STEAM stained-glass window: Integrating the arts, mathematics and technology into a *maker* experience

Vitral STEAM: Integrando artes, matemática e tecnologia numa experiência *maker*

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Abstract. This article presents an experience conducted within the context of STEAM education and maker culture, involving the creation of an artistic stained-glass window, based on a newly discovered aperiodic tiling, using a single irregular piece with 13 sides. Combining digital resources (GeoGebra) and physical resources (laser-cut acrylic pieces), two pre-service Mathematics teachers faced geometric construction challenges and explored symmetries, patterns and visual properties, considering the Gestalt principles — unification, segregation, symmetry and simplification. In this context, the qualitative and exploratory study was guided by the following research question: How can the process of geometrically constructing an irregular figure, motivated by a contextualized problem, evolve with the support of technology and from the perspective of form perception (Gestalt Theory)?. The task was designed as an open activity, valuing autonomy, creativity and interdisciplinarity among Mathematics, Arts, and Technology. Throughout four construction cycles, an evolution in geometric reasoning, technological mastery in tool use, and visual perception was observed among the participants. The final result not only solved a practical problem, but was consolidated as an artistic and pedagogical product, reinforcing the transformative potential of STEAM practices in Mathematics teaching.
Keywords: GeoGebra; irregular polygon; stained-glass window; aperiodic tiling; STEAM education; teacher education.

Resumo. Este artigo baseia-se numa experiência realizada no contexto da educação STEAM e da Cultura *Maker*, a partir da criação de um vitral artístico com base num ladrilhamento aperiódico recém descoberto, utilizando uma única peça irregular de 13 lados. Combinando recursos digitais (GeoGebra) e físicos (peças em acrílico, cortadas a laser), duas estudantes de Licenciatura em Matemática enfrentaram desafios de construção geométrica e exploraram simetrias, padrões e propriedades visuais à luz dos princípios da Gestalt — unificação, segregação, simetria e simplificação. Nesse cenário, a pergunta que guiou a pesquisa, de caráter qualitativo e exploratório, foi como o processo de construção geométrica de uma figura irregular, motivado por um problema contextualizado, pode evoluir com o auxílio da tecnologia e sob a perspectiva da percepção da forma (Teoria da Gestalt)?. A tarefa foi concebida como uma atividade aberta, valorizando a autonomia, a criatividade e a interdisciplinaridade entre Matemática, Artes e Tecnologia. Ao longo de quatro ciclos de construção, observou-se uma evolução do raciocínio geométrico, do domínio tecnológico no uso das ferramentas e da percepção visual das participantes. O resultado final não apenas solucionou um problema prático, mas consolidou-se como produto artístico e pedagógico, reforçando o potencial transformador de práticas STEAM no ensino da Matemática.
Palavras-chave: GeoGebra; polígono irregular; vitral; ladrilho aperiódico; Educação STEAM; formação de professores.

Introduction

Recent research on geometric solutions for the three-dimensional coverage of epithelia (Gómez-Gálvez et al., 2018) has rekindled efforts to explore and analyze such structures, which form a "mosaic" in both three- and two-dimensional space. In addition, investigating patterns of regularity and the applications of mathematics, particularly geometry, across different fields is often neglected in schools to accommodate extensive curricula that are distant from students' reality (Berry & Larson, 2019). Exploring mosaics in basic education could naturally trigger integrated STEAM approaches in schools, but it is also an opportunity to promote open tasks in classrooms and thus encourage creativity in school settings.

This article is part of research investigating *Maker* culture and STEAM education, combining current technologies, such as GeoGebra and laser cutters, to create materials for pedagogical use, especially for teaching and learning mathematics, arts, or for entertainment (games or logical challenges). In this text, we focus on discussing an experience with an open task in which mathematics was used to create artistic constructions with the aid of digital and non-digital resources, with the final objective of developing a "stained glass" to cover a window based on a pattern discovered in March 2023 (Smith et al., 2024). This pattern covers the plane infinitely with the same irregular part and aperiodically, that is, without exhibiting any arrangement that repeats continuously[2].

For this task, two students in the mathematics teaching degree course were required to explore the mentioned pattern to build a mosaic that would cover a window at the institution where they study. In addition to solving a real problem (the window had no curtain and, at the same time, did not depend on the passage of light), the proposal aimed to foster integration (**A**rts and **M**athematics, supported by **T**echnology) as a methodological strategy to explore different concepts, both geometric and technological (with GeoGebra and laser cutter), in the development of an artistic product. In this context, the research was guided by the following question: How can the process of geometric construction of an irregular figure, motivated by a contextualized problem, evolve with the help of technology and from the perspective of the perception of form (Gestalt theory)?

