Identifying difficulties in problem solving: Contributions of a pedagogical tool for educators

Identificação de dificuldades na resolução de problemas: Contribuições de uma ferramenta pedagógica para educadores

Sofia Rézio

Instituto Superior de Contabilidade e Administração de Lisboa; Centro de Estudos Interdisciplinares em Educação e Desenvolvimento (CeiED)

Portugal

asrezio@iscal.ipl.pt

Abstract. Numerous education specialists contend that problem-solving constitutes a fundamental mathematical competence that should be cultivated by students from elementary through higher education. The efficacy of problem-solving instruction is influenced, among other factors, by educators' capacity to critically examine the errors that occur throughout problem-solving processes. Within the context of a training course with educational psychologists and teachers, a pedagogical tool was conceived and implemented to facilitate the identification of students' difficulties in problem-solving. The study sample comprised educators in training, and a qualitative research methodology was employed. The findings underscore several potentialities. According to these educators, the tool enables more targeted interventions, aiding in the determination of whether difficulties stem from deficiencies in language comprehension, an inability to execute calculations, insufficient conceptual understanding, or impediments in decoding mathematical language. Although, applying the model to an entire class was considered challenging, the benefit of focusing on students with greater difficulties was highlighted.

Keywords: mathematical problem solving; mathematical language; difficulties in problem solving; teacher training.

Resumo. Diversos especialistas em educação argumentam que a resolução de problemas constitui uma competência matemática essencial que os alunos devem desenvolver ao longo do seu percurso educativo, desde o ensino básico até ao ensino superior. A eficácia do ensino voltado para a resolução de problemas está, entre outros fatores, associada à capacidade dos professores em analisar os erros cometidos durante as tentativas de resolução. No contexto de um curso de formação com psicólogos educacionais e professores, foi concebida e implementada uma ferramenta pedagógica destinada a facilitar a identificação das dificuldades enfrentadas pelos alunos na resolução de problemas. A amostra do estudo é composta por educadores em formação, empregando-se uma metodologia de investigação qualitativa. Os resultados apontam para diversas potencialidades. Na opinião dos

Quadrante: Revista de Investigação em Educação Matemática 35(1) 126-153
https://doi.org/10.48489/quadrante.41852
Recebido: maio 2025 / Aceite: janeiro 2026

educadores, a ferramenta permite intervenções mais direcionadas, auxiliando na identificação das dificuldades que podem decorrer nomeadamente: da incompreensão da linguagem, da incapacidade de realizar cálculos, da falta de entendimento conceitual ou de obstáculos na descodificação da linguagem matemática. Embora a aplicação do modelo a uma turma inteira tenha sido considerada desafiadora, o benefício no enfoque em alunos com maiores dificuldades foi realçado.
Palavras-chave: resolução de problemas de matemática; linguagem matemática; dificuldades na resolução de problemas; formação de professores.

Introduction

Research on mathematics problem-solving in elementary education over the past five decades highlights its growing relevance in the field. Suseelan et al. (2022), through a bibliometric analysis of studies published between 1969 and 2021, reveal a global distribution of research across five continents, with the United States emerging as the most prolific contributor. The increasing number of publications suggests continued growth and influence within mathematics education. Their scientific mapping identifies four main research domains: arithmetic and mathematical representations; teaching and learning through word problems; cognitive and affective aspects of problem-solving; and algebraic problem-solving, with particular emphasis on the teacher's role in problem-solving pedagogy.

In the context of this research, the author posits that a teacher's effectiveness in imparting problem-solving skills is contingent, among other factors, upon their proficiency in scrutinizing the errors encountered during problem-solving processes. Such analysis can provide direction for the enhancement of their pedagogical methods. Nevertheless, this undertaking is challenging due to the multitude of factors that may influence these errors.

Within the framework of the *Advanced Specialization in Dyscalculia and Low Mathematics Performance: From Identification to* Intervention *(E-learning)*, launched in October 2023 in Lisbon, Portugal, a cohort of trainees enrolled in the module, entitled "Problem Solving: A Communicative Approach" expressed their need for a pedagogical tool. They asked specifically, "In what ways can we specifically identify the difficulties each student encounters when solving a problem, through the application of a pedagogical tool?". Responding to this need, the main research goal of the present study is to develop a tool capable of generating reports that identify students' challenges in solving mathematical problems. To operationalise this goal, specific objectives have been established focusing on the assessment of students' performance in key areas, such as comprehension of mathematical language, notation, or concepts; the operationalization of concepts and procedures; the employment of strategies for problem-solving; and the formulation of solutions to problems.

Problem solving is universally recognised as an essential aspect of everyday life, highlighting the importance of developing the skills required to address diverse challenges. From an early age, educational settings confront children with complex situations, particularly in mathematics, where difficulties often emerge and may be difficult to identify. According to Palhares (2004), problem-solving enables the acquisition of new mathematical concepts and skills by engaging students with challenging tasks that require the understanding and application of strategies, properties, and mathematical relationships. A problem arises when a student cannot immediately respond using existing knowledge, making problem-solving an active learning process that justifies its central role in the mathematics curriculum.

In Portugal, the *Essential Learning for Mathematics at the Basic Education Level* (Ministério da Educação, 2021) assert that the notion of mathematical literacy, as underscored by the Organisation for Economic Cooperation and Development (OECD), pertains to the ability for mathematical reasoning and the ability to interpret and employ mathematics in addressing real-world problems. This proficiency is deemed essential for empowering individuals to engage in society in an informed, contributive, and autonomous way.

Although processing information during problem-solving may present challenges to cognitive functions, it remains a critical component. Consecutive curricular evaluations in Portugal have emphasized the importance of problem-solving across diverse disciplines, aiming to achieve a greater consistency between educational outcomes and real-world applications. The *National Mathematics Program of Basic Education* (Ministério da Educação e Ciência, 2013) highlights this necessity: "It is essential that students do not complete this educational phase (first four years of schooling) with the ability merely to answer questions requiring immediate responses." Both national and international research, including the *Trends in International Mathematics and Science Study* (TIMSS), indicate that in 2011, 60% of Portuguese 4th-grade students were unable to progress beyond this basic level. In this regard, a variety of international guiding documents have been disseminated to assist educators in enhancing students' problem-solving abilities. For instance, the guide provided by CORE (2013) (Consortium on Reaching Excellence in Education) offers a set of guidelines for educators on how to advance students' mathematical problem-solving skills from grades four through eight. These guidelines encompass the design and implementation of problems in whole-class instruction, aiding students in monitoring and reflecting on their problem-solving processes, promote the use of visual representations, expose learners to diverse problem-solving strategies, and assist them in recognizing and articulating mathematical concepts and notations.

