Young children's embodied computational thinking developed with touchscreen mathematics applications

Pensamento computacional corporificado de crianças desenvolvido com aplicativos matemáticos em ecrã tátil

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Abstract. This study focuses on the computational thinking (CT) competence young children develop in a touchscreen application, TouchCounts, in a mathematical problem-solving context, as well as the way CT competence were demonstrated by children verbally and with their bodily actions. We adopt four widely accepted CT components—problem decomposition, pattern recognition, algorithmic thinking, and abstraction—that are particularly relevant for young learners, and we selected episodes of two children working with TouchCounts during a number pattern task to analyse their hand movements performed on (and off) the touchscreen, that are not conventionally recognised as CT, but could potentially be reframed through the lens of CT. Our results demonstrate children's CT development with touchscreen applications as an age-appropriate pedagogical approach which enables young learners to decompose problems into smaller steps, develop algorithmic thinking, recognize recognise patterns, and handling abstract concepts. This study contributes toward understanding the incorporation of tactile movements and bodily actions (on touchscreen devices) in learning and communicating computer science concepts—as a form of embodied CT, as well as how it can be supported in non-traditional programming environments.

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Resumo. Este estudo centra-se na competência de pensamento computacional (PC) que as crianças desenvolvem ao utilizar uma aplicação com ecrã tátil, TouchCounts, num contexto de resolução de problemas matemáticos, bem como na forma como as competências de PC foram demonstradas pelas crianças verbalmente e através de ações corporais. Adotamos quatro componentes de PC amplamente aceites—decomposição de problemas, reconhecimento de padrões, pensamento algorítmico e abstração—que são particularmente relevantes para jovens alunos, e selecionamos episódios de duas crianças a trabalhar com o TouchCounts durante uma tarefa de padrões numéricos para analisar os movimentos de mãos realizados no ecrã tátil (e fora dele), que não são convencionalmente reconhecidos como PC, mas que poderiam ser reinterpretados através da perspetiva do PC. Os nossos resultados demonstram que o desenvolvimento do PC com aplicações em ecrãs táteis constitui uma abordagem pedagógica adequada à idade, que permite aos alunos decompor problemas em etapas menores, desenvolver pensamento algorítmico, reconhecer padrões e lidar com conceitos abstratos. Este estudo contribui para a compreensão da incorporação de movimentos táteis e ações corporais (em dispositivos com ecrã tátil) na aprendizagem e na comunicação de conceitos das ciências da computação—como uma forma de PC corporificado, e explorando como isso pode ser apoiado em ambientes de programação não tradicionais.
Palavras-chave: pensamento computacional; educação matemática; educação infantil; cognição incorporada; ecrã tátil; TouchCounts.

Introduction

In the last decade, computational thinking (CT) has gained international acknowledgment as an essential 21st-century skill, with its integration into the compulsory education systems of numerous countries taking place (NRC, 2012; ISTE, 2011; Zhang & Nouri, 2019). This integration reflects the growing understanding of CT's significance and its potential to equip future generations with competence for visualizing and analyzing problems (Einhorn, 2012). First, to coin the term, Wing (2006) emphasized that CT is a mode of reasoning required for problem-solving, which goes beyond being a programming skill exclusively for computer scientists but as a skill applicable to daily life for everyone. Meanwhile, CT exhibits analytical thinking traits that share close connections with mathematical thinking, engineering, and scientific practices (Henderson et al., 2007). As such, research studies on CT have received increasing attention, especially regarding its integration into K-12 educational contexts (e.g., Grover & Pea, 2013; Ye et al., 2023). In fact, this integration is not new to the 21st century and can be traced back to the constructionist vision of Seymour Papert in the 1970s, namely his advocacy of improving children's mathematics learning through a programming environment, LOGO (Clements & Battista, 1989; Papert, 1980). The interplay between CT and mathematics is further developed by researchers such as Weintrop et al. (2016), who illustrated the resemblance of CT practices and those of scientific and mathematical endeavors, as demonstrated in data handling, modeling and simulation, computational problem-solving, and systems thinking.

As essential thinking skills that can be applied to diverse problem-solving situations (Kalelioğlu et al., 2016; Lye & Koh, 2014), it is suggested that CT should be grasped by everyone starting from early years of education in a similar manner as early literacy and mathematics (Wing, 2006). Accordingly, the global movement of cultivating students' CT skills has penetrated early childhood (Heintz et al., 2016; Seow et al., 2019), with the purpose of not only preparing the next generations to accommodate the digital and AI-empowered society that is yet to come but also providing a powerful way for young children to express themselves through creative making-driven activities (Resnick et al., 2009). In addition, the importance of early CT has also been underscored by research suggesting positive effects on children's cognitive development (e.g., executive function, mathematical concepts) and socio-emotional aspects (e.g., collaboration, communication, resilience) (Bers et al., 2022; Brainin et al., 2022; Sung et al., 2023). To achieve the learning outcomes, increasing tools have been invented to make CT more accessible to young children with puzzle-like, graphical, or robotic programming interfaces (Ching et al., 2018; Rich et al., 2024), alongside various instructional approaches for CT integration into early childhood STE(A)M settings (Zhang et al., 2023), including narrative-based tasks (Bakala et al., 2021) and child-centred play (McCormick & Hall, 2022). Despite the growing interest, CT learning can be challenging for young children when grasping abstract CT concepts, which may be related to their lack of both experience with computing and the requisite language to describe computational processes and practices (Angeli & Valanides, 2020; Sung et al., 2017; Zhang et al., 2024). As research focusing on CT development in early childhood contexts is still in its infancy (Acevedo-Borrega et al., 2022; Ching et al., 2018), more scholarly efforts are needed to explore how to teach young children with appropriate instructional approaches and how abstract CT concepts are enacted by young learners in different tool-based learning contexts.