STEAM Education and *Maker* Culture

The approach through *Maker* Culture in the school environment can be presented in various ways, but it mainly involves students designing, developing, modifying, and sharing products to solve problems (Chen & Lin, 2019; Doorman et al., 2019; Paganelli et al., 2016). Products refer to digital and non-digital artefacts elaborated within the *maker* environment, that is, the *makerspace* (Yin et al., 2020).

The maker culture approach, anchored in hands-on activities, emphasizes design and is typically implemented through STEAM education. Among these approaches, we highlight the promotion of curiosity, the stimulation of pleasure in learning and the appreciation of unique solutions (Kurti et al., 2014). When such activities address multiple subjects but focus on one, for example, mathematics, they are sometimes labelled as an integrated steaM activity[3] (Ortiz-Laso, 2023; Namukasa et al., 2023; Stohlmann, 2018). Practical activities promote students' involvement in the learning process rather than having them assume only passive roles as listeners (Chen & Lin, 2019). Generally, such activities are seen as a way to contextualize STEAM-related content through real-world problems, allowing for multiple approaches and solutions (Diego-Mantecón et al., 2022; Hsu et al., 2019). In particular, English (2020) highlights their role in helping students not only develop new knowledge and disciplinary skills but also understand the links between subjects.

Practical activities within STEAM education have predominantly received attention in engineering education studies, with less exploration of their impact on mathematics teaching and learning (Diego-Mantecón et al., 2022; English, 2020). As noted by English (2020), this approach is generally defined in terms of problem solving in engineering and technology, using the engineering design process. When adopting a mathematics learning approach, emphasis is also placed on problem solving, as it offers students the opportunity to engage with mathematical concepts, employ diverse representations, and engage in critical reflection (Doorman et al., 2019).

To this end, Ortiz-Laso et al. (2023) adapted one of the categories of Diego-Mantecón et al. (2019) and Blanco et al. (2021), 'creating art with mathematics', and proposed its application through practical activities. In particular, Ortiz-Laso and collaborators conducted refined activities over three years to promote the use of geometric content in creating works of art in a technological creation environment (Ortiz-Laso et al., 2023). In this sense, Doorman et al. (2019) and Harron et al. (2022) also note that it is challenging for teachers to design and implement practical activities that align with institutional practices and learning objectives.

In this study, the integration of STEAM practices and Maker Culture is articulated through the concept of the open task (Sullivan, 2018), as both emphasize valuing creative processes, addressing real problems, and exploring multiple, non-prescribed solutions (or resolutions).

Thus, the proposal to build the stained-glass window, as an open task, favors the development of the sTeAM[4] practice by promoting an experience in which mathematics, the arts, and technology are intertwined to give meaning to the final product. In addition, it allows students to assume the role of creator rather than just a breeder, engaging with maker principles in the school environment.

Tiling, visual perception, and technology in geometric and artistic creation

Gestalt theory gained strength at the beginning of the 20th century through the work of Max Wertheimer, Wolfgang Köhler, and Kurt Koffka (Gomes Filho, 2009; Lefrançois, 2008), and offers important contributions to the understanding of how individuals perceive and organize visual information. In the context of mathematics teaching (more specifically, geometry), this approach is particularly relevant, as the way the content is presented can significantly influence how students understand the objects of study, in particular geometry.

From a Gestalt perspective, the arts are based on the principle of *Prägnanz* (simplification). That is, in the formation of images, the factors of balance, clarity and visual harmony constitute for the human being a need and, therefore, are considered indispensable—whether in a work of art, in a graphic piece or in any other type of visual manifestation (Gomes Filho, 2009). According to Gestalt, the processes of perception and imagination do not obey logical or rational rules, but are guided by prototypical patterns (familiar figures are more easily identified) and, at the same time, can introduce a certain ambiguity (Kus & Newcombe, 2025).

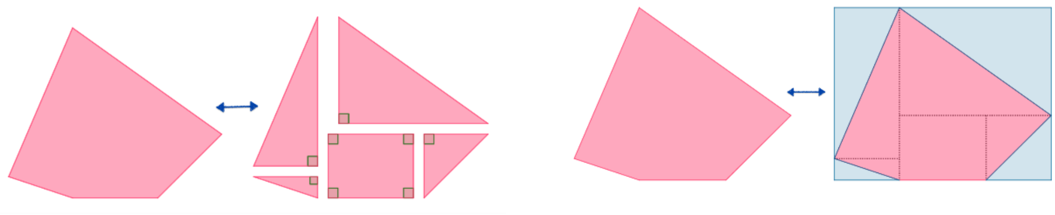
According to Lefrançois (2008), the central concept of Gestalt theory is insight, understood as the sudden perception of the relationships among the elements of a problem situation. This is relational thinking, in which the subject mentally reorganizes the components of a problem until they form a coherent, meaningful structure. Thus, learning is not about memorizing isolated data, but about perceiving the structure and internal relationships of concepts, which strengthens the Gestalt's connection with constructivist approaches (particularly, in the scope of this work, those favored by the combination of geometry and the arts), which foster active, less rote learning.