Problem-solving is widely regarded as a primary objective of mathematics education (Kusumadewi & Retnawati, 2020; Martaningsih et al., 2022). However, problem-solving related skills are often underdeveloped in elementary students, particularly due to limited exposure to learning tools. Several authors highlight that problem-solving is a key mathematical competency from elementary school onward (Yeni & Wahyudin, 2013) and plays a crucial role in developing higher-order thinking skills (Nurkaeti, 2018). Kusumadewi and Retnawati (2020) identify recurring difficulties students' problem-solving, including unfamiliarity with problem-solving tasks, challenges in identifying relevant information, difficulties in translating problems into mathematical representations, selecting appropriate strategies, and performing arithmetic operations. Complementing this perspective, Hadi et al. (2018) attribute students' low proficiency to insufficient verification of concepts, strategies, and calculations, resulting in fragile knowledge structures.

Pólya (1945) proposes strategies to address such challenges, including comparing with similar problems, working backwards, decomposing problems into smaller parts, and using analogies. Pólya's (1962) work has provided valuable insights for structuring pedagogical interventions, improving the way teachers respond to and learn from students' errors. Kojo et al. (2018) investigated how teachers guide students during mathematical problem-solving and communicate with them, indicating that teachers can support learning through several questioning techniques. Such guided questioning helps uncover students' reasoning processes, misconceptions, and strategic gaps, thereby making their problem-solving difficulties more visible. Consequently, understanding how students respond to problem-solving tasks and teacher prompts is essential for identifying the specific obstacles they encounter and for designing instructional strategies that effectively address these challenges.

Kirkland and McNeil (2021) assert that mathematics word problems offer students the opportunity to apply their learning from mathematic classes to real-world contexts. These authors suggest that conventional textbook word problems can potentially be rephrased to diminish a meaningless approach. If teachers cultivate the practice of questioning the text that presents the problem, it is posited that they will be better equipped to reformulate that text in different ways, thereby providing students with a clearer understanding of the problem context.

It is imperative to clarify that the presented pedagogical experience does not pretend to be a teaching-learning methodology, but rather the development of a pedagogical tool, which might eventually be integrated into such a methodology, thereby assisting in the recognition of students' difficulties.

The problem-solving section encompasses theoretical frameworks, emphasizing the significance of mathematical vocabulary and the cognitive processing of mathematical information. This is essential for understanding the errors and challenges faced by students in mathematical problem-solving. The subsequent section outlines the methodology employed in the training module.

Problem solving

Vocabulary and concepts

As stated by Mahmud et al. (2020), the use of precise mathematical language is among the fundamental competencies in the acquisition of mathematical knowledge, comparable to other skills such as procedural proficiency and problem-solving acumen. Consequently, these scholars contend that the increase of linguistic mathematical capabilities constitutes a pivotal element in the mathematical pedagogy at the elementary level. Effective mathematical communication is characterized by the capacity to accurately transmit information and to comprehend and implement mathematical notation and language. The ability to discern the narrative of a problem and to identify the corresponding mathematical model are separate proficiencies that should be cultivated in students.

In the domain of mathematics, exercises and problems may be presented to students either in numerical or textual formats. For instance, one might instruct 'Calculate 2×10,' or pose the problem 'Knowing that John possesses ten books and Mary possesses twice that amount, calculate the number of books Mary holds.' As Nurharyanto and Retnawati (2020) assert, narrative-type test items pose greater difficulty than other test item types due to their indirect presentation of the mathematical model. Instead, they offer a narrative that requires comprehension on the part of the students. Scholars have examined this subject in an effort to elucidate and justify it further. The interpretation of concepts and the ability to discern keywords within problem narratives are fundamental elements that underscore the importance of vocabulary students must be able to acquire and use effectively.

The ability to use mathematical vocabulary either partially or completely elucidates the relationship between general cognitive abilities and students' performance in narrative-type problem-solving, thereby underscoring its significance. Consequently, Powell et al. (2021) recommend that educators should deliver explicit instruction in the mathematical vocabulary presently employed and intended for future use. This approach will facilitate students' success in acquiring mathematical language. Language constitutes a structured communication system, encompassing grammar and its elements, with vocabulary being one of the most autonomous components. Marzano (2004) proposed a collection of techniques to assist students in enhancing their mathematical vocabulary, including informal explanation, paraphrasing in their own words, constructing pictures, diagrams, or illustrations, incrementally augmenting their knowledge, regularly revisiting terms, and engaging in enjoyable game-like activities. Indeed, these techniques offer valuable directives for educators, and their incorporation into teacher training presents a fertile platform for the discussion of ideas and pedagogical practices.

According to Proença et al. (2022), students in elementary education face considerable challenges in learning mathematics through problem-solving tasks. These difficulties are mainly related to problem comprehension, which arises from an inadequate conceptual understanding and insufficient familiarity with the vocabulary employed in mathematical contexts. Further impediments include the application of formulas and the execution of algorithmic procedures. The authors emphasize that problem-solving should not be regarded merely as a review of content; rather, instructional strategies ought to concentrate on the advancement of mathematical concepts and procedures.

Brain and mathematical processing

As previously noted, the use of mathematical language is crucial for cultivating problem-solving abilities; however, how does mathematical reasoning transpire within the student's cerebral processes? What elements does it encompass? Can it elucidate the challenges students face when solving problems? Indeed, solving problems involve a complex and dynamic activation of multiple brain regions collectively working to process information.

It is established that mathematical processing requires a range of cognitive faculties, including different forms of attention, working memory, abstract conceptualization, number sense, estimation, and mental arithmetic, as well as the ability to process mathematical information alongside possessing a conceptual comprehension of numbers and their relationships (Salimpoor, 2018). These cognitive demands placed on students as they engage in learning and process mathematical information should be effectively supported by the pedagogical strategies employed by mathematics educators.

The study of brain research, as noted in *Differentiated Teaching* (Davies, 2025), can significantly enhance the pedagogical approaches employed in instructing students in complex mathematical problem-solving, particularly for those experiencing difficulties. Accordingly, there are specific recommendations addressed to educators:

- The attainment of expertise in problem-solving needs consistent practice. Educators are advised to designate 8-12 minutes within their daily schedule to concentrate on problem-solving activities, selecting merely 1-2 word problems each day;
- Students who encounter both challenges and adequate support demonstrate improved outcomes: identify problems that align closely with the boundaries of students' Zone of Proximal Development, use scaffolding or modelling techniques with more complex problems to encourage risk-taking, and consistently offer feedback and assistance—conduct daily reviews and discussions of the work;
- Innovativeness and diversity are essential for enhancing engagement. The practice of word problems is not obligated to align precisely with the mathematics lesson of the day. Provide occasions for practicing identical skills

or strategies in varied formats, and alter the language and/or subject matter in word problems to facilitate the generalization of skills among students;

- Interest and emotional engagement enhance retention and skill acquisition: identify mathematical word problems that align with students' interests and link real-life scenarios and emotions to the practice of story problems. Consider implementing a weekly thematic approach to integrate and reinforce practice throughout the week;
- Fostering student autonomy enhances both confidence and independence. It is crucial to allow students sufficient time to address the problem individually or in pairs. It is advisable to avoid concentrating excessively on one exclusive method of problem-solving. Instead, educators should facilitate opportunities for collaborative learning and knowledge sharing. Additionally, it is imperative to offer suitable support in order to cultivate autonomy among all learners, such as by orally presenting the problem;
- Students have to be instructed that the process involves failure and subsequently recuperation. Allocating sufficient time for the examination of mistakes and reflection is advised. It is crucial to acknowledge and encourage effort and developmental progress at least at the same level, if not more, than precision, particularly in the early stages.