Although the debate about the definition and model of early CT continues (Bers et al., 2022), one widely recognised viewpoint embedded in relevant research is that CT resides within our intellectual capacities and underscores the human ability to think logically and solve problems in a systematic, algorithmic manner (Aho, 2012). Whereas the value of problem-solving, decomposing complex tasks, and structuring logical sequences is imminent, this focus on the 'invisible' mental gymnastics of computing may hinder our understanding of the role of embodiment in the CT process. For example, the most oft-cited definition of CT, provided by Wing (2011), defines CT as "the thought processes involved in formulating problems and their solutions, so that the solutions are represented in a form that can be effectively carried out by an information-processing agent" (p. 20), which situates CT at mental activities rather than physical actions. To date, various educational paradigms within CT tend to portray CT as primarily mental exercises, potentially overlooking the embodied nature of cognition. Informed by the recent embodiment turn in the cognitive sciences (Clark, 1999), our bodies play an active role in our cognitive processes, shaping and reinforcing mental concepts through tangible interaction. With regards to CT, this would mean acknowledging the link between physical manipulation and cognitive comprehension of CT concepts. While the investigation of embodiment has proven valuable in disciplinary learning, such as mathematics (Alibali & Nathan, 2012), it has not yet been extensively explored in computing education, especially for young learners (Almjally et al., 2023). This raises the research gap of how CT can be interpreted through the lens of embodied cognition and how this interpretation can potentially be integrated into CT education to improve learning, especially in early childhood contexts.

To address the aforementioned research gap, we introduce a new perspective on CT, termed embodied CT, as the main focus of this study. This take on CT aligns with the work of Manches et al. (2020), who viewed gestures as metaphors for computational concepts; for example, the left-to-right transverse gesture was seen as time-based process in "for loops." Building on this notion, we define embodied CT as leveraging the physical presence of our body parts and harnessing our bodily capabilities, such as mobility, as constitutive of enacting CT. As such, embodied CT, when applied to instructional settings, could be potentially beneficial for early learners who may lack traditional computing experience and the language to communicate computational concepts. For example, a recent empirical study by Sung et al. (2017) with 66 kindergarten and first-grade students has demonstrated that full-body, embodied activities combined with computational perspective-taking improved programming skills in novice young learners using Scratch Jr, revealing the beneficial role of body in eliciting CT activities. In this paper, we make use of an interactive and multimodal touchscreen application, *TouchCounts* (TC), which, as its name implies, counts the number that the user touches on the touchscreen where the multitouch function is enabled. From this environment, we explored episodes of young children interacting with the touchscreen application in a mathematical problem-solving context. In summary, we investigate the generative potential for embodied expressions of CT when young children participate in our designed number pattern tasks in TC, as guided by two research questions: (1) *What level of CT competence did young children develop in the multimodal environment, TC, in a mathematical problem-solving context?* (2) *How were these CT components demonstrated by children verbally and with their bodily actions*?

Research background

Computational thinking and tools

To date, there is a lack of consensus on the definitions and frameworks of CT due to its evolving nature as researchers accumulate knowledge and academics develop explicit definitions for their research (Shute et al., 2017). Notable models in the field include the framework proposed by Brennan and Resnick (2012), which consists of three facets of CT: foundational concepts (e.g., Sequences, Loops, Parallelism), CT practices that build on conceptual knowledge (e.g., Being incremental and iterative, Testing and debugging), and the use of computing as a means of communication, or CT perspectives (e.g., Expressing, Connecting, and Questioning). However, this model is limited to the context of Scratch and focuses primarily on coding (Zhang & Nouri, 2019; Falloon, 2016; Shute et al., 2017). Other scholars, such as Lye and Koh (2014) and Weintrop et al. (2016), have specifically proposed more specific examples suggesting how CT can be integrated into K-12 education, such as data practices, modeling practices, and computational problem-solving practices, aligning with Wing's (2006) perspective that CT extends beyond computer science.

Despite the inconsistencies in the definitions of CT components and the varying focuses of conceptual and componential definitions, there are still commonalities among frequently used terms, such as 'sequencing,' 'conditional logic,' and 'loops' can be conceptually categorized under 'algorithm design' (Boom et al., 2022). As such, in this study, we select the four fundamental CT components that are suitable for interpreting the findings of the study. These components include (1) problem decomposition, (2) pattern recognition, (3) algorithmic thinking, and (4) abstraction. These components align with the definitions provided by other researchers in the field (Angeli & Giannakos, 2020; Cansu & Cansu, 2019). While we acknowledge that certain studies identify additional aspects of CT, such as 'parallelism,' 'data collection,' and 'modeling' as outlined by Shute et al. (2017), we argue, following Ezeamuzie and Leung (2022), that these aspects are less common and not universally applicable of all computational problems.

The four concepts of CT, namely problem decomposition, pattern recognition, algorithmic thinking, and abstraction, are interconnected stages in the computational problem-solving process. First, problem decomposition involves breaking down complex problems into smaller, manageable components (Kilpeläinen, 2010; Shute et al., 2017). After decomposing a problem, one can apply systematic processes at a later stage, such as algorithmic thinking, to design step-by-step procedures or algorithms for each sub-problem, thereby enabling efficient problem-solving. Pattern recognition plays a crucial role in this process by helping one identify recurring patterns or structures within data or information. It allows for the development of algorithms or procedures that can be applied to similar instances or situations. With the problem decomposed and the pattern identified, one is able to model complex systems with simplified models or representations, thereby developing higher-level perspectives that capture the most crucial aspects (Brennan & Resnick, 2012), which is also referred to as abstraction. Finally, the implementation of the problem-solving process is activated by algorithmic thinking, which is a "series of steps that control some abstract machine or computational model without requiring human judgment" (Denning, 2017, p. 33), rather than a random sequence of steps (Ezeamuzie & Leung, 2022). It encompasses the ability to analyze and understand the problem space, identify appropriate operations, and devise efficient strategies for problem-solving (Grover & Pea, 2013).