Thus, Gestalt does not prioritize what students should learn, but how they should learn. The focus is not on the answers, but on the processes that lead to them; the same answer can result from different processes of understanding that involve perception and organization of the structure in question (Kus & Newcombe, 2025).

More specifically, in geometry teaching, the Gestalt principles that can be explored or developed are those of *unification*, *segregation*, *symmetry* and *simplification*, since they guide how visual stimuli can be organized to facilitate the understanding of structures (or shapes). These principles are briefly described below:

Unification: refers to the tendency of the human mind to group visual elements that seem to belong to the same set or context. Visual unification fosters a global understanding of the content, rather than the fragmented memorization of isolated rules. For example, identifying the congruence of sides and angles in a given polygon is a necessary step in recognizing it as a regular polygon.

Segregation: refers to the perceptual ability to separate, identify, put in evidence or highlight formal units in a compositional whole or in parts of this whole. Using colors, auxiliary lines or points, shadows, textures, among others, can help better perceive which information belongs to the same group and which should be analyzed separately. In problem-solving situations, for example, visual clarity helps identify relevant data from secondary information. Below is an example illustrating how the pictorial representation enables different strategies for calculating the area of an irregular polygon, depending on how the figure is interpreted (Arnal-Palacián, 2021). One strategy is to decompose this polygon into simpler geometric figures, such as a rectangle and four right triangles, as shown in Figure 1(a). Another strategy is to "surround" the polygon with a rectangle (Figure 1(b)) and compute its area as the difference between the area of the rectangle and the sum of the areas of the complementary right triangles that delimit it.



(a) (b)

Figure 1. Representations to favor the understanding of the area of an irregular polygon

Symmetry: it is related to the mind's search for balance and stability in perceived forms. When presenting mathematical content, especially in topics such as geometry, algebra[5], and functions, the use of symmetries can make it easier to identify patterns and properties. The recognition of symmetry in graphs, expressions or figures, in addition to supporting the development of logical reasoning, can foster an intuitive experience. For example, at their earliest ages, children have already awakened a sense of symmetry when playing with games of building blocks or Lego assembly.

Simplification (Prägnanz): considers that the perceptual system tends to organize stimuli as simply and coherently as possible. In geometry (and mathematics in general), this principle encourages presenting content clearly, reducing visual noise and focusing on the essentials. Well-organized and visually "clean" materials reduce cognitive overload, facilitating students' focus on mathematical reasoning itself.

These principles not only organize visual perception but also promote conceptual understanding by emphasizing the internal relationships among mathematical elements. As Batista et al. (2017) point out, by adopting pedagogical practices based on the Gestalt, the mathematics teacher instigates the student to "see beyond the exposed", encouraging research, questioning, and the active construction of knowledge.

The application of Gestalt to the teaching of mathematics implies a repositioning of the teacher's role: from transmitter of content to organizer of learning experiences. As stated by Burow and Scherpp (1985), the educator must organize environmental stimuli in a way that favors the student's perceptual process. This organization is not limited to the visual aspect, but extends to the structuring of tasks, the language used, and how concepts are connected.

Based on Gestalt principles, the teaching of mathematics should favor the discovery, the perception of relationships, and the construction of meanings. Instead of focusing only on rules and procedures, the focus shifts to the recognition of structures and the mental reorganization of problem situations, in line with the foundations of insight thinking described by Köhler (1925, 1929, cited by Lefrançois, 2008). This perceptual and constructive dimension sets the stage for the work with complex geometric patterns, such as tiling, in which shape, symmetry, and structural relationships become even more evident.

In the context of tiling (or tessellation), the relationship between geometry and the arts materializes in visual constructions that fill the plane in an orderly or non-trivial way. Tiling is present in everyday life — in floors, walls, and works of art — and consists of filling a two-dimensional surface without overlapping polygons or leaving gaps. They can be classified as periodic or aperiodic (Barros, 2016; Neto et al., 2023).

Periodic tessellations are pavements that, when translated, overlap, keeping the tiles aligned and juxtaposed. In contrast, the *aperiodic* ones are tessellations in which there is no repetition of a pattern, only by translations, and they can cover the plane without intermediate spaces or overlaps (Amaral et al., 2014). They can also be classified as regular, semi-regular, demi-regular, or irregular.