In mathematics, it is imperative for the integration of knowledge that students possess the ability to read and interpret, along with knowledge and reasoning skills necessary for problem-solving (Figure 1).

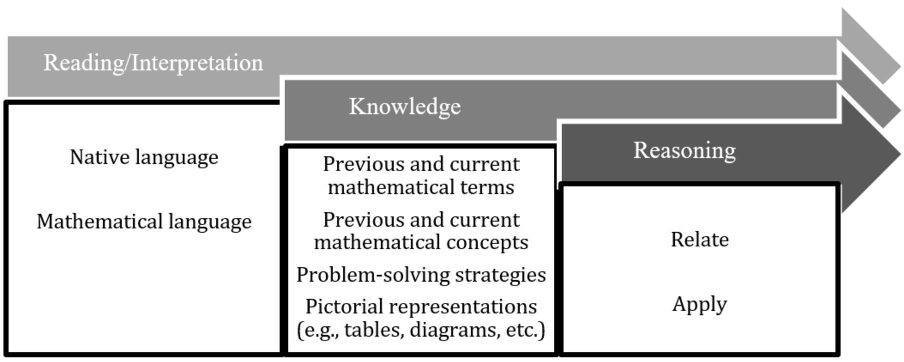

Figure 1. Problem-solving's activation

This text does not encompass an in-depth examination of the research concerning the acquisition of the native language and its association with mathematical language, so, this topic will not be addressed herein.

Nevertheless, the processes of reading and interpretation are facilitated through language, encompassing not only native but also symbolic or mathematical forms, which synergistically integrate and complement one another in the formulation of a problem statement. Scientific knowledge is disseminated using terms, concepts, strategies, and representations. When a student engages this knowledge during and after reading and interpretating the problem, it can be asserted that they are engaging in reasoning, with the capability to interrelate and apply this knowledge. The more efficiently this activation of knowledge occurs, the more rapidly reasoning can be developed and structured.

The guide *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades* published in 2021 (Fuchs et al., 2021) emphasizes the significance of systematic instruction by teachers during interventions to foster students' comprehension of mathematical ideas. This guide further underscores the necessity of imparting clear and concise mathematical language and supporting students in utilizing this language to effectively communicate their grasp of mathematical concepts. Additionally, it stresses the provision of deliberate instruction regarding word problems to augment students' understanding of mathematical principles and enhance their ability to apply mathematical concepts.

The integration of the aforementioned recommendations, alongside the identification of students' specific challenges, enhances the support that educators can offer to their students to enhance their problem-solving performance. Numerous reports outline pedagogical experiences within this domain. Table 1 underscores various literary works highlighting researchers who have identified students' difficulties in problem-solving and their potential causes.

The challenge of understanding the problem is prevalent throughout these scientific reports. However, what exactly does the student struggle to understand? Is it the linguistic elements, the mathematical terminology, or other components? These questions are deemed significant. Addressing these questions is essential, as doing so can enhance our understanding of how to assist students in overcoming such challenges.

Palhares (2004) argues that students' frequent struggles with problem-solving are not attributable to a lack of mathematical knowledge but rather to the ineffective application of such knowledge. Consequently, understanding models of problematic situations and employing suitable problem-solving strategies can be advantageous for each model of problem categorization.

Students' ability to represent a problem has significant importance in mathematical modelling. Nasrun and Akib (2023) assert that representation is fundamental in mathematics, serving as a foundation for students to comprehend and apply mathematical concepts. These authors seek to elucidate how students generate various representations when solving word problems. The research instruments employed included word problems and interview sheets, nevertheless some students employed only one or two forms of mathematical representation. The findings reveal that several representations were generated, including verbal–written, images, and symbol representations, and also that students engaged in processes of translation, integration, and evaluation to reach solutions. Furthermore, some students demonstrated a level of mathematical literacy, which significantly assisted them in the representation process, particularly when solving narrative-type problems. Similarly, Irenika et al. (2018) conducted a study aimed at describing the design outcomes of problem-solving questions that can be used to assess types of mathematical thinking representation. Their results illustrate how students engage with symbolic, numerical, and visual representations when solving such tasks.

Table 1. Students' difficulties in problem-solving and their potential causes, identified by some researchers.

Nurharyanto and Retnawati (2020)	Yeni and Wahyudin (2013)	Nurkaeti (2018)	Sulisyani et al. (2021)
Difficulties			
reading	do not understand the questions in the problem (comprehension)	difficulty of understanding the problem	understanding the problem
comprehension	lacking in understanding mathematical concepts and procedural steps	determining the mathematical formula/concepts that is used	planning aspects
process	mistakes in representing problems in mathematical models	making connections between mathematical concepts	implementing a plan
encoding	not happy to reevaluate the answers that have been written to check the validity of the answers	not reviewing the correctness of answers with questions.	re-checking
Causes			
students' semantic skills have not yet been explored in-depth, consequently, the students might commit errors in the subsequent stages	processing skills errors and encoding errors	this happened because the problem presented is in a story problem, which is rarely studied by students	lack of understanding questions errors in determining difficulties in the calculation process low encoding power

Proença (2022) developed a study with pre-service teachers to examine their capacity to distinguish between mathematical abilities and mathematical knowledge within the realm of problem-solving. The significance assigned to cultivating this ability is intrinsically linked to the pedagogy and acquisition of mathematics via problem-solving strategies. This approach enables future educators to effectively link students' abilities and knowledge with the problem-solving stages, thereby mitigating student difficulties.

Method

The above theoretical foundations highlight the central role of problem representation in both students' problem-solving performance and teachers' ability to interpret students' reasoning. This theoretical grounding supports the trainer's decision to implement Stage 1, focused on problem modelling, within the training experience. While students may experience difficulties during problem-solving, teachers often face parallel challenges in identifying and interpreting the specific obstacles students experience throughout the problem-solving process.

The need to identify students' difficulties in problem-solving was conveyed by the trainees, as they believed that they would be better equipped to assist students in enhancing their performance, if they act in an effective manner. The trainer did not encounter any suitable model for the intended purpose that was not too generalist, therefore she undertook the challenge of developing and implementing one with the trainee teachers. This phase represents stage 2 of the training experience.