In terms of computational tools, a number of scholars highlight a crucial aspect: the selection of programming tools should depend on individual student characteristics, including their stage of development and prior knowledge (Pea & Kurland, 1984). For instance, block-based programming is often considered more appropriate for younger students or novice programmers due to its inherent characteristics that facilitate CT without the need for intricate coding (Sáez-López et al., 2019). Conversely, text-based programming, with its higher level of abstraction and cognitive demands, is commonly recommended for advanced students and experienced programmers (Ye et al., 2023). In the early years, tangible designs such as T-Maze (Wang et al., 2014) and digital Montessori-inspired manipulatives (Zuckerman et al., 2005) that employ physical interfaces rather than screens are also becoming increasingly common (Manches & O'malley, 2012). This suggestion is also supported by a wealth of empirical findings that age-appropriate programming tools yielded benefits in terms of improved cognitive, problem-solving, and sequencing skills among young learners (Kazakoff & Bers, 2012). Furthermore, exposure to on-screen or robotic programming has been found to significantly foster the development of both domain-specific knowledge and the skills typically associated with CT (Sung et al., 2017). These insights suggest that designing suitable learning environments that promote explicit thinking processes is key to introducing programming in early education and supporting students as they navigate the complexities of learning to program.

In this study, we draw upon and explore how these conceptual and empirical works of CT can potentially be interpreted and applied in an interactive touchscreen medium for mathematics learning, thereby extending its reach into embodied settings. Given the lack of consensus on CT definitions, we adopt four widely accepted CT components—problem decomposition, pattern recognition, algorithmic thinking, and abstraction—that are particularly relevant for young learners (Angeli & Giannakos, 2020; Cansu & Cansu, 2019). In turn, our primary objective is to identify and analyze a series of hand movements performed on (and off) the touchscreen that is not conventionally recognized as CT but could potentially be reframed through the lens of CT. By doing so, we aim to address existing gaps in making CT accessible to young children by bridging the divide between abstract CT concepts and tangible experiences.

Embodied cognition

Traditional assumptions of cognition consider cognition to be solely a function of the brain or mind, detached from the body and its sensorimotor interactions with the environment

(Fodor, 1975). To illustrate, one may agree that we learn how to dance or play a musical instrument through bodily activities but not to devise a strategy for a math problem. This line of thinking relates to the traditional and dominant conception that mathematical cognition is a cognitive process centered on abstract symbol manipulation, detached from sensory perception and physical action (Lakoff & Núñez, 2000). Consequently, contemporary education, encompassing STEM fields, has been constructed upon formal symbol systems (exemplified by advancements in programming languages and algebraic notations), wherein individuals employ symbols to communicate and manipulate abstract concepts in a comprehensible manner (Kwon et al., 2022). More recently, scholars have examined and recognized the significant influence of the human body on cognition and behavior and explored the interconnections between the body, environment, and cognitive processes (Barsalou, 2008; Varela et al., 2017). To illustrate, the enactivist view of cognition suggests that thinking of 'abstract' ideas is fundamentally a modal process that shares neural, sensorimotor, experiential, and cognitive capabilities with actual dynamical corporeal being (Abrahamson et al., 2020). Particularly, this research trend also promotes physical engagement of the body with tools in contextually meaningful environments (e.g., de Freitas & Sinclair, 2013) as a new avenue for thinking and learning, responding to the need to consider multisensory and multimodal forms of cognition in contemporary education.

One emerging area of such research is the use of gestures as an effective methodology for designing educational studies. In the context of mathematics education, gesture and multimodality studies reveal how mathematics learning is embodied and used to ground abstractions to the physical environment, support simulated action of conceptual ideas, and invoke conceptual blends and metaphors (Abrahamson et al., 2020). Building on investigations that gestures can serve as powerful mediators of both concrete and abstract concepts (McNeil, 1995), scholars have identified a wide range of teachers' and students' use of gestures with(out) tools, potentially complementing and bridging verbal communication about mathematics (Alibali & Nathan, 2012; Ng, 2016; Núñez, 2006). This perspective aligns with the developmental trajectories of children and offers a scaffold for building their CT skills. Furthermore, this movement towards embodied learning is perpetuated by the advent of innovative technologies and interfaces that allow for physical movements, such as gestures, touch, and body positioning, as input in interactive digital environments.

Motivated by the embodiment turn and the body of literature in embodied cognition, we aim to leverage these insights to shed light on the importance of integrating bodily interaction in CT education. In view of this, we propose the idea of embodied CT that the bodies interact and collectively contribute to demonstrating CT. This approach acknowledges the holistic nature of cognition, incorporating sensory, cognitive, and affective dimensions to enact CT through physical and tactile engagement with environments. In this paper, we focus on gestures, touchscreen actions, and human-touchscreen interactions as enactors of embodied CT. Regarding touchscreen actions, Yeung and Ng (2023) have characterized different touch taxonomies, such as temporality and a number of inputs, in association with different touchscreen actions, such as swiping, dragging, and tapping, where they can potentially convey conceptual meanings as users interact on touchscreens. Building upon this foundation, we introduced TC as an embodied computing tool which generates visual and auditory feedback in response to user interactions, potentially affording an environment that users can enact computational concepts through tactile, multimodal experiences. In our analyses, we present episodes depicting a participant's interaction with the tasks designed. We also adopt an embodied CT lens to interpret these episodes as human bodies enacting CT in action.

Methodology

Context and participants of the study

The data source of this study comes from a larger research project that examined children's mathematical communication during a series of teaching interventions designed in various multimodal learning environments (e.g., with physical manipulatives and screen-based applications such as TC, TouchTimes, and GeoGebra). Taking place in Hong Kong, the researchers conducted nine 1.25-hour sessions of mathematics lessons with eight children at a local community centre over the span of nine weekends. Each week, the sessions were allocated at three separate time slots for two to four children each. At each session, three researchers worked with their assigned child or pair of children at different tables in the same room. In this paper, we report on two of the eight participants: Student A and B (pseudonyms). Both of them were six years old, first-grade multilingual learners who spoke Urdu as a home language and English at school and had knowledge of some vocabulary in the local language of Cantonese. Due to the limit of word count, we particularly highlight Student A's discourse because her attendance and learning progress at the sessions was the highest among the children over the nine sessions. All children provided parental consent to participate in the study.