Regular tessellations are created by repeating the same regular polygon, in three possible types: with equilateral triangles, squares, and hexagons (regular). This phenomenon stems from the fact that the interior angles of the polygon used must be 360° dividers so that they completely cover the plane, without gaps or overlaps. *Semi-regular*, or Archimedean, tessellations are those that have two or more types of regular polygons, all vertices having the same configurations. *Demiregular* tessellations are those in which the tiling is composed of two or more distinct regular polygons, but the vertices do not all have the same configuration (Barros, 2016). Finally, *irregulars* are those that are neither regular, nor semi-regular, nor demiregular (in other words, that are not composed entirely of regular polygons).

The pattern explored in this work is a particular case of irregular aperiodic tiling, from a base polygon identified as "*Einstein*"[6] or "hat"[7]. Santos Neto et al. (2023, p.1) present "two ways of constructing these polygons, the first of which is based on the juxtaposition of copies of a particular kite and the second consists of constructing, with a ruler and compass, the polygonal line formed by the sides of each of these polygons". The exploration was carried out using GeoGebra. Still addressing studies on this irregular polygon, which has 13 sides, Carniel et al. (2023, p. 1) developed an experience aiming to highlight how the arts and mathematics can be integrated into pedagogical proposals that foster the creative development of students.

In this context, the idea of building the stained-glass window was to extend the look on the approach of Carniel et al. (2023), in contrast to Santos Neto et al. (2023), in order to identify how the recognition of the geometric patterns involved impacted the construction process of the base polygon for the tiling in question, exploring a tessellation with an irregular polygon, to form a stained-glass window with a 13-sided polygon in which it was possible to fit identical pieces, in a non-trivial way, which demanded the search for a solution in materials that deal with the theme. This analysis also considered the circumstances of the combined use of physical and digital resources as constructs for both mathematical, technological, and artistic education (especially in relation to the use of resources such as GeoGebra and laser cutters).

In the stained-glass window project, the use of technologies served as the link between geometric-visual reasoning, grounded in Gestalt principles, and the materialization of the artistic work, enabling the construction of elaborate patterns and the creative exploration of space. This scenario is advocated in the literature to the extent that technologies not only expand human capacities but also profoundly transform ways of thinking, producing knowledge, and relating to the world.

Pierre Lévy (1997), in discussing intelligence technologies, shows how orality, writing, and computer science are different forms of organizing thinking and collective memory. According to him, there is no separation between subjects and technical objects — we are shaped by the tools we produce and, at the same time, we shape the world through them. Computer science, in this context, does not replace orality or writing, but transforms them, giving rise to new forms of expression, such as digital writing, hypertext, and network communication.

More recently, authors such as Borba and Villarreal (2005) propose that writing and orality begin to assume new configurations in the digital environment: we speak of tertiary orality and secondary writing, pointing to a new status of language and cognition. These transformations are not neutral, as they imply changes in teaching and learning — especially in mathematics.

Mathematical production, historically marked by pencil, paper, and blackboard, is influenced by dynamic digital environments, in which thinking relies on simulations, animations, interactions, and visualizations that were not possible before. According to Healy and Sinclair (2007), digital technologies challenge traditional conceptions of mathematics as a purely symbolic and abstract language, as they introduce more visual, interactive, and manipulative modes of exploration.

An example of this is the use of GeoGebra. With it, mathematical reasoning becomes less linear, allowing experimentation with conjectures, visualization of properties, and direct manipulation of mathematical objects (Zulatto, 2002). The way mathematics is learned and done is transformed: there is a transition from "thinking with pencil and paper" to "thinking with digital objects", which also implies epistemic transformations.

As Borba et al. (2020) point out, when operating in digital environments, students begin to build mathematical meanings in articulation with the actions carried out on and with technologies. It is a form of distributed cognition (Noss & Hoyles, 1996), in which the subject, technology, and knowledge form an integrated system. In this sense, thinking mathematically with GeoGebra is not the same as thinking mathematically without it: the very nature of mathematical activity changes. Thus, technology is not only an auxiliary tool, but an active agent in the constitution of mathematical thinking and doing.

Methodology

This study was conducted within the scope of the Tutorial Education Program (Programa de Educação Tutorial - PET) of a mathematics teaching degree course at a federal public educational institution, whose objective is to promote integrated teaching, research, and extension actions within teacher education. The experiment was developed during periodic meetings with two undergraduate students linked to the Program, in which guidelines for the task to be performed, conceptual discussions, experiments with digital tools, and practical construction processes were articulated.