The 10-hour training module, entitled "Problem Solving: A Communicative Approach" featured within the *Advanced Specialization in Dyscalculia and Low Mathematics Achievement: from Identification to Intervention [E-learning],* involved the participation of four psychologists and two elementary school teachers. The psychologists are engaged in elementary educational institutions, specialized in the domain of development and learning, particularly with students encountering mathematical learning challenges. All six participants are based in Portuguese territory and constitute the subject of this research, deliberately selected for their status as trainees. In addition, one elementary school teacher – who was not enrolled in the training programme - was invited by the trainer to implement the tool. Overall, this study includes seven teachers/educational psychologists.

A qualitative research methodology was employed to examine these educators and their experiences with the tool. This approach included the use of data triangulation to support the tool's construction, drawing on multiple data sources (Given, 2008), in line with the exploratory nature of the study. These data sources are detailed in Stage 2 of the methodology and include Documents A, B, and C.

Following an exhaustive review of relevant research and literature, the trainer-researcher structured the training program into two distinct stages:

Stage 1: Enhance trainees' understanding of models, terminology, and theoretical constructs;

Stage 2: Develop and implement an educational instrument aimed at diagnosing students' challenges in problem-solving.

In Stage 1, trainees collaboratively developed reflective questions addressing vocabulary, concepts, prerequisites, and problem-solving strategies. In Stage 2, this process fostered their capacity to produce and implement four documents, thereby establishing the pedagogical model for the tool.

Stage 1. Models, Vocabulary, and Concepts

The trainer conducted a thorough review of 3rd- and 4th-grade textbooks, selected because trainees indicated that students face the most challenges in problem-solving at these grade levels. Based on this review, the trainer identified and categorized a range of problems.

This led to the recognition of six distinct categories, accompanied by their respective resolution models, as depicted in Table 2, provided herein.

Table 2. Problems of 3rd and 4th grades textbooks

Class	Calculations with operations	Pattern	Counting combinations
Model	Operations	Regularities	Combinatorial calculus
Example of a problem	Problem 1	Problem 2	Problem 3
	A rectangle has a length equal to twice its width. What is the value of its' perimeter?	The trains of two railways depart from the same station in Porto. One railway departs every 30 minutes and the other every 40 minutes. The first departure is simultaneous. After how much time will there be a new simultaneous departure?	Peter went out for dinner and has to choose one meal and one dessert. He has 3 different meals and 3 different desserts to choose from. How many combinations of meal and dessert can Peter make?
	Parts of the whole	**Numerical sequence**	**Final result is known**
	Problem 4	Problem 5	Problem 6
	A chocolate bar weighing 200 grams was divided among 3 friends. Anthony ate one-fifth, Anne ate one-half, and Johanna ate the rest. How many grams of chocolate did Johanna eat?	The sum of Leo and Theo's ages is 25 years. Leo is 7 years older than Theo. How old is Leo?	A number was multiplied by 6, then 24 was added to it, resulting in 114. What was the initial number?

The trainees were encouraged to collaboratively formulate a set of questions designed to facilitate reflection on vocabulary, concepts, prerequisites, and resolution strategies inherent to each problem. These types of questions allow teachers to foster active learning among students, aligning with the objectives of instruction as outlined in the constructive learning theory. On one hand, this reflective practice enables teachers to consider the nature and diversity of questions they might introduce in a classroom discussion; on the other hand, it also enhances their awareness of potential questions that students might raise when confronted with a problem.

The inquiries developed by the six trainees were systematically categorized by the trainer into four distinct classifications: (1) language, (2) concept/definition, (3) operationalization of concepts/definitions, and (4) resolution strategy, as illustrated in Table 3.

Table 3. Categorization of questions enumerated by Teacher Trainees

Domains	Language	Concept/definition	Operationalization of concepts/definitions	Strategy/ representation
Problem 1	How would you rewrite this same problem using different words?	What is a rectangle? What is the length of a rectangle? And the width? Do they vary with its position? What is double? What is the perimeter of a rectangle? What are the dimensions of a rectangle?	Can rectangles with different dimensions have the same perimeter? Does the problem have more than one solution? (What does "more than" mean?)	What representation can you make of the problem?
Problem 2	What does "simultaneous" mean?	What are prime numbers; relatively prime numbers; multiples, least common multiple between two numbers; greatest common multiple between two numbers...?	In what unit of time can we answer? What is the most suitable unit of time for responding? Why? What numerical relationship exists between the values considered in the statement and the value obtained in the response to the problem? Explain.	What representation can you make of the problematic situation?
Problem 3	What does the term 'combinations' mean?	What do you understand by combination?	What numerical relationship do you find between the given values and the obtained result?	What representation can you make of the problematic

			How much would the answer to the problem increase if the number of different dishes increased by one unit? Suppose the number of different desserts increased to 5. How would you change your answer?	situation? Can you think of another one? What does the numerical expression $3 \times (3+1)=(3 \times 3)+(3 \times 1)$ mean in the context of the described situation?
Problem 4	What does "divided" mean?	What is a fraction? How do you translate mathematically "Anthony ate one-fifth"?	What operation should be performed to solve the problem? Why? Is it the only one? Present the answer in kilograms. Explain the unit conversion.	What representation can you make of the problematic situation?
Problem 5	Does " Leo is 7 years older than Theo " mean "he has exactly seven years more than Theo" or "he has at least seven years more than Theo"? Saying " a number is worth more than 7 " have the same meaning as saying " a number is worth at least 7 "?	What is addition? What does "more than" mean?	What is the opposite of saying "more than 7"?	What representation can you make of the problematic situation?
Problem 6	What does "obtaining" mean? Is this a mathematical term?	What does "multiplying" mean? What is the difference between the terms "multiplication" and "product"? What does "adding" mean? What is the meaning of the equal sign?		Outline the described situation

An additional perspective by the trainer is that this stage of training aims to enhance empathy between teachers and students with respect to the challenges faced by students.

Stage 2: Identification of difficulties

When confronted with the challenge of developing a pedagogical tool to discern students' difficulties in problem-solving, the trainer considered theoretical contributions, the diversity and specificities of existing problems, and the participation of both students and teachers in the process. The resulting conclusions are supported by the triangulation of collected data, which in qualitative research enables the researcher to gain a comprehensive understanding of the phenomenon (Adami & Kiger, 2005) and construct findings that are less susceptible to bias.

The objective of the tool is to reflect linguistic difficulties, deficiencies in prerequisites, students' awareness of these issues, and their performance in solving the proposed problem. In addition, the tool prepares a final report format that can be composed by the educator.

The trainees were asked to generate four documents, which were implemented in the sequence outlined below, thereby establishing a pedagogical model for the tool:

(1) Document A: serves as an a priori document, which the student is required to complete before the problem presentation. This document evaluates the student's comprehension of language, notation, concepts, operationalization, and problem-solving strategies. In each domain, the educator formulates questions with caution, ensuring that the questions do not inadvertently guide the student to the correct solution of the problem. This document, once completed by the student, is submitted to the teacher.