Given the nature of the informal learning setting and small class size, the teaching intervention took the form of semi-structured, task-based teaching sessions (Hunting, 1997). Our task design was informed by the following considerations. First, they generally began with open-ended, low-floor, and high-ceiling tasks to familiarize the children with the tool-based environment and to assess the children's baseline knowledge, and then progressed to more content-specific tasks to assess their development of target skills and knowledge. Second, the tasks were designed to foster exploratory learning; this was achieved in a multimodal environment via inviting the children to tinker with different actions with the physical manipulatives (e.g., M&Ms, coins, cards, base-10 blocks) or digital tools (e.g., TC, TouchTimes). Third, the tasks were designed to focus on competence beyond procedural arithmetic operations at their grade level. For example, we designed tasks with physical manipulatives to investigate the children's conceptual learning of multiples and multiplications, which were beyond their curricular learning at the time of study. The sessions were videotaped to capture the participants' and researcher's verbal expressions and bodily movements and were transcribed for data analysis.

The task

In response to the research questions posed, we selected and analysed two sessions around two tasks related to problem-solving with counting and multiples. Prior to introducing the main task to the participants, a series of pre-tasks were administered to acquaint them with TC. An example of a pre-task was to count the number of unit blocks (less than 10) in unit cubes with the use of TC's Counting World1 (Figure 1). To illustrate, TC generates yellow discs on the touchscreen interface, accompanied by auditory cues on the number, based on the users' touchscreen actions with their fingers. When the users continue to tap or multitap on the screen, it further prompts the appearance of corresponding yellow discs with the sound of corresponding numerals (e.g., a single tap followed by a multitap of three fingers produce, in succession, one disc and the sound of "one", followed by three more discs and the sound of "four").

Figure 1. The participants were asked to count the number of cubes with TC

The main task consists of two levels. The first level is centered on a rectangular model constructed from unit cubes, as depicted in Figure 2. Participants were tasked with devising efficient strategies to determine the total number of blocks of the rectangular model without resorting to laborious one-by-one counting while utilizing TC (Figure 2).

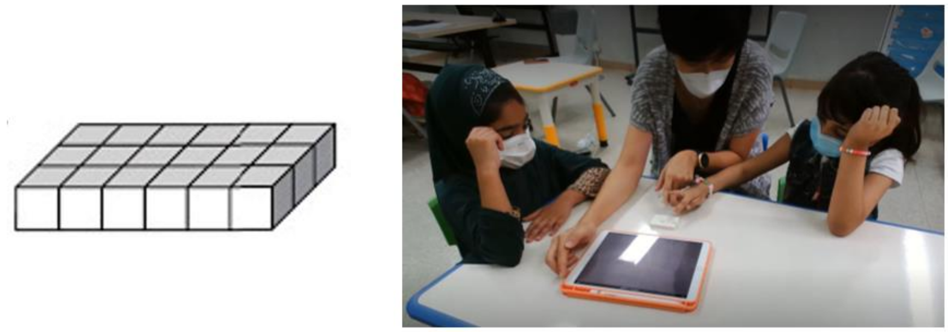

Figure 2. (left). An example of the rectangular model task given to the participants. (right) A participant devising a strategy of finding the number of unit blocks in a random rectangular model without one-by-one ‘counting’

As mentioned, the tasks were designed to support conceptual learning of mathematics beyond what the students had learned at school. During the study, we assessed that the participants possessed basic numeracy skills, such as counting to 20 and performing addition with single-digit numbers. They had not received any formal instruction on the addition of two-digit numbers and multiplication, which were involved in the tasks. Such designs allow the researchers to examine the children’s developing CT (in the form of problem-solving skills, such as decomposition, abstraction, etc.) alongside mathematics learning.

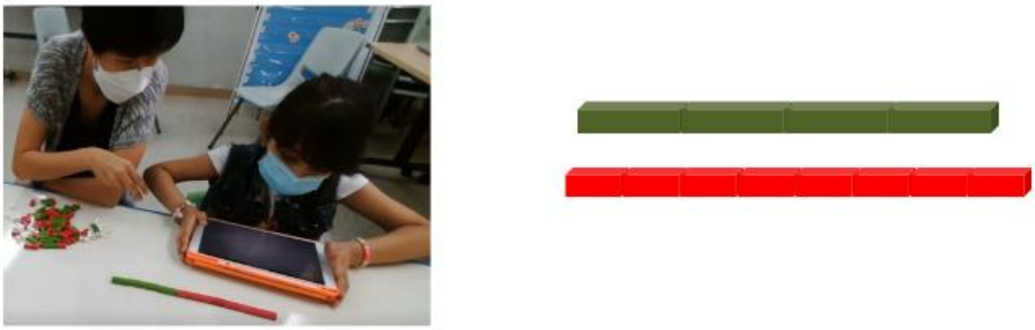

Figure 3. (left) The second task in which participants were presented with a rectangular model featuring two classes, each comprising different colours and units attached to a single block. (right) An example of the rectangular model task given to the participants

Data Collection and Analysis

The data collected in this study comprised mainly a series of video recordings of the children’s interactions with each other as a pair and with the researcher as a triad as they engaged in the task-based teaching sessions. A digital camera was placed in front of the triad to collect these recordings, while screen recording was activated to capture all screen activities with TC. Paper drafts and worksheets were also gathered to complement data that could not be gained by video recording.

