be learned is known in advance by those who design, manage, and implement various learning activities. However, they are not sufficient when entire systems of collective activities, such as work processes and organizations, need to redefine themselves in a continuous process of optimization (Engeström, 1999; Engeström & Sannino, 2010). At the basis of expansive learning, various pivotal theoretical principles can be identified, mainly coming from the Russian school. We focus on two of them that have inspired our design:

(P1) The dialectical method of ascending from the abstract to the concrete, developed by Davydov (1990): in the learning activity, the initial simple idea is transformed into a complex object, into a new form of practice and learning leads to the construction of theoretical concepts. In this context abstract means partial in relation to a whole.

(P2) In tune with the Bachtin’s (1982) idea of multiple voices, all the voices (academic or not) of the system must be involved and utilized, thus expansive learning is intrinsically a multi-voice process of debate, negotiation, and orchestration.

Methodology

The experimental design

Twenty-six students aged fourteen, attending the second year of an Italian Liceo Scientifico Scienze Applicate (grade 10) were involved in a pilot study.

The students were given the following mathematical problem, concerning algebraic thinking:

Choose four consecutive natural numbers, multiply the two intermediate numbers, multiply the two extremes, and subtract the results. What do you get? (Mellone & Tortora, 2015).

This problem was posed in the context of a fictional story; thus, the mathematical problem was presented as a story problem (Verschaffel, Greer, & De Corte, 2000).

The design foresees that students are engaged in a story problem whose characters personify the above cognitive roles. Thus, there are four peers and an expert (the Guru). According to the principle of moving from the abstract to the concrete (P1), the four characters were presented to the students by depicting their characteristics. These give a simple and partial idea of them, which will then have to take their complete and personal form when the student takes charge of the personification of the various characters. The introduction of the characters is shown in Figure 1. Note that some features refer to the scenario of the story problem; this is the case of 'extra-terrestrial life' since the story problem concerns interactions with aliens.

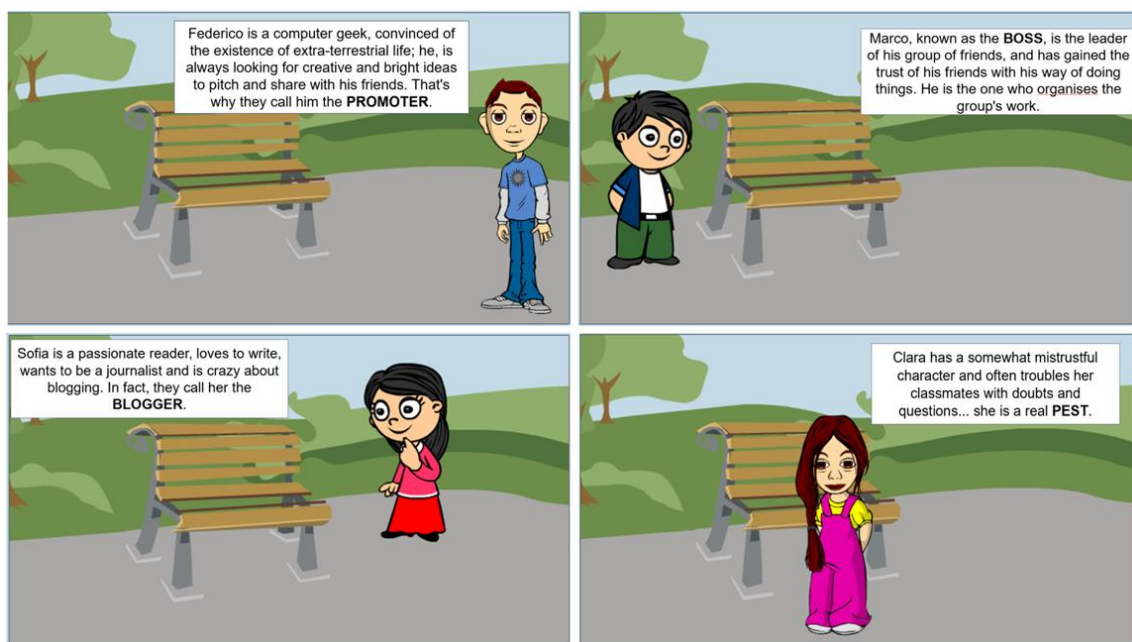


Figure 1. The introduction of the characters

According to (P2), during problem solving, space is given to different 'voices', i.e., forms of thought and discourse that represent a character's point of view (Bartolini Bussi, 1996), simulating the inner discourse occurring among the various cognitive functions existing and operating in a problem solver. The story problem developed along various episodes (Albano et al., 2020), each of them corresponding to specific phases of the problem-solving process: exploring, conjecturing, formalizing, proving. The scenario was presented as comic strips acting as the canvas so that each episode of the story takes its unique and original expression when the Actors take charge by chatting in the digital environment.