From a methodological point of view, the research is characterized as qualitative and exploratory (Bogdan & Biklen, 1994), aligned with STEAM education and *Maker* Culture perspectives, as it investigates how digital technologies (GeoGebra and laser-cutting resources) can be integrated into teacher education through interdisciplinary activities. The choice of a qualitative approach is justified by the interest in interpreting the meanings students attribute throughout the project's development, as well as in understanding the challenges, advances, and learning in the context of maker culture.

Initially, the meetings (between the teacher-researcher and students) were aimed at exploring the mathematical problem at hand, centered on the aperiodic tessellation discovered by Smith et al. (2024), involving the construction of tiles composed of a single irregular 13-sided polygon. After a first identification of the piece and recognition of its basic elements (sides and angles), the participants began to investigate the possibilities for constructing the figure in GeoGebra, facing challenges related to the rigidity of the shapes, symmetry, and replicability. This process involved multiple attempts at digital prototyping until a viable version was reached, allowing the creation of tools within the software and optimizing the reproduction of the pieces from two entry points.

Once the digital construction of the unitary part was completed (and the resource to replicate it was available), the parts were juxtaposed to simulate the assembly required for the window coverage, respecting the tessellation-based fittings and using triangular meshes as a support for alignment. The unit part file was, in turn, converted to DXF (Drawing Exchange Format), a format compatible with laser cutters. Using laser cutting resources, the pieces were produced in colored acrylic, enabling the assembly of a stained-glass window installed in one of the institution's windows. The choice of material and space was aimed not only at meeting practical demands (light control and privacy), but also at transforming the window into an aesthetically attractive object with pedagogical potential for the dissemination of mathematics.

The data analyzed were recorded in participants' logbooks, teacher-researcher[8] notes, screenshots, GeoGebra files saved online, and photographic records. The analysis involved examining the construction processes, the strategies adopted in the different phases of the project, and the learning opportunities identified. Following the principles of qualitative research (André, 2005; Stake, 2010), we sought to interpret the data in the light of the literature, seeking to understand the decisions made during the different phases of construction, the obstacles faced, and the learning involved, both in the mathematical, technological, and pedagogical scopes.

Our experience: the stained-glass window

The motivation for the stained-glass was to cover a window (for which there were not enough curtains), giving it an artistic character, since the window faces a corridor with a constant flow of students and teachers. Thus, the idea was to value a common space to attract passersby's attention and turn an artistic piece into an instrument for disseminating mathematics, since the intention is to place information about the mosaic next to the stained-glass itself.

Combined with the recent discovery already mentioned (Smith et al., 2024) and the interest in presenting a sTeAM proposal to students of the mathematics teaching degree course—who were already accustomed to the maker space of the institution—it was proposed that the pieces were modeled in GeoGebra and cut in acrylic for the composition of the stained-glass window. An example of composition with the part is shown in Figure 2.

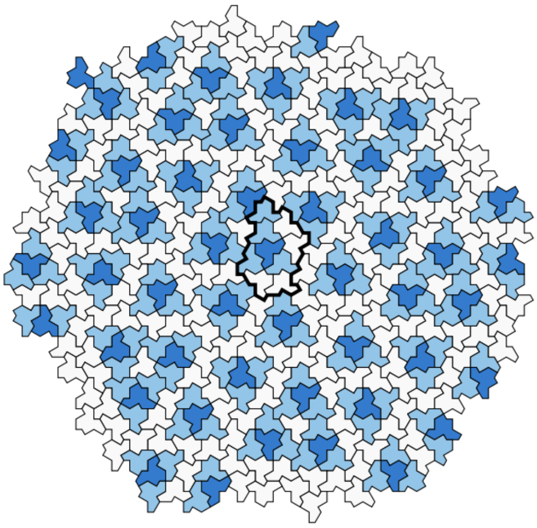


Figure 2. Tiling with the Einstein piece (Smith et al., 2024)

There were some challenges with the construction; initially, it was necessary to build the *Einstein* piece, and the students did not know how to create it in GeoGebra because they had no step-by-step reference. To this end, it was essential to explore the physical pieces[9]. It was necessary to carry out research and attempts to understand 'what was behind the construction of the piece' and, only then, start the process of its construction. During this journey, four prototype tests were carried out until the piece's model was obtained, and later a tool was created to digitally build the mosaic in GeoGebra, without always having to recreate the piece's step-by-step process. Throughout the attempts, the students learned to use software tools to create the prototype and to associate mathematical elements with the process. A brief discussion of the four construction cycles of the students is presented in the following section.

Data discussion: building the stained-glass window as a geometric, artistic, and technological practice

The construction process for the stained-glass window developed by the students evidences a learning path deeply aligned with STEAM education, maker culture, and principles of visual perception. From the students' logbook, it was possible to identify that the constructions evolved, as inconsistencies and even opportunities for simplification were tested and documented at various stages.