Regarding data analysis, we employed a multimodal discourse analysis approach to examine three aspects of the data collected: (1) the problem-solving strategies, specifically

CT components, adopted by the participants, (2) the multimodal behaviours observed during the problem-solving process, and (3) the interaction patterns between the participants and the research instruments, including the touchscreen application (TC) and unit blocks. To conduct the multimodal discourse analysis, we reviewed and transcribed the recording sessions in their entirety (Hatch, 2002). Next, the first author selected representative episodes that aligned with the selection criteria. Employing constant comparative strategies as outlined by Strauss and Corbin (1990), we looked for common patterns within these representative episodes that related to our defined CT components. These commonalities were then analysed and presented with respect to the CT components, while data triangulation was performed by seeking supporting or non-supporting evidence from various data sources. To ensure the reliability of our findings, the research team watched the video recordings a second time, specifically examining the extent to which the identified general patterns aligned with the collected data.

Results

In this section, the results are presented, aligning with the four components of CT outlined in the Research Background, namely decomposition, algorithmic thinking, abstraction, and pattern recognition, to answer the two research questions posed. We carefully select episodes that exemplify embodied CT in terms of the four components of CT delineated to address the research questions posed and achieve our overarching goal of advancing this concept. Our participant-oriented approach delves into how verbal cues and bodily actions are employed by participants to enact their CT. By organizing these sections chronologically, we track the evolution of participants' embodied CT and problem-solving strategies as they progress over time.

Decomposition

First, we present three episodes of students working on a counting block problem, where we highlight how students enact decomposition in CT processes. The three episodes showcase the participants engaging in a problem-solving task involving a rectangular block with dimensions 5 × 7, where the objective is to determine the total number of unit blocks it comprises (Figure 4). The first episode captures participants' initial reactions to the problem, the second episode delves into the strategies devised by the participants, and the third episode depicts the execution of these strategies by the participants.

Figure 4. Researcher posed a problem to two Grade 1 students on finding the number of unit blocks in a random rectangular model without one-by-one 'counting'

Episode 1. Early reaction to the problem [1:04:10–1:04:54]

Researcher:	So look at the big rectangular block here. How many unit blocks are there in total?
Student B:	So many. *(Pointing fingers to the rectangular block, attempting to count)*
Researcher:	Don't count!
Student A:	*(Putting five fingers on TC)*
TouchCounts:	Five
Student B:	*(Putting another five fingers on TC)*
TouchCounts:	Ten
Students A & B:	*(Repeatedly putting five fingers on TC)*
Researcher:	How do you know when to stop?
Student A:	I don't know.

Initially, the participants attempted to count the blocks individually, but as aforementioned in methodology, this task requires participants to refrain from any one-by-one counting. When the two participants encountered this problem, their initial resolution was to take turns and tap all five fingers on the touchscreen device in a continuous manner. This iterative process continued, showing that the participants took action to decompose the total number of blocks into increments of five, even if they were not yet clear on when to stop. As a result, their repeated tapping on the touchscreen underscores their early enactment of decomposition ability by hands-on breaking down the problem into sub-problems or more manageable pieces. However, when questioned about their stopping point, both students exhibited confusion, emphasizing the necessity of devising a more sophisticated decomposition and problem-solving strategy.

In the second episode, the participants engaged in the same task as the first episode, yet they devised a decomposition strategy with various capacities of their finger gestures to determine the total number of units in the given scenario.

Episode 2. Devising a strategy [1:04:50–1:05:59]

Researcher:	So, how do we know the total number of cubes here? Student A, can you show me?
Student A:	*(Putting five fingers on TC)*
TouchCounts:	Five
Student A:	Oh wait. *(Restart TC)*

Researcher:	*(Intervening Student B to put another five fingers on TC)* Wait... Student A said wait.
Student A:	*(Putting a finger across a line of blocks)* One, Two, Three, Four, Five, Six, Seven (see Figure 5).
Student B:	Five, Ten, Eleven..., I think it [total number of cubes] is seventeen.
Researcher:	You sure?

(a) (b) (c)

Figure 5. (a; left) The student pointed at the number of units in the first line of blocks. (b; middle) The student stayed in a line of blocks. (c; right) The student pinched upon the line of blocks and counted the iterative units as she swiped across the rectangular model

Identifying a need for decomposition, Student A devised a three-step gesture scheme to decompose the rectangular model. First, she used a swift pointing gesture (Figure 5a) to enumerate the number of blocks in the width of the rectangular model, arriving at a count of 5. As the student also realized that another key to solving the problem is to recognize the multiples of 5 to be counted, Student A used her index finger to hold on and cover the set of blocks that represents one unit of 5 (Figure 5b). Lastly, Student A pinched the line across the set of blocks, counting the iterative units as she carefully swiped across the entirety of the model's structure (Figure 5c).

This gesture scheme, particularly the physical action of tracing the iterative units, is thus seen as an embodied action of more mature and developed substantive decomposition, where the participants not only broke down the problem into smaller parts with their hands to facilitate a systematic approach to counting, but the participants also identified the essential elements required for a solution. Even though Student A initially proposed a strategy, Student B interjected and ultimately suggested a count of seventeen cubes. This exchange exposes a fluency gap in counting and a lack of proficiency in multiples among the participants. Hence, it underscores the necessity for them to implement the strategy with TC to arrive at the correct solution.

In the third episode, the participants employed substantive decomposition as a problem-solving strategy during their hands-on interaction with TC.

Episode 3. Executing the strategy by decomposition [1:04:10–1:07:00]

Student B *:*	*(Randomly put fingers on TC)*
Researcher:	This is messy. Let Student A try it.
Student A:	One *(Putting five fingers on TC)*

TouchCounts:	Five!
Student A:	Two *(Putting five fingers on TC)*
TouchCounts:	Ten!
Student A:	Three *(Putting five fingers on TC)*
TouchCounts:	Fifteen!
...	
Student A:	*(Iterating the process of putting five fingers on TC)* Seven
TouchCounts:	Thirty-Five
Student A:	(Total number of blocks is) Thirty-Five!
...	
Researcher:	Student B, can you explain what student A did?
Student B:	I don't know
Researcher:	Student A, can you explain what did you do?
Student A:	I counted there are one *(pointing to the unit of blocks)*, two *(pointing to the unit of blocks)*, three *(pointing to the unit of blocks)*, four *(pointing to the unit of blocks)*, five *(pointing to the unit of blocks)*, six *(pointing to the unit of blocks)*, seven *(pointing to the unit of blocks)*, so I tapped seven times (on the screen).
Student A:	I did not count (them one by one), but I counted this, this, and this *(referring to the three steps of the gesture scheme)*.