The students were split into two groups of 5 students each and four groups of 4 students each, plus an adult (i.e., the teacher/researcher) who personifies the Guru, named Gianmaria, who is the uncle of Federico. According to the inside-out model proposed by

Albano, Coppola, and Dello Iacono (2021), for each episode of the story problem, the students are engaged at different levels:

- a level where one group acts as Actors so that each student in the group takes on one of the cognitive roles described above;
- a level where the other groups act as active and aware Onlookers of the Actors in the story: each student takes on the task of observing a specific character in the story and reflects on how the observed character behaves with respect to both the mathematical problem and the role the character is playing.

Note that in the groups of 5 students, the role of the Pest was duplicated since we assumed that such role was the most significant one. When moving on to the next episode, two types of role changes are triggered, one at group level and one at individual character level. On the one hand, a new group takes the position of Actors and the group that was composed of Actors in the previous episode becomes an Onlooker group. On the other hand, in each group there is a turnover of characters, whereby each participant takes the role of a character they have not yet taken on either as Actor or Onlooker. Through this double turnover, the design aims to allow all participants to experience the different characters and to promote reflection on their own actions and others' actions, looking at them from a different viewpoint, in which they identify and, at the same time, are detached, moving on to a *reflection on the action*.

The pilot was carried out as a curricular activity in January-February 2020. Both the researcher and the teacher carrying out the experimentation attended the sessions; the role of the expert (Guru) was played by the researcher whilst the teacher observed. The first two episodes were experimented in school labs; then an afternoon slot of time was agreed to have a new meeting with the class and the expert, in order to continue with the story. Other afternoon meetings were then scheduled so that the students, always interacting remotely, would complete the episodes of the story together with the expert.

Data collection and analysis

We analyse the personal logbooks of the participants. The analysis of the Guru's role is out of the scope of this article, and we refer to some preliminary analysis of the Guru's role in Pierri (2022). At the end of each episode of the story, each student was required to fill in a steered personal logbook (Figure 2) with some reflections on the role played/observed. The aim of the personal logbook is to enable students to become aware of the characteristic cognitive functions represented by the characters as well as (in the long term) the importance of the development of all the roles as parts of themselves. The student indicates the role played in the episode and how she played it (as actor or onlooker). Then she is expected to answer to questions slightly different depending on the participation modes, as we report in the following.

Aliens arrive - Episode 1

Due to a spacecraft malfunction, a group of aliens are forced to land on Earth and send a message. The machine built by Federico receives the message and the four friends ask Uncle Gianmaria to help them... decipher the message from the aliens



<p>In this episode I played as (action and role): <i>(put an X in place of the dot)</i></p>	
<p>(x) Actor Boss (.) Actor Promoter (.) Actor Blogger (.) Actor Pest</p>	<p>(.) Onlooker Boss (.) Onlooker Promoter (.) Onlooker Blogger (.) Onlooker Pest</p>
<p style="text-align: center;">I was an Actor:</p> <p>Describe how your character played her role and what her contribution was. Do you think your character's contributions were helpful in achieving the objective? Why? Would you have done something differently? Why?</p>	<p style="text-align: center;">I was an Onlooker:</p> <p>How do you think the character you observed played his role? What was her contribution? Do you think her interventions in the story were helpful in achieving the objective? Why? What would you have done in her shoes?</p>
<p>Here are these my reflections and remarks:</p>	

Figure 2. Reflection grid given in the Personal Logbook

We collected 25 logbooks, which were analysed according to the theoretical lens given by the definition of narrative identity. Thus, we analysed the logbooks by looking for each role at the actual identity and at the designated identity as emerged by the narratives authored by actors and onlookers. Therefore, through our theoretical lens we look at excerpts which shed light on factual descriptions for actual identity or on descriptions of desirable or necessary characteristics for the designated identity.