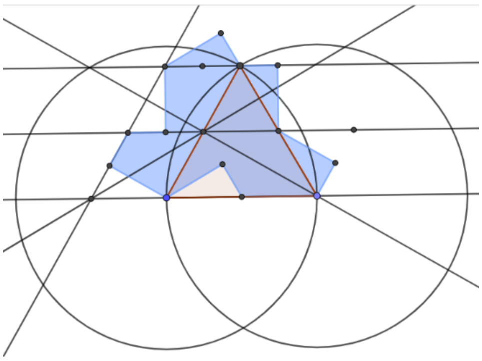
This identification enabled observation not only of the gradual mastery of technological tools but also of advances in understanding the geometric properties of the piece and in the ability to visually organize the tiling pattern. The advancement of this ability dialogues with the Gestalt principle of *Prägnanz*, by showing how perception seeks structured and familiar forms, an essential element for reading and building the tiling explored in the proposal. Therefore, this beginning already indicated how the project integrated, seamlessly, mathematics, arts, and technology, reaffirming the objective of the proposal and establishing the scenario for the analysis of the following cycles, in addition to signaling the character of an open task that would allow the students multiple explorations in the maker environment.

For this initial exploration process, it is worth pointing out the nature of the proposal as an open task (Sullivan, 2018), to the extent that the resolution path was not previously defined and the students had the opportunity to explore different strategies, mobilize knowledge and make creative decisions throughout the construction cycles. This open character reinforces the integration of STEAM practices and the maker environment as spaces that value authorship and the resolution of real problems.

In the *first cycle*, the predominant focus was on mastering GeoGebra's functionalities, with emphasis on software operability (how to use the tool to build the polygon from two given starting points) and on trying to create a replicable model, so that they could reproduce the pieces on a large scale without having to repeat the construction for each new tile. As they were concerned with the tool's operational issues, the students did not focus as much on the construction's rigor; when they finished, they realized that the formed part lacked stiffness when the initial points were dragged. The maker character of this stage is evident in the investigative posture and the autonomous use of tools, even if the initial products have not passed the "dragging test" (Zulatto, 2002).

The loss of the invariant properties of the figure, far from being a limitation, reflected a typical iterative design process in the STEAM context, in which ideas are prototyped, tested, and improved in successive cycles, with room for error as a constitutive part of learning (Doorman et al., 2019). In addition, this initial cycle laid the groundwork for more structured thinking in later cycles, showing how technology served as a mediating tool between the idea and its materialization.

In the *second cycle*, the incorporation of geometric elements, such as parallels and bisectors, indicated an advance in the perception of the regularities of the piece and in the pursuit of rigidity in digital construction. The identification of deformations when moving free points aroused a closer look at the criteria necessary to maintain the properties of the figure, even after the process of dragging these points. Here, the students' practice engages directly with the principles of visual organization as they try to structure a coherent, stable pattern, in line with the perceptual foundations of geometric design. Records provided by the students after the second construction cycle are shown in Figure 3, including the deformations of the pieces on the right when the free points (initial objects) in blue were moved.


(a)

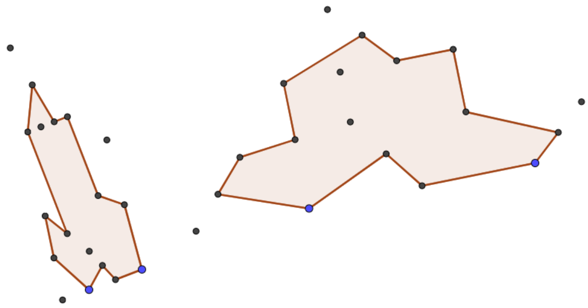

(b)

Figure 3. Representations obtained by the students in the second construction cycle

At this stage of the construction process, the use of parallels and bisectors not only helped assessing the geometric rigidity of the piece but also aligned with the perceptual principles of unification and symmetry, favoring a visual organization that made the piece part of a coherent whole within the tiling pattern. This perception reinforces the relationship between geometric reasoning and visual design in the project, marking a transition from a technical focus to a more integrated approach that connects mathematics and the arts, preparing the ground for the increasing complexity of the following cycles. In addition, this moment allowed the students to mobilize, albeit intuitively, the Gestalt principle of good form, to the extent that they sought patterns that favored clarity and visual stability, both fundamental to the proposed tessellation.

During the *third cycle*, still working autonomously but seeking additional references online, the students used the triangular mesh as a support resource to better identify convenient symmetries and partitions. Thus, the use of the triangular mesh and support for reflected symmetries mark a turning point in the process: the perception of regularities and relationships among the components of the piece was enhanced, demonstrating the role of graphical representations and visual reasoning in the construction of mathematical knowledge.