(2) Document B: the problem to solve;

(3) Document C: involves a self-assessment by the student concerning his comprehension of terms or symbols, identification of relevant data, necessary knowledge, and identification of problem’s question. It also encompasses the definition of the strategy employed and supplementary information the student chooses to provide. For each of these categories, the teacher poses questions. The student fully completes this document and submits it;

(4) Document D: constitutes a report identifying the challenges encountered by students. This report considers the triangulation of data from documents A, B, and C, focusing on aspects such as language proficiency, notation conventions, conceptual clarity, strategic approach, and problem-solving responses.

The categorization illustrated in Table 3, has facilitated a uniform sectional structure within documents A, B, and C. The tool is illustrated in Figure 2 and was distributed to the trainees.

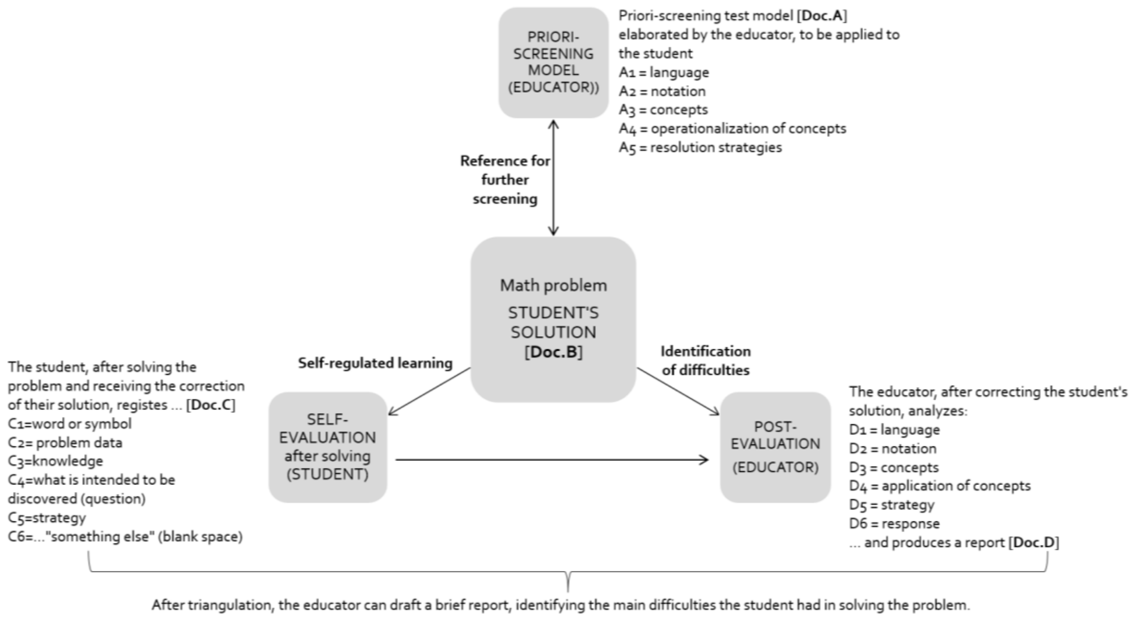

Figure 2. A pedagogical instrument for diagnosing the difficulties faced by students in problem-solving

An illustrative problem suitable for nine to ten-year-old students (4th grade) has been provided:

> Peter and David operate a lawn-cutting business. On Tuesday, Peter is available to cut 2/3 of Mrs. Silvia's garden lawn, while David is available to cut only 1/4 of the same lawn.
>
> a) What fraction of the lawn can the two friends cut on Tuesday?
>
> b) What fraction of the lawn Peter cut more than David?
>
> c) Assuming the lawn area was 1800 m^2, what portion of the lawn would remain uncut by Peter on Tuesday?

The documents labelled as A, C, and D, which were referenced in the preceding model (Figure 2), are exhibited in Appendices A, B and C. Document A, the *Priori Model Test*, is presented in Appendix A; document C, *Student Learning Self-assessment*, is in Appendix B; and document D, *Report on the Identification of Students' Difficulties*, is presented in Appendix C.

To familiarize the trainees with the model, they were organized into two working groups, each composed of one teacher and two educational psychologists. One group played the role of the teacher, while the other took on the role of the student, and the tool was applied accordingly. In the end, the groups switched roles and reapplied the tool. This activity lasted about 2 hours. After this training session, one of the trainees applied the same model with a student, which constitutes the example of application presented in this study. The student filled out documents A, B, and C, and the teacher organized the data presented in Table 4.

Table 4. Application of documents A, B, and C. (NA means not applicable; NR means no response).

	Document A (Diagnosis)	Document B (Student's resolution)	Document C (Student self-assessment)	Observations
Language				
"One part of something"	Identifies "half of something" as possible but not "a quarter of something" or "less than something".	NA	Identifies, correctly describing, what was being asked	Understands the language, the meaning of mathematical terms, and what is asked in the problem.
"more than"	Gets all correct options right.			
surface	Recognizes that a surface is a portion of a plane.			
Notation				
Fraction	Doesn't recognize the representation of a fraction because identifies division (perhaps thinking about the meaning of the operation, so the failure is in the meaning of the word "represents").	In items a) and b), it provides a fraction as an answer, although it responds incorrectly.	Indicates not understanding the symbol 3:4.	Confuses the representation of the division operation with the representation of a fraction.
m^2(square meters)	Recognizes the usual correct identification but doesn't recognize that m^2 is equivalent to $m \times m$.			Does not understand the meaning of square meter.
Concepts				
Fraction	Chooses the correct option that a fraction means a part (numerator) of the whole (denominator).	In items a) and b), it does not seem to have acquired the concept of a fraction, as it does not correctly apply equivalent fractions to operationalize what it intends.	Recognizes that they should have knowledge about "solving" fractions.	Recognizes that a fraction is part of a whole but does not understand the meaning of equivalent fractions.
m^2(square meters)	Doesn't have the concept of $1m^2$.			

Operationalization of concepts and operations	Correctly identifies the calculation of 4/5 of 20 (chooses the correct option for the operation in question). Correctly associates the expression "more than" with the addition operation.	In items a) and b), it does not correctly convert one fraction into another equivalent, in order to add or subtract two fractions. NR in items c)	NA	Does not know how to add or subtract fractions (due to not understanding the meaning of equivalent fractions).
Strategy	NR	It only uses operations (does not resort to any image).	Mental reasoning.	Did not identify any appropriate strategy to solve problems involving fractions; however, designates mental reasoning as the strategy used.
Answer	NA	It responds incorrectly to items a) and b). It does not respond to item c), failing to calculate a fraction of the total area.	NA	Does not calculate a fraction of the total area; however, correctly identified the calculation of 4/5 of 20 in the chosen option.