Data Analysis

In this section we will analyse the perceptions about the roles played as actors or onlookers as described in the participants' logbooks.

The role of BOSS

In the following we will see some excerpts concerning the role of Boss, from the actor's point of view.

Some descriptions concern the management of the group: student S1 individuates the ability of engagement of the peers, student S12 refers to strong features such as the ability to give orders. They can be seen as two sides of the same coin; the latter can be seen as a

specific way of involving the other characters. It is worth mentioning the adjective “smart” used by student S12, which shows the student’s appreciation for this way of leading the group. On the other hand, the latter can also be interpreted as an assumption of responsibility by the Boss, who sometimes plays its role by making decisions for others and thus giving orders.

Student S1: My character tried to involve the other characters and helped to propose ideas, in order to reach a solution. I think my character’s interventions were helpful in reaching the goal because she proposed useful ideas. No, I probably wouldn’t have done it any other way because I think I contributed in the right way to reach the final goal, also explaining the reason for any mistakes in writing a formula.

Student S12: My character played smart by giving hints and orders.

Furthermore, the role of Boss may concern management also from the affective point of view. Indeed, Student S16 refers to activities such as motivating that seems to be considered as pivotal for being a ‘real’ Boss, while helping seems to be something usual and taken for granted.

Student S16: I think I played well, helping out when needed, and above all motivating the team like a real boss.

Let us now analyse some excerpts from the onlooker point of view. In the following, the onlooker notices a change in the student’s actions playing the role of the Boss. Indeed, she emphasises a turning point that is identified when the Boss begins making decisions. These decisions become essential in achieving the solution. Thus, the onlooker underlines the importance of being more engaging since the beginning. We cannot discard the hypothesis that such initial lack of presence and incisiveness of the Boss was due to an initial struggle to get into the role.

Student S5: I think that in the beginning she was not very present and engaging, she sometimes proposed something, but she did not stand out as a leader, and she was not incisive. Later on, she made firm decisions and actually she managed to reach a solution which was precisely what she proposed. In her shoes I would have tried to be more engaging from the beginning. She doesn’t quite stand up to Gianmaria.

In the following excerpt we again find that the emphasis is placed on decision-making skills: indeed, the contribution made in taking charge of situations is recognised as a major one. The competences of engaging the whole group and of making each member an active participant are also noted as good features of the actor playing the role of Boss.

It is worth noting that further features have been individuated, which are related to argumentative skills (justifications of her ideas, explanation of errors detected in others' statements), as in the following excerpt.

Student S9 I think the Boss, the role I observed, played her role well, involving the whole group and making useful statements, explaining why her ideas were the way they were. She contributed a lot by taking charge of the situation, getting everyone involved, explaining why the formula given by the Pest was wrong and talking to Gianmaria about the final solution, even though she didn't intervene at all in the final questions, leaving everything in the hands of the Pest. I also think that her interventions were very useful and that in her shoes I would have behaved in the same way.

The following excerpt criticizes the lack of involvement of the Boss in conversations and in helping the group but, at the same time, it should be noted that the onlooker realized the difficulty of this role, which is an important metacognitive reflection: everyone would like to be Boss, since it is generally seen as the most important figure in a group, but this onlooker shed light on the great effort this role requires.

Student S11 Boss did not fulfil her role by letting Promoter do what was her job. He helped little in solving the problem due to her absence from almost the entire chat. I realize that it is not an easy role, but she could certainly have been more involved in the conversations and thus helped the group in finding the solution.

The following two excerpts again emphasize the importance of the Boss' management of the group, which they did not find in the observed Boss.

Student S15 Boss did not play her role very well, because she was not very present in the chat, actually she spoke little, and helped her teammates little in the search for a solution. Her interventions were few and not very decisive in solving the problem, in fact she did not know how to organize the group and did not give good ideas. If I had been in her shoes, I would have tried to help the group with some ideas or by giving some suppositions and hypotheses to the promoter. In conclusion, I would like to emphasize the good work of Pest who helped Promoter a lot to come up with a good response.

Student S17 The team is not working, they are slow, someone is needed to take over and this someone should be Boss. Boss keeps wasting time and after a total waste of time she just writes "yes indeed". In my opinion you can't go any slower than this. They got the formula, but Boss didn't realize the mistake made, if they sent this to Gianmaria after all this time it would mean they hadn't understand anything.