The episode commenced with Student B's initial, random approach to the problem. Then, Student A intervened and demonstrated a more structured gesture scheme as a decomposition strategy: she decomposed the rectangular blocks into smaller and manageable units of five by sliding each unit of five out (Figure 6a), pointing to count the number of units (Figure 6b), and finally thereby recognizing there were seven units of five. Despite lacking a concrete understanding of multiples, Student A began by multitapping with her five fingers on the touchscreen (Figure 6c). The participants' use of five-fingered multitapping suggests they recognised the association between the structure and configuration of the rectangular model and the total number of units enclosed by the rectangular model, as the participants continued to tap five on the touchscreen to assemble the structure of the rectangular model. Student A repeated the process by multitapping five fingers on the touchscreen for another time, which was audibly confirmed as ten by TC. Continuing the process, Student A arrived at the correct total count of thirty-five cubes, demonstrating the successful utilization of the gesture scheme as the decomposition strategy. Throughout the process, the participant's use of a five-finger multitap enacts the counting of the width of the rectangular model, while the number of multitaps enacts the counting of the length of the model (which is 7). Student A not only utilized the gesture scheme to decompose the rectangular model for her body to process but also engaged her body as *capabilities*, incorporating her mobility to move systematically up and down, enacting repetitive units to construct the rectangular model against TC.

(a) (b) (c)

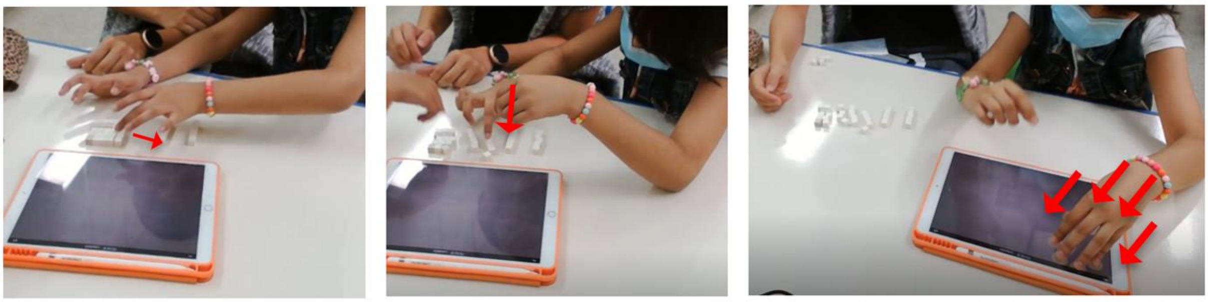

Figure 6. (a; left) The student employed sliding action to separate each unit of five. (b; middle) The student employed pointing gestures to count the multiples of five. (c; right) The student multitapped with five fingers seven times on TC

Algorithmic Thinking

In this section, we revisit the problem-solving processes in the third episode of the previous section, with a different focus on the algorithmic thinking demonstrated by the participants, emphasizing Student A's sequences of bodily actions. In addition, we extend Maches et al.'s (2020) notion of gestures as metaphors for computational concepts, with particular attention to how bodily actions could potentially enact algorithmic thinking in the context of TC. Specifically, we describe how the two distinct notions of bodily engagement: body as *capabilities* and body as *resources*, constitute the enactment of algorithmic thinking.

After devising the decomposition strategy, Student A first used her left hand to pose a "5" in the air for 3 seconds (Table 1). This moment witnessed that Student A was anchoring an initial value with her body as an initialization, a common step to proceed to a state where the program starts to run. In this case, her fingers (body) become her *resources* (or repository) to "store" the initializing value in her body for further computation.

Table 1. An illustration of storing an initializing value to A's body

Explanation of child's multimodal expressions in TC	Finger Action
Student A starts by holding a "5" with her left hand, maintaining this gesture consistently throughout the action phase.	

Following initialization, the gesture of holding the number five transitioned into an action as she tapped her left hand against TC. Consequently, TC responded by chanting "5".

After this action, she raised her right hand with one single finger extended. The process continued as she multitapped her left hand on the screen, subsequently raising another finger with her right hand. By repeatedly tapping her left hand against TC, Student A enacts the core idea of a for loop—an algorithmic construct that repeats a set of instructions a certain, known number of times. In this case, the repeated multitapping (of her left hand) enacts the reiterated execution of adding 5, as she moved her hand up and down against TC (Table 2). This tangible action allows for a more immediate and intuitive enactment of the loop concept.

Table 2. An illustration of child-tablet interaction and the corresponding computational concepts being enacted

Explanation of child's multimodal expressions in TC	**Finger Action**
Student A taps her left hand with five fingers against TC while TC chants "5." Following the tapping, Student A uses her right hand to signify "1," indicating that the iteration has occurred once.	
Consequently, Student A taps TC with her right hand again, adding an additional finger with each iteration.	

It is also worth examining the contrasting roles of A's both hands at this moment (Table 2). In particular, her left hand showcased how she leveraged her body as a *capability* to enact algorithmic thinking, while the right hand exemplified her body functioning as a resource in the same process. As A interacted with the touchscreen using her left hand, the number stored in her body remained constant, emphasizing the hand's role as wherewithal for executing algorithmic processes. After that, each tap with her left hand enacted an iterated step in the computation, showcasing how she leveraged body movements as capabilities to enact the algorithmic processes. Concurrently, her right hand enumerated the iteration count, extending one finger at a time to track the number of iterated units. Unlike the left hand, her right hand remained stationary, underscoring its function as a repository that changed with each incremental count. To illustrate, her right hand maintained a fixed position without physical movement. Meanwhile, it also signified that her right hand acted as a *resource* (repository) that adapted with each count, indicating that while the hand itself stayed still, its role in the execution of the algorithmic processes evolved as the counting advanced.