The examination of table 4 reveals the following findings.

Teacher: In terms of language, the student accurately identifies and articulates the inquiry presented. With reference to the expression one part of something he interprets it as potentially half of something but not as a quarter of something or less than something. Concerning the expression more than, he selects all correct options. He comprehends that a surface represents a portion of a plane and understands the language, the meaning of mathematical terms, and the nature of the problem posed. He properly associates the expression more than with the addition operation. In terms of notation, the student confuses the representation of the division operation with a fraction. He does not grasp the symbol 3/4. However, he accurately computes 4/5 of 20, choosing the correct option for the operation. He fails to recognize the representation of a fraction because he interprets the division operation symbol as the fraction symbol. In items a) and b), he provides a fraction as a response,

> although inaccurately. He acknowledges the need to understand solving fractions to appropriately address the problem. He correctly selects that a fraction signifies a portion (numerator) of a whole (denominator). Nevertheless, he does not appear to have a full comprehension about the concept of a fraction, as he does not apply equivalent fractions to his operations. He recognizes that a fraction constitutes a part of a whole, but he does not understand the concept of equivalent fractions. He understands the concept of a square meter, correctly identifying it but failing to recognize that m^2 is equivalent to m×m. He does not employ any specific strategy, despite identifying one known as mental reasoning relying solely on operations without incorporating any visual supports. His responses to items a) and b) are incorrect. He leaves item c) unanswered, unable to compute a fraction of the total area, but he accurately identifies the calculation of 4/5 of 20 in the selected option.

Subsequently, two additional simulations were conducted with different problems. Each group produced the four documents and applied them to the other group. The session culminated in a reflection about the experience of creating and utilizing such a tool. These simulations occupied four hours of work.

After the training, the trainer requested that an elementary teacher (designated by teacher S) implemented the tool with her class of twenty-four third-grade students. Together, they designed four documents addressing the following problem:

> All the houses in the owl neighbourhood are white, green, or brown. There are twice as many white houses as there are brown ones. Additionally, there are five more brown houses than green ones, and there are exactly seven green houses. What is the total number of houses in the neighbourhood?

These documents will not be included in this text, as their inclusion would result in excessive length. In the next section we present the results.

Results

There is a clear alignment between the categories addressed in Table 3 and documents A and D, supporting the integration between stages of training: language, notation, concepts, operationalization, and strategies.

Trainees indicated that the limitations of the proposed tool include: the significant amount of time demanded from teachers when implemented on a large number of students, due to the personalized nature of the report; the careful attention required from teachers when formulating questions in document A; and the presentation of the problem being unattractive to less proficient students. Several strengths have been identified:

- Following the triangulation of documents A, B, and C, it facilitates the conclusions in document D;

- Acquiring an understanding of students' awareness regarding their performance, offers a valuable perspective on the functioning of triangulation and the cognitive processes of the students;
- Implementing the model in a classroom presents challenges, primarily due to the substantial number of students; however, after identifying those students facing greater difficulties, it becomes highly logical to undertake this form of intervention;
- The use of this tool facilitates a more precise intervention than previously achievable;
- Ascertaining whether each student's challenges stem from insufficient comprehension of language, a deficiency in calculation skills, a lack of conceptual knowledge, or difficulties in decoding mathematical language;
- Cultivating awareness among educators to facilitate the development of more insightful instructional cues for students;
- Presenting a student with previously unrecognized aspects of the problem;
- Deemed an effective instrument for application in psychological assessments and more personalized follow-up, while enhancing the trainees' ability to orally identify difficulties that they had previously attempted to express in writing.

To better understand the impact of its implementation, some results obtained from Teacher S's class are presented. These results are exclusively related to the concept of "more than", one of the mathematical concepts included in the problem presented to the students (Table 5).

With reference to Table 5, numerous findings pertaining to the concept of "more than" can be emphasized:

- A single student accurately responded to the inquiries concerning language comprehension, the operationalization of the concept, and the problem-solving section, despite the individual's self-assessment of not having comprehended the concept;
- Six students demonstrated a lack of comprehension regarding the language; however, they successfully responded to the operationalization of the concept and the problem-solving section associated with this concept. Furthermore, two of these students acknowledged their lack of understanding of the concept itself;
- Five students provided incorrect responses to the questions pertaining to language, operationalization, and the problem-solving section;
- Five students, despite their incorrect responses to the questions pertaining to language and operationalization, successfully provided correct solutions to the subsequent problem-solving section.

Table 5. Results concerning the concept "More than" (C=Correct; I=Incorrect)

	Doc A		Doc B	Doc C
	Post-Screening	Model Test	Problem	Student Learning Self-assessment
Student	Language	Operationalization	Resolution	(student says he does not understand)
1	C	C	C	X
2	I	I	C	
3	I	C	I	
4	I	C	C	X
5	I	C	C	
6	I	C	C	X
7	I	C	C	
8	I	C	I	
9	I	I	C	
10	I	C	C	
11	I	I	I	
12	I	I	C	X
13	C	I	C	
14	I	C	C	
15	I	C	I	X
16	I	I	I	
17	C	C	I	X
18	I	I	I	
19	I	I	C	
20	I	C	I	
21	I	I	C	
22	C	C	I	X
23	I	I	I	X
24	I	I	I	X

According to teacher S, who is not aware of any comparable tool, the significance of this one lies in its ability to identify and comprehend specific challenges students face in understanding the problem. It also enhances the teacher's sensitivity in crafting a problem statement that is clearer to the student. As strengths, she acknowledges that this pedagogical tool enables:

- the organization of cognitive reasoning in an elementary student, facilitating the identification of the stages of problem resolution;
- to present a student with nuances that he had not previously recognized, such as the meaning of certain terms in mathematical language.

In her opinion, it is suggested that it would be beneficial for the student to have document B available when completing document C.

The following section presents the discussion of results and the formulation of conclusions.

Discussion and conclusions

The reviewed literature highlights key points of analysis regarding students' difficulties in problem-solving, which can be summarized as follows:

- mathematical language/notation/vocabulary (Kusumadewi & Retnawati, 2020; Mahmud et al., 2020; Marzano, 2004; Nurharyanto & Retnawati, 2020; Nurkaeti, 2018; Powell et al., 2021; Sulisyani et al., 2021; Yeni & Wahyudin, 2013);
- mathematical concepts (Hadi et al., 2018; Nurkaeti, 2018; Palhares, 2004; Yeni & Wahyudin, 2013);
- operationalization of concepts (counting operations) (Hadi et al., 2018; Kusumadewi & Retnawati, 2020; Nurharyanto & Retnawati, 2020; Nurkaeti, 2018; Palhares, 2004);
- strategies (Hadi et al., 2018; Palhares, 2004; Sulisyani et al., 2021).

In stage 1 of the method, the proposed formulation of questions by trainees is supported by questioning techniques mentioned by Kojo et al. (2018) and Kirkland and McNeil (2021).