We note that Student S17 relates the poor group performance to the lack of someone in control of the group.

The critiques in the following excerpt highlight various facets of the Boss' role at different levels.

Student S20 Initially, Boss seems switched off, she doesn't seem to be responsive, she doesn't seem to give the right push to the team, she doesn't seem to be convinced of what she is saying, she doesn't seem to play her role in the right way, correctly; she doesn't seem to respond, not giving the right support and encouragement to the team, she seems inactive, as if she was offline. There was a certain point where she seemed to be 'dead'... she didn't write anything for about 40 minutes. She seems not to be an integrated part of the team, as if she were an extra; she does not seem to be committed enough, even though she plays the most important role, being Boss. Her contribution to the story inherent to the achievement of the goal, also taking into account her interventions in the story, was not much, nothing, she seems to give implicit answers, already said before or already known, repeating the same things several times, making the storyline boring, she seems to agree most of the time, not proposing her own hypothesis. Not giving any real help to the team, disappearing into thin air most of the time. If I had been in her shoes I would have tried to identify myself more with the character, trying to play it in the best possible way, trying to give more support, more encouragement to the team, giving and proposing various ideas, hypotheses, coordinating the work done and helping the team... feeding the discussion, talking more often and proposing various things, not always the same things, so as not to make the discussion boring and also speed up the storyline, in theory. She was not focused.

As seen, for the first time there is an explicit reference to the story: the Boss actions or the lack of the Boss actions can impact on the storyline. The onlooker noted that the way the Boss acted, by means of few and poor interventions in the discussion, made the storyline boring. Her way of engagement also did not give any real help to the team in solving the problem. Indeed, it seems that her poor interventions may depend on her poor knowing as perceived by the onlooker ("she doesn't seem to be convinced of what she is saying"). This seems to agree with the fact that she always agreed with what others said and with the fact that she was not proactive with her own ideas but repeated things that had already been said. Also from the affective viewpoint, Boss failed in giving her support and encouragement to the group.

It is also interesting to underline two more notes of this onlooker. One concerns the remark that Boss is conceived not as an isolated role but as an integrated part of the team ("She seems not to be an integrated part of the team, as if she were an extra"), which is exactly in tune with the underlying design. Another note concerns the features needed to play this role: in order to be Boss, among other things perceived as the most important role, one has to be committed and focused ("she does not seem to be committed enough, even though she plays the most important role, being Boss. [...] she was not focused").

In the following excerpt we can see the appearance of the word 'leader', as a true interpretation of the Boss' role. As leader, the onlooker underlines two aspects: Boss takes care of the group from an emotional point of view (she encourages them) and shares her reasoning, i.e., from a cognitive point of view, she does not impose her thinking but invites the group to reason with her. Moreover, she takes the responsibility of the group's work in front of Gianmaria's requests.

Student S24 Boss behaves appropriately (as a true leader of the group) by encouraging her teammates and sharing her reasoning with them. Boss, at Gianmaria's request, correctly summarizes what has been said so far. Boss gives a general account, once again, detailing every step taken.

The last excerpt highlights a further important feature of the Boss' role: being Boss does not mean to be perfect ("she asked for advice") or to know everything ("She was not afraid to express her doubts and uncertainties). This seems a key element that contributes to the proper development of this role and dispels the myth of Boss as superior to others in terms of knowledge and also of self-assurance.

Student S22 In my opinion, Boss played her role well, because she was very active and answered every question put to her. She was also the one who asked for advice and considerations regarding the formula that was to be sent to her uncle. She was not afraid to express his doubts and uncertainties about the formula, even advising what to add and what not to add. In his shoes, perhaps I would have helped the others a little at the end when Gianmaria asked many questions to better understand.

The role of PEST

In this section we will focus on the role of the Pest. We will start from the actor's point of view. Both the following excerpts highlight the Pest's contribution to reach the solution of the problem. Moreover, it is made evident that her contribution makes sense to the extent that it is perceived by the peers (all or some of them) and together they process it to arrive at a solution.

Student S11 I think I played my role well: I asked enough questions and gave the cue for the solution, which was then found together with my classmates. I got confused when Gianmaria proposed the same solution but in a different way: I thought the aliens only wanted symbols and not numbers too.

Student S15 I think I played my role well by making good remarks and providing a helping hand to Promoter in answering to the problem. I was also involved in the dialogue with Gianmaria and Promoter. I would have done it all over again and not changed anything because I think I did good things for the character I was.