As she repeated this process, her body adapted to and synchronized the rhythm of left-hand tapping and right-hand extending (Figure 7), culminating when the value in her right hand matched the desired number of taps (noted in the decomposition stage). This rhythmic and coordinated two-handed action serves as a tangible enactment of the iteration construct of loops—a fundamental concept in various programming languages. The synchronization of these actions provides another evidence of the intertwined roles and utilization of our bodies as *capabilities* and *resources* to enact abstract computational concepts and algorithmic thinking.

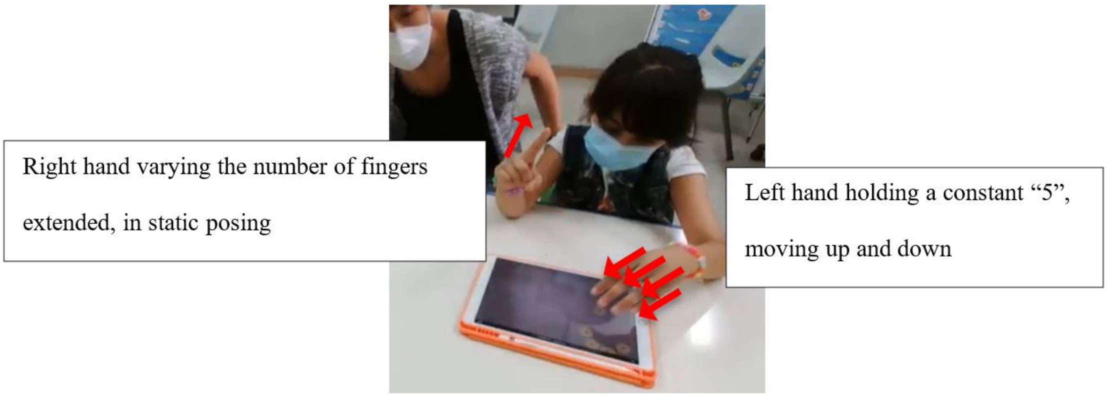

Figure 7. A pictorial analysis of the two-handed action in enacting algorithmic thinking

Abstraction

We present an episode where the students demonstrate abstraction in their problem-solving approach in the following transcript.

Episode 4. Abstraction [1:10:00–1:12:22]

Researcher: Student A, did you say something?
Student A: *(taking some random cubes out)* I don't know how many blocks there are, but I know the answer.
Researcher: Oh, so next time when I give you something (rectangular blocks), you know the answer (total number of cubes)?
Student A: Yes!
Researcher: So, I make a shape for you, and you know the answer?
Student A: Yes.
Researcher: Wow! So, now I will make this shape for you *(making another set of rectangular blocks)* (see Figure 8a). Have you not seen this shape before?
Student A: No
Researcher: How many blocks are there?
Student A: *(sliding one unit of 3 blocks out; multitapping three fingers on TC)* One
TouchCounts: Three
Student A: Two
…
Student A: *(sliding another unit of 3 blocks out; multitapping three fingers on TC)* Nine
TouchCounts: Twenty-seven

Student A: (The total number of cubes is) Twenty-seven.
Researcher: You are right. The answer is twenty-seven.

(a) (b) (c)

Figure 8. (a; left) The researcher showed the student a random shape. (b; middle) The student counted the units of three. (c; right) The student tapped three fingers with her right hand and counted the number of iterative units with her left hand

As shown in Episode 4, after being presented with a series of "number of blocks" tasks, Student A expressed that she would be able to figure out the total number of blocks without individually counting them whenever presented with a new shape. We infer that Student A was thinking about the overall problem beyond familiar shapes and patterns by ignoring specific details of the problem, a form of abstract thinking. To validate Student A's statement, the researcher presented a new shape made of rectangular blocks that Student A had not seen or interacted with before (Figure 8a). In response, Student A applied the gesture scheme developed earlier and quickly recognized she needed to slide out blocks of units of three, contrasting with the previous examples where she was presented with units of five. She thereby deduced the number of iterative units she needed to perform on TC (Figure 8b). She multitapped three fingers nine times on TC (Figure 8c). With the TC's audio prompts of 27, Student A affirmed that the total number of cubes of the given shape was 27. This example not only highlights Student A's ability to decompose problems, recognize patterns, and engage in algorithmic thinking, but also showcases her capacity to transcend the physical arrangement of familiar blocks and focus on the higher-level structure and pattern underlying the problem. In so doing, she disregarded the specific block arrangement, evidenced by her statement that whenever she was given some rectangular blocks, she could know the count. Therefore, we see Student A's enactment of abstraction in this episode. Overall, this episode illustrates an example that abstraction is not necessarily confined to the realm of intangible concepts but can be made concrete by encouraging students to enact generalizations with physical objects.

Pattern Recognition

Pattern recognition involves the capacity to classify objects into various classes or categories based on recognizable patterns. In the following episode, we showcase the pattern recognition abilities exhibited by the students in the second level of the task, where the students were tasked to determine the total number of units within the rectangular model composed of two types of blocks: red blocks symbolizing 3 units each (Figure 9a), and green blocks representing 2 units each (Figure 9b).

Episode 5. Distinguishing the colors and respective unit values of blocks [1:20:50–1:22:50]

Researcher:	This one is harder as you have two colors – you want to try?
Student A:	Yes.
Researcher:	Go ahead!
Student A:	(*Counting the number of red blocks)*
Student A:	(*Multitaping two fingers on the screen with her left hand and raising one finger on her right hand.)*
TouchCounts:	Two
Student A:	(*Multitaping two fingers on the screen with her left hand and raising one finger on her right hand repeatedly, doing so a total of eight times; Figure 9a)*
TouchCounts:	Four – Six – Eight – Ten – … – Fourteen – Sixteen
Student A:	So, eight times already… So…
Student A:	(*Counting the number of green blocks; Figure 9b) (Then, multitaping three fingers on the screen with her left hand and raising one finger on her right hand repeatedly, doing so a total of four times)*
TouchCounts:	Nineteen – Twenty-Two – Twenty-Five – Twenty-Eight
Student A:	Twenty-Eight!