Regarding stage 2, the active participation of educators in the development of the tool, underscores the critical significance of the teacher's role in problem-solving learning, as emphasized by Suseelan et al. (2022).

According to Nurharyanto and Retnawati (2020), narrative-type problems must first be comprehended. Their complexity contributes to the increased difficulty students encounter in resolving such problems and the challenges teachers face in understanding students' difficulties. The aim of this study was to develop a pedagogical tool for teachers to use, which enables the identification of students' difficulties in problem-solving, thereby ensuring the achievement of both general and specific objectives. This tool was created by the trainer teacher and disseminated among other teachers during training sessions.

The perception of trainees regarding the use of the tool indicates that it is possible to ascertain whether students' difficulties arise from a lack of prior knowledge, an inability to comprehend the language or concepts within the narrative, or a challenge in applying knowledge or strategies. This model has met the expectations of trainees within this training module. In their view, although it may require more time at the initial stage (defining the model's documents), time is subsequently conserved through a clear assessment of students' difficulties in problem-solving, which facilitates an efficient intervention.

In the opinion of teacher S, the pedagogical tool offers an account of the general challenges faced by the class, but more significantly, it provides insights into each student's individual difficulties. Through this model, S articulated that it enables the identification of students who require additional attention in language, operationalization, or contextual problem-solving. Furthermore, it facilitates the development of self-awareness concerning knowledge and performance, thereby underscoring the tool's robustness. Yeni and Wahyudin (2013) asserted a deficiency in the comprehension of mathematical concepts, which aligns with the observation that only four students in the class understood the language.

This study is subject to certain limitations, specifically the limited number of teachers providing feedback regarding the pedagogical tool and the implementation of the tool with a relatively small sample of students.

In future research, it would be advantageous to replicate the model with a more substantial dataset from teachers and to apply the tool to a larger group of students. This approach would facilitate the generation of additional suggestions for refinement and allow for a more comprehensive assessment of the model's efficacy. As noted by Proença et al. (2022), forthcoming research could investigate pedagogical proposals that are explicitly designed to promote the development of concepts and procedures, both before and during problem-solving activities. Consequently, future enhancements to the pedagogical tool could consider its integration into a teaching-learning methodology, rather than being merely employed as a diagnostic instrument.

The significance of this study lies in its contribution to the ongoing efforts to enhance the effectiveness of mathematical problem-solving instruction. By developing a pedagogical tool that enables educators to identify specific difficulties that students face in problem-solving, the study provides a practical and evidence-based approach to improve teaching practices. The tool not only facilitates more targeted interventions but also enhances the educators' ability to offer individualized support, ensuring that students' needs—whether related to language comprehension, conceptual understanding, or operationalization—are accurately addressed. Furthermore, the study’s findings highlight the potential for this tool to foster a deeper awareness among educators, promoting a more nuanced approach to teaching problem-solving skills. Given the pivotal role that problem-solving plays in the development of mathematical competence, this research serves as a crucial step towards improving both instructional strategies and student outcomes in mathematics education.

Acknowledgments

I would like to express my gratitude to Teacher S for implementing the pedagogical tool with her students in class B, 3rd year, during the 2023/2024 academic year.

References

Adami, M., & Kiger, A. (2005). The use of triangulation for completeness purposes. *Nurse researcher, 12*, 19-29. https://doi.org/10.7748/nr2005.04.12.4.19.c5956

CORE (2013). *Problem solving in elementary math: Participant handout* (Version 1.0). Consortium on Reaching Excellence in Education. https://www.corelearn.com

Davies, R. (2017, 16 august). *Teaching math problem solving: 6 research-based approaches that work*. Differentiated Teaching. https://www.differentiatedteaching.com/build-math-problem-solving-skills/

Fuchs, L. S., Bucka, N., Clarke, B., Dougherty, B., Jordan, N. C., Karp, K. S., & Woodward, J. (2021). *Assisting students struggling with mathematics: Intervention in the elementary grades (Practice Guide)*. National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. https://ies.ed.gov/ncee/wwc/PracticeGuide/26

Given, L. M. (2008). The SAGE encyclopedia of qualitative research methods (2 vols.). SAGE Publications.

Hadi, S., Herman, T., & Hasanah, A. (2018). Students' difficulties in solving mathematical problems. International Journal of Educational Science and Research, 8(1), 55-63. https://scispace.com/pdf/students-difficulties-in-solving-mathematical-problems-50m6iro8rx.pdf

Irenika, I., Susanti, E., Hiltrimartin, C., Hapizah, H., & Meryansumayeka, M. (2019). Design of problem-solving questions for measuring student's mathematical thinking type representation. *Journal of Physics*: Conference Series, 1318, 012102. https://doi.org/10.1088/1742-6596/1318/1/012102

Kirkland, P., & McNeil, M. (2021). Question Design Affects Students' Sense-Making on Mathematics Word Problems. *Cognitive Science, 45*(4), 1-29. https://doi.org/10.1111/cogs.12960

Kojo, A., Laine, A., & Näveri, L. (2018). How did you solve it? – Teachers' approaches to guiding mathematics problem solving. *LUMAT: International Journal on Math, Science and Technology Education*, *6*(1), 22–40. https://doi.org/10.31129/LUMAT.6.1.294

Kusumadewi, C. A., & Retnawati, H. (2020). Identification of elementary school students' difficulties in mathematical problem-solving. *Journal of Physics: Conference Series, 1511*(1), 12-31. https://doi.org/10.1088/1742-6596/1511/1/012031

Mahmud, M., Yunus, A., Ayub, & A. F. M., Sulaiman, T. (2020). Enhancing Mathematical Language through Oral Questioning in Primary Schools. *International journal of Learning, Teaching and Educational Research, 19*(5). https://doi.org/10.26803/ijlter.19.5.24

Martaningsih, S. T., Maryani, I., Prasetya, D. S., Purwanti, S., Sayekti, I. C., Abdul Aziz, N. A., & Siwayanan, P. (2022). STEM Problem-Based Learning Module: a solution to overcome elementary students' poor problem-solving skills. *Pegem Journal of Education and Instruction*, *12*(4), 340–348. https://www.pegegog.net/index.php/pegegog/article/view/1856

Marzano, R. (2004). *Building background knowledge for academic achievement*. Association for Supervision and Curriculum Development.