Let us see the viewpoint of the onlookers. Often, they highlight the Pest's feature of posing questions and the consequences of those questions. Student S6 focuses on the simplicity of the questions posed at the end. She points out that, when the questions are very simple, no doubt arises. This makes clear that, even if posing questions is a task required by the Pest, the questions posed should produce specific outcomes consisting in raising doubts, in promoting reflections on the validity of the solving process as well as of the results or conjectures emerged. So, questions expected by the Pest are not simply any sentence finished with a question mark.

Student S6 Initially, she played well, posing questions but also contributing to the solution to the problem (always raising some doubt). But in the end, she spoke little and the questions she asked were very simple, so no doubt arose.

In the same strand, Student S22 appreciates questions asking for explanation and justification ("What do you mean?", "Why?"), because of their potential of fostering interactions among teammates which lead to the understanding of the conjecture emerged (referred as 'theorem' by Student S22).

Student S22 I do not find Pest participating in finding the solution to the theorem. She is not very active on chat. On the other hand, I find her questions, such as 'what do you mean?', 'why?', fair, and in doing so Boss or someone else explains the solution well and helps others understand the theorem. In her shoes I would have been more active on chat, asking for more information.

Finally, also Student S24 refers the usefulness of the Pest's interventions and their impact in terms of the reasoning induced within the group.

Student S24 Pest plays her role correctly, asking her peers for information. I think her interventions were very useful in the course of the story, because she induced (through her questions) her teammates to reason. [...] Pest is decisive in her interventions. Following proper 'pair work', the boys managed to solve the question.

The theme of posing questions which promote reflection is also addressed by Student S10. Note that she talks about 'simple' questions in a different sense from Student S6: here it is intended as 'divide and conquer' ("more questions, perhaps even simpler ones, so that they would have arrived at the answer sooner"). Moreover, as seen for the Boss, in the excerpt from Student S11, there is an awareness of the effort required to play the role of the Pest, i.e., to pose good questions is individuated.

Student S10 I think the character I observed could have done better, but I understand that it was not very easy. Her part was crucial, even though she almost wrote more statements than questions. I think her interventions were important for reflection. If I were her, I

would have asked more questions, perhaps even simpler ones, so that they would have arrived at the answer sooner.

The following last excerpt, by Student S20, adds some emotional aspects. She notes the initial Pest's uncertainty in playing her role ("She does not seem to know how to proceed"). Such uncertainty can be ascribed to the intrinsic difficulty of the role of the Pest, already emerged from the Student S6 excerpt. In addition, it can be ascribed to the performance anxiety as seems to be suggested at the end by an advice given for improvement ("to be less timid, shy, write without any fear").

Student S20 Initially Pest does not seem very convinced, she does not seem to know how to proceed, she seems clueless, silent, fearful. She, in playing the role of Pest, seems to have identified herself perfectly to her role; being sceptical, asking several questions, wanting confirmation, proposing counterexamples..., etc. She seems to be genuinely committed, wanting to achieve the final goal, helping the team. [...] Pest alternates between good and bad moments, moments of silence and moments of talking... she staggers. [...] She initially seemed to give very good support and help to her team. [...] Honestly, Pest played a good role, there is no great criticism of her, I would have behaved almost like her. Only one advice, to be less timid, shy, write without any fear.

The role of BLOGGER

Let us see, at this point, some excerpts about the role of the Blogger. Here is one of the most interesting excerpts from an actor:

Student S4 [...] I also believe that the interventions of my character were useful because otherwise we would not have obtained the final formula. I wanted to change something: when I wrote the message to Gianmaria, if I did not forget to write the result and other things, then he would know more about what we did.

Student S4 reflects on what she did, and she realizes that her communication to Gianmaria lacked important and useful information. It emerged that the incompleteness of the reported information did not allow Gianmaria to understand the solving process implemented by the group ("what we did").

The following excerpt comes from an onlooker:

Student S24 She didn't play badly, but she could have done much better. At the beginning she was not very active but, in the end, she helped to give the formula for the final answer, even if previously she tried several times but with vain attempts. Finally, when she sent the email to uncle Gianmaria she was not very precise, she did not write an important part of the formula. In her shoes I would have thought more and finally I would have given the solution in less time.