Figure 9. (a; left) The student inputting the number of green blocks to TC. (b; right) The student counting the number of green blocks in the collection of blocks

Through their engagement in the preceding task activities and their interactions with TC, the participants exhibited their adeptness in discerning distinct patterns associated with the two distinct block classes. They recognized that the two classes attached to the block, distinguished by visual cues such as the colors of the blocks and the distinct numerical values assigned to each block, need to be handled separately. In one instance, the student first stored "2" in her right hand and proceeded to multitap against TC while spotting the quantities of green blocks in the model, concurrently extending her left-hand finger after each tap. Upon completion of inputting the value of green blocks, she reset the value in her left hand to zero (initializing the iteration count) and instantly adjusted the value stored in her right hand to "3" to account for the differing values of the red blocks. Subsequently, she multitapped her right hand against TC, raising one finger on her left hand with each tap. This episode demonstrated Student A's adept pattern recognition skills, facilitated by her eye-hand coordination in discerning block colours and values and adapting the stored values in their hands to correspond with the specific block class. In summary, the student's competence in adjusting the inputs stored in her right hand by manipulating the number of fingers to be positioned and the frequency of repetitions with her left hand on the touchscreen exemplified relational thinking about the number of blocks, as well as advanced use of TC to achieve the task. Her capacity to flexibly modify the initial value in her right hand and determine the required repetitions for her left hand based on the unique characteristics and quantities associated with each block class underscored her ability to effectively recognize and flexibly apply patterns.

Discussion and conclusion

This study contributes toward understanding the incorporation of tactile movements and bodily actions (on touchscreen devices) in learning and communicating computer science concepts (Maches et al., 2020) and how it can be supported in non-traditional programming environments. In particular, we have shown the use of a touchscreen application to approach CT development as an age-appropriate pedagogical approach that enables young learners to decompose problems into smaller steps, develop algorithmic thinking, recognize patterns, and abstract processes. To address these points, our study demonstrates that young learners' movements and tactile actions on touchscreens serve functions beyond merely operating with tools: they can actually enact CT concepts with their bodies as capabilities and resources. As shown in this study, Student A coordinated her two-hand movements to perform loop concepts, which are crucial for enacting algorithmic thinking. Thus, we characterize our designed CT context as fostering a form of embodied CT, which is more accessible to young learners and provides a rich, corporeal, and multimodal environment that supports concretizing, mirroring, and expressing the abstract language of computing. This work also goes beyond other similar research by integrating embodied cognition into CT without merely considering gestures as representations or metaphors for CT (e.g., Manches et al., 2020; Almjally et al., 2023). Instead, it highlights the notion that our bodies offer resources and capabilities to enact CT effectively when interacting with embodied computational tools. As a culmination of research in embodied cognition, multimodality, gesture, and computing education (e.g., Grover & Pea, 2013; Maches et al., 2020), our work further advances how we may examine CT learning, especially in early childhood contexts, which as Niebert et al. (2012) states, this understanding requires embodiment.

Our contribution of embodied CT also draws attention towards technology-rich learning in the area of human-computer interaction (HCI), building upon existing research on various forms of physical manipulations (e.g., gestures) with digital (e.g., touchscreens) and immersive (e.g., virtual reality) environments regarding the role of the HCI in learning. In mathematics, this line of research is exemplified by Abrahamson and Trninic(2015), who utilized motion sensor technology to support the learning of concepts such as ratio through defined hand movements with the technology. While some efforts have been put into incorporating embodiment in delivering computational learning environments, such as using bodies to trace the actuation of the virtual characters (Daily et al., 2014), it seems to be primarily interpreted as creating programming sequences through engaging body-based interfaces, rather than leveraging understanding of how specific computational concepts can be physically enacted. Hence, there is room for research, in which learners may explore and manipulate computing concepts through different tactile actions within technology-rich contexts, such as the touchscreen, which has become a prevalent mode of HCI in the modern day. As shown in this study, touchscreens afford users to execute a loop command by physically drawing or gesturing a loop action in a tangible manner. Meanwhile, users can determine the iterative units to execute the algorithm based on the number of loops they have drawn. This hands-on approach not only enhances the engagement of young learners but also provides them with a tangible and intuitive means to interact with computational concepts. Through this enactment, children can develop understanding of CT principles by directly manipulating and visualizing algorithmic constructs. Hence, we encourage future research to explore these possibilities of embodied CT in designing interactive and intuitive learning experiences that bridge the gap between abstract computational concepts and physical interactions.

This study presents empirical evidence for the embodied nature of computing concepts, focusing specifically on mathematics education. By investigating how participants express their computational concepts through touchscreen actions, it sheds light on their embodied cognition in mathematical problem-solving in computational environments. Meanwhile, due to the length constraints of this paper, we have not been able to capture more of the peer interactions and have focused on the embodied CT of a single participant for the sake of nuanced analysis. Instead, we have taken up a qualitative study to examine a small sample size of participants with specific demographics (first-graders from a particular school). However, we consider the insights gained as meaningful and potentially representative of the experiences of learners across classroom contexts. Future research with more diverse participant groups could help validate and expand on these results. Additionally, we encourage more empirical evidence of the embodied nature of exhibiting CT concepts. This would facilitate the development of pedagogical strategies and design frameworks for embodied computing, both as a theoretical framework and a practical approach. Ultimately, this study contributes to advancing theories of embodied cognition in computing education, with the aim of improving mathematics and CT learning outcomes in this domain.

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Notes

[1] A video demonstration of TC can be found at https://www.youtube.com/watch?v=7xD-pqnsce0

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