In the construction process, they once again used an equilateral triangle as a reference; from this, they reflected it across its sides and, again, reflected one of the triangles obtained by the previous reflections. After the reflections, they demarcated two bisectors of each of these four 'new triangles' to delimit the remaining contour of the piece, as illustrated in Figure 4. The manipulation and exploration of the physical parts at this stage of construction (sometimes even scratching them) were essential for them to understand the angles and lengths of the contours of the part, as well as the existing symmetries.

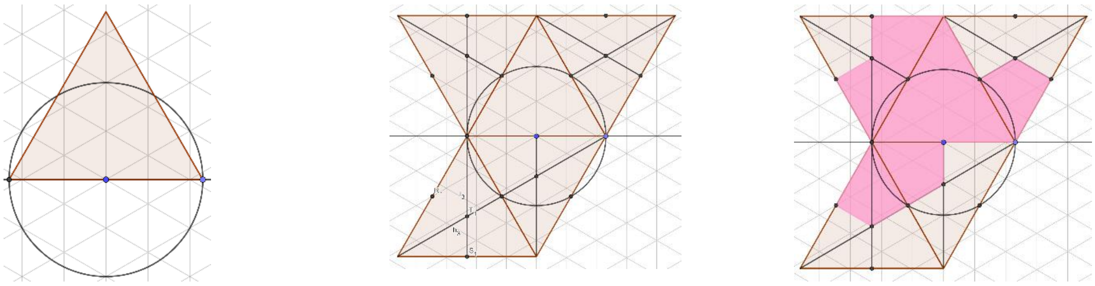


Figure 4. Representations obtained by the students in the second construction cycle

The construction process[10] started with an equilateral triangle juxtaposed with the quadrilaterals defined by the center, the midpoints of two sides of the triangle, and by the vertex of intersection of these two sides. These quadrilaterals define the third parts of the original triangle and were fundamental for the fourth and last construction cycle. In addition, this construction evidenced an important fact that was to recognize that the perimeter of the piece had only three measures of different sides, being them half of the side of the original triangle (six appearances, in red, in the figure below), the apothem of the original triangle (six appearances, in blue, in the figure below) or twice the apothem (the latter measure appearing on only one of the 13 sides of the piece, in purple, in Figure 5 below).

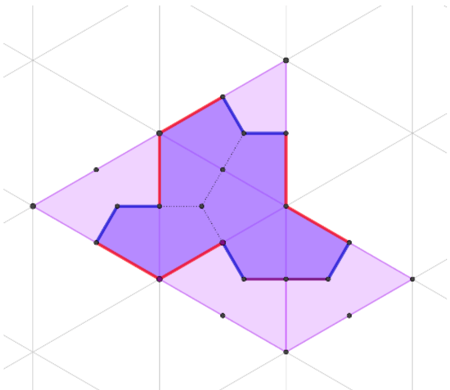


Figure 5. Representation of the piece built with emphasis on the different sides

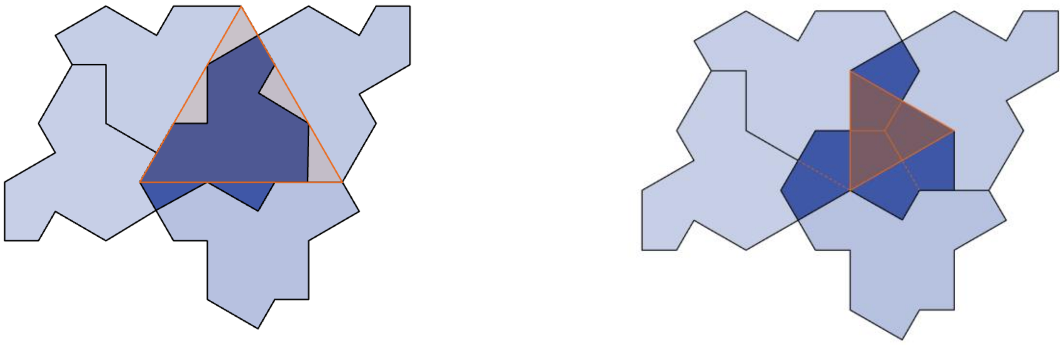
The process developed in this cycle shows how the combined use of physical and digital resources supports the construction of meaning, as outlined by the framework of distributed cognition (Noss & Hoyles, 1996), and reinforces STEAM practice. The construction, which began with the triangle and its reflections, contributed to the perception of spatial patterns and symmetries in the plane, while mobilizing geometric modeling and problem-solving competencies, in line with the formative objectives of STEAM practices (Ortiz-Laso et al., 2023). This cycle marks a stage of greater integration among the mathematical, artistic, and technological dimensions, consolidating the sTeAM practice and showing how the use of graphical representations and symmetries strengthens the role of visual perception as a mediator in geometric learning.