Therefore, an important feature of this role emerges. The Blogger is not simply someone who slavishly copies what others have done, as a scribe, but in putting it back in plain sight she critically reconsiders of what she is writing. The onlooker notes the lack of an important part of the formula and therefore stresses the need to think more. It is worth noting the use of the terms ‘not very precise’, whose meaning concerns the lack of an ‘important part’, that is, the idea of being precise seems to be linked to the correctness and the completeness of the answer. Analogous remarks concern the following excerpt.

Student S2 [...] The participation of Blogger, however, was not always constant but still helped and when she had to send the email to Gianmaria asked the group how to write different from 0, in order to be precise in the answer. In her shoes I would have tried to have a more constant participation and maybe I would have asked more questions to the group to see if the reasoning was correct, but I think she helped to find the solution and I believe that having the role of Blogger I would have checked the formula further before sending the email, but during the conversation with Gianmaria she recognized the mistake made and proposed the correct formula.

Also, Student S2 underlines the need of checking what was written (“the formula”) before sending it to Gianmaria, which means once again that the Blogger is considered not just a scribe, but she is required to think about the validity of the content of the text she is writing. It is worthy to note that the dialogue with Gianmaria activated the Blogger’s thinking so that she was able to recover the erroneous formula. The following excerpt is in the same strand, with a slightly different shade.

Student S1 I think the character I observed played her role quite well, proposing ideas to help other characters. She helped to find a solution by proposing to find a theorem to describe in words what is expressed with a formula. Yes, most of her speeches were useful to achieve the goal because she was also able to summarize in the right way the reasoning followed and also she explained her considerations on the app proposed by Gianmaria. In her shoes, I would have done more or less the same.

Indeed, the above onlooker emphasizes that the usefulness of the Blogger’s work in solving the problem is due to the fact that she was able to correctly summarize the reasoning followed. Two key factors then emerge: the ability of a correct synthesis (so, again, not a simple copy) and the focus on reasoning.

The role of PROMOTER

In this section, we focus on the role of the Promoter. Let us see some actor’s excerpts. The first one considers ‘understanding’ as essential to the successful playing of this role.

Student S2 I took part in the first episode playing the role of Promoter, one of the tasks I had was to talk to Gianmaria who gave us suggestions

to find the solution. I think my interventions were quite helpful in reaching the goal because I tried to understand how to fill-in the blanks with numbers and also tried to understand why the calculations were done and how they were done.

The following excerpt focuses on the contributions given by the Promoter in terms of proposing new ideas. A bias also emerges: one can propose ideas only when those ideas are correct! Indeed, this is not in the mathematician mood and experience.

Student S9 I think I played well in my role as Promoter by coming up with ideas, some of them right and useful while others were wrong, which I wish I had not done.

Let us see some excerpts from the onlookers' viewpoint. The most cited feature regards interventions consisting in proposing ideas for solving the problem (Student S13) and suggesting the use of further tools (Student S11), as shown in the following excerpts.

Student S13 Yes, she has played her role well, she has tried several times to propose new ideas and has also corrected one of her teammates. Her interventions were very important for the development of the problem, almost fundamental, because she proposed the formula that everyone thought correct and corrected the formula of her teammate making her understand what she had done wrong. In her shoes, I would have done the same thing, I would have proposed the formula and in case one of my teammates had made a mistake and I had sensed the mistake I would have corrected it too, making her understand what she did wrong.

Student S11 Promoter played her role decently allowing the group to note an important detail for the future solution of the problem that the operations were between medium and extreme, advising the group to use a spreadsheet to verify the accuracy of the calculations. After recommending the spreadsheet, however, the Promoter's participation in the problem goes down, but we do not neglect that her considerations and advice were important to achieve the solution to the problem. In her shoes, I would probably try to keep the group focused on one hypothesis at a time instead of 3 at the same time, so as to improve the response then given to uncle Gianmaria making it complete so as to eliminate any doubt in this regard.

Let us now see some excerpts concerning the special relationship between the Promoter and the Guru. The following two excerpts look at the opportunity to have a private and privileged communication channel from two opposite sides. In fact, Student S15 seems to scold the character observed for not taking advantage of the opportunity of having an expert to whom she can turn.