Ministério da Educação. (2021, August 19). *Despacho n.º 8209/2021*. Diário da República, 2.ª série, n.º 161. https://diariodarepublica.pt/dr/detalhe/despacho/8209-2021-169831748

Ministério da Educação e Ciência (2013). *Programa Nacional de Matemática do Ensino Básico. Programa e Metas Curriculares Matemática Ensino Básico.* https://www.dge.mec.pt/sites/default/files/Basico/Metas/Matematica/programa_matematica_basico.pdf

Nasrun, Prahmana, R.., & Akib, I. (2023). The Students' Representative Processes in Solving Mathematical Word Problems. *Knowledge*, *3*, 70-79. https://doi.org/10.3390/knowledge3010006

Nurharyanto, D., & Retnawati, H. (2020). The difficulties of the elementary school students in solving the mathematical narrative test items. *Jurnal Prima Edukasia, 8*(1), 29-39. https://doi.org/10.21831/jpe.v8i1.29969

Nurkaeti, N. (2018). Polya's strategy: an analysis of mathematical problem solving difficulty in 5th grade elementary school. *EduHumaniora|Journal of Basic Education Cibiru Campus*, *10*(2), 140. https://doi.org/10.17509/eh.v10i2.10868

Palhares, P. (2004). *Elementos de matemática para professores do ensino básico*. Lidel.
Pólya, G. (1945). *How to Solve It: A New Aspect of Mathematical Method*. Princeton University Press.
Pólya, G. (1962). *Mathematical Discovery: On Understanding, Learning, and Teaching Problem Solving* (Vol. 1). Wiley.
Powell, S., Bos, S. & Lin, X. (2021). The assessment of mathematics vocabulary in the elementary and middle school grades. In A. Fritz, E. Gürsoy, & M. Herzog (Eds.), *Diversity Dimensions in Mathematics and Language Learning: Perspectives on Culture, Education and Multilingualism* (pp. 313-330). De Gruyter. https://doi.org/10.1515/9783110661941-016
Proença, M. (2022). Habilidades matemáticas na resolução de problemas: Análise da compreensão de futuros professores. *Bolema*, *36*(74), 1135–1157. https://doi.org/10.1590/1980-4415v36n74a09
Proença, M., Maia-Afonso, É., Mendes, L., & Travassos, W. (2022). Dificuldades de alunos na resolução de problemas: Análise a partir de propostas de ensino em dissertações. *Bolema*, *36*(72), 262–285. https://doi.org/10.1590/1980-4415v36n72a12
Salimpoor, V. (2018, 7 november). *The role of working memory in math processing. Cignition*. https://www.cignition.com/blog/the-role-of-working-memory-in-math-processing
Sulisyani, D., Subekti, E. E., Wardana M. S. (2021). Students' Learning Difficulties Review from Mathematics Problem-Solving Ability in Third-Grade Elementary School. *Indonesian Journal of Educational Research and Review 4*(2), 345-351.http://dx.doi.org/10.23887/ijerr.v4i2.30310
Suseelan, M., Chew, C.M., & Chin, H. (2022). Research on mathematics problem solving in elementary education conducted from 1969 to 2021: A bibliometric review. *International Journal of Education in Mathematics, Science, and Technology (IJEMST), 10*(4), 1003-1029. https://doi.org/10.46328/ijemst.2198
Yeni, E. M., & Wahyudin, T. H. (2020). Difficulty Analysis of Elementary School Students in Mathematical Problem Solving in Solutions. *International Journal of Scientific & Technology Research, 9*, 44-47. https://api.semanticscholar.org/CorpusID:216466826

Appendix A

DOCUMENT A. Priori Model Test
Choose the options that seem correct to you, for each question (note that you can choose more than one).

A1) Language
1.1. Saying "one part of " can mean:
a) Half of something
b) Twice of something
c) A quarter of something
d) Less than something
e) Greater than something

1.2. Johanna has 8 candies and has 4 candies "more than" Mary. So, we can state that:
a) Mary has 4 more candies than Johanna
b) Mary has 12 candies
c) Johanna has 8 candies
d) Mary has 4 candies less than Johanna
e) The two friends, together, have 12 candies

1.3. A surface can be:
a) A portion of a plane
b) A portion of a straight line
c) What delimits a cube
d) What delimits a sphere
e) What delimits a rectangle

A2) Notation
2.1. A fraction can be represented by:
a) 3:4
b) 3,4
c) $\frac{5}{4}$
d) 5-4
e) $\frac{4}{5}$

2.2. 3 square meters can be represented by:
a) 3 m × m
b) 3 m^2
c) 3 m × 2
d) 3 m+2
e) 3^2 m

A3) Concepts
3.1. Eating $\frac{4}{5}$ of a chocolate means that…:
a) If you divide the chocolate into 4 parts, you eat 5 of those parts
b) If you divide the chocolate into 5 parts, you eat 4 of those parts
c) If you divide the chocolate into 9 equal parts, you eat 4 of those parts
d) If you divide the chocolate into 4 equal parts, you eat 5 of those parts
e) If you divide the chocolate into 5 equal parts, you eat 4 of those parts

3.2. A square meter is the measurement of a rectangular surface with:
a) 1 meter in length and 1 meter in width
b) 0.5 meters in length and 0.5 meters in width
c) 1 meter in length and 0.5 meters in width
d) 1 centimetre in length and 1 centimetre in width
e) 0.5 centimetres in length and 0.5 centimetres in width

A4) Operationalization of Concepts

4.1. Calculating $\frac{4}{5}$ of 20 euros is computed by:
a) $\frac{4}{5} \times 20 = \frac{4 \times 20}{5} = 16$ euros
b) $\frac{1}{5} + 20 = \frac{4+20}{5} = 4.8$ euros
c) $\frac{4}{5} \times 20 = \frac{4 \times 20}{5 \times 20} = 0.2$ euros
d) $\frac{4}{5} \div 20 = 0.8 \times 20 = 1.6$ euros
e) $4 \times 20 + 5 \times 20 = 180$ euros

4.2. When you find the expression "more than" in the text of a problem, you think of:
a) subtracting
b) adding
c) multiplying by 2
d) dividing
e) multiplying by 10

A5) Resolution Strategies

To solve problems with fractions, which of the following representations seems most appropriate to you?
a) Table
b) Tree diagram
c) Number line
d) Sets
e) Correspondences
f) Other:

Appendix B

DOCUMENT C. Student Learning Self-assessment

Answer thoughtfully to the following questions:

C1) Word or symbol

Mark the expressions or symbols that you didn't understand…

part more than surface m^2

C2) Problem data

Explain in your own words which data were provided in the problem?

C3) Required knowledge

In your opinion, which mathematical knowledge would you need to solve the problem?

C4) What is to be discovered

Explain in your own words what numerical calculations were asked?

C5) Strategy

What strategy did you use to solve the problem?

C6) Add any comments you consider important about understanding/lack of understanding you had of the problem.

Appendix C

DOCUMENT D. Report on the Identification of Students' Difficulties (presented in a pre-formatted table)

	Document A (Diagnosis)	Document B (Student's resolution)	Document C (Student self-assessment)	Observations
Language				
"One part of something"				
"more than"				
surface				
Notation				
Fraction				
m^2(square meters)				
Concepts				
Fraction				
m^2 (square meters)				
Operationalization of concepts and operations				
Strategy				
Answer				