Student S15 Promoter played well her role, showing her ideas and also giving good indications to her teammates. Her contribution was important in finding the solution to the problem, she also asked several questions to better understand some things. Without her, they

would not have found the 2 solutions, so her contribution was all in all important. If I were in her shoes, I would have done the same things because they were all really right and made with a good logical criterion. Promoter could talk more with Gianmaria when she wanted to know about the explanations on the answer, this was the character's flaw.

On the contrary, Student S6 seems to point out that the observed character has settled on the possibility of asking an expert and therefore has not reasoned sufficiently.

Student S6 She played well, she contributed a lot and was able to answer Gianmaria's questions. Her interventions in the story were useful in achieving the goal because she managed to find a solution when there was a stalemate. She played well, maybe I would have thought a little more before asking the uncle for help, but otherwise she was good.

Discussion and conclusions

The outcome of the analysis presented in the previous sections allows us to outline the profiles of the roles/characters as they emerged from the students' experience in terms of actual identity or of designated identity.

For what concerns the role of the Boss, it seems that almost all students believe that an important feature is the capability of making decisions and giving orders. Sometimes they observed it, sometimes they note the lack of it because they expected it. This ability contributes to manage the group of peers, to involve each member of the group as an active participant, to take care of the persons in terms of support and encouragement, and to favour the problem-solving process. It has been noted that, when this ability is missing, the team does not effectively work, both for the success of the problem solving and for the storyline (e.g., the story becomes boring). From the metacognitive point of view, some interesting remarks emerge. First of all, the students seem to become aware of the effort required to play the role of the Boss. A further facet refers to the whole mathematical identity: in fact, it emerges that the Boss is expected to be one integrated with others in a single team, according to the underlying design, which assumes each character as a specific cognitive role of one entity (i.e. the mathematician). Finally, the myth of the Boss as being perfect or omniscient is debunked.

Regarding the role of the Pest, almost all the students agree on the importance of her capability of posing questions. It is noted that the questions posed by the Pest should be able to activate the teammates in terms of promoting discussions and reflections starting from those questions. These can lead the group to reason, to split the problem at stake into sub-problems that are simpler to solve, to grasp the key points in order to go forward in the problem-solving process. Also, in the case of the Pest, the students shed light on the effort

required by this role, mainly linked to pose 'good' questions. The development of such capability for students cannot be given for granted and needs attention from the educational point of view. Finally, it was evident that some students may experience performance anxiety.

In relation to the role of the Blogger, some interesting features arose, all of them converging on the importance of communication, in terms of usefulness and correctness of the information communicated. Indeed, in some cases, the students noted that the Blogger did not report essential parts of the group's work and outcomes. This generated an imperfect understanding by the expert, preventing her from making an effective contribution to the group. On the contrary, the correct summary of the problem-solving process implemented by the group, in terms of reasoning, theoretical tools used, partial results obtained, can offer insights to proceed towards the achievement of the goal. The remarks on bad communication and its consequences have brought the students to a key metacognitive awareness: the Blogger is not a scribe; she is expected to be a critical writer.

In what concerns the role of the Promoter, all the students focused on her capability of proposing ideas useful to solve the problem at stake, although under the bias that for being proposed these ideas should be necessarily correct. Controversial notes can be found with respect to the opportunity of asking the expert for help, by exploiting the privileged and private communication channel. Two different views appear, which pose an interesting question to discuss with the students: to what extent should we rely only on our own resources (knowledge and skills) and when is it legitimate to go and seek help from external resources?

It is worth noting that being required to play a certain role does not prevent the student to sometimes behave according to other roles. This is in tune with the underlying idea that these roles are part of a single person. However, we highlight the importance that the students become aware of these roles inside themselves and that they recognize them. It is also important to recognize that the proper functioning of each role is necessary for the group, as a whole, to function.

The profiles of the cognitive roles allow us to answer to (RQ1): the combination of all the profiles give the flavour of the mathematical identity which students are constructing experiencing all the roles.

The double-level structure of the inside-out model, actors and onlookers, implemented thanks to the digital environment, helps to promote the students' awareness of the different facets of the mathematical identity, experienced according to different cognitive functions involved in solving a problem. The onlooker point of view with its reflection on action seems to allow students to grasp some pivotal metacognitive aspects of the various roles. This result constitutes an answer to (RQ2).

The above discussion on the results obtained encourages us to continue the research presented in this article. We plan to deepen the analysis of the reported experience, moving

to the teacher's exploration of the students' logbooks as didactical resources in classroom discussions.

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