The *fourth cycle* was consolidated after a discussion with the advisor about the previous construction, which also allowed for a new construction based on an initial equilateral triangle and the symmetry of its thirds (generated by the segments that join the center to the midpoint on each side). This moment represented the synthesis of the process: the students' work evolved into an adjusted and more "economic" construction, in which the reflections were applied directly to the thirds of the triangle, eliminating unnecessary steps and organizing the pattern clearly, that is, the reflections made were no longer from the original triangle to later define the contour, but rather, directly from the third parts (quadrilaterals defined by the center, midpoints of two sides of the triangle, and by the vertex of intersection of these two sides), considerably reducing the construction steps.

It is important to note that this construction cycle was a natural evolution of the previous cycle, to the extent that it was evidenced that the 'protrusions' beyond the original triangle were the result of symmetries of the third parts of the triangle. In addition, this advance shows how the maker environment associated with the use of technologies creates conditions for learning, enabling the materialization of complex patterns in artistic and mathematical practice.

This advance not only reduced the operational complexity of the construction but also illustrated the development of students' perceptions and thinking by reorganizing steps, recognizing patterns, and optimizing processes—central elements of STEAM practices (Doorman et al., 2019). This final cycle reinforced the full articulation among the geometric, technological, and artistic dimensions, demonstrating how the maker environment can mediate the development of complex competencies, including mathematical and design thinking processes.

From the different construction cycles developed, we noted that the geometric construction process is closely linked to the identification of parts in relation to their symmetry patterns (translation, rotation, and reflection) and their composition. These contrasts are highlighted in the representations in Figure 6.



(a) (b)

Figure 6. Representation of the part contrasting different forms of perception

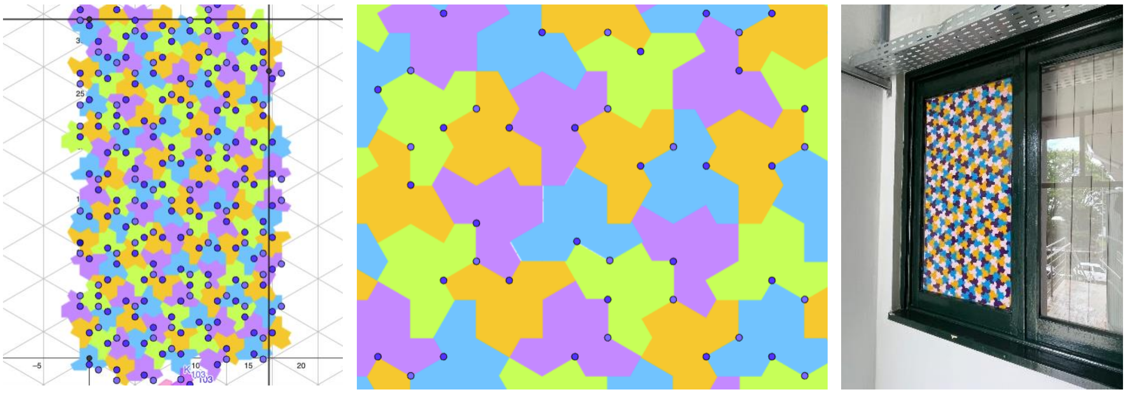
While the first two cycles were more closely associated with a design of the part by difference and the transfer of regions from a triangle[11], the last two cycles had constructions designed by adding symmetrical parts (identical to the thirds of the original triangle). Figure 6 (a) shows that the protruding part of the detached piece, in relation to the orange triangle, is congruent to the missing parts in relation to the same triangle. In Figure 6 (b), parts obtained by reflections of the thirds of this triangle are added to the original reference triangle.

This possibility of strategies reveals how experience facilitated the transition between more analytical and more synthetic approaches to geometric reasoning, an essential characteristic in teaching integrated with the arts and technology.

To this end, physical manipulation of the material was fundamental to establishing geometric criteria in digital construction. It is also evident that the adopted strategies affect the construction process through the use of the tool (technological development) and can involve more or fewer construction stages. It is also important to note that both constructions differ from those presented by Santos Neto et al. (2023), since they did not use an equilateral triangle as a basis. In the first version presented by the authors, the same quadrilateral that serves as the basis in Figure 6(b) is constructed, but from the polygon that defines it rather than as a third of an equilateral triangle. This comparison reinforces the creative potential of the project by exploring alternative constructive paths, highlighting how the open task promoted original solutions and the development of autonomy by students, in addition to reinforcing the principle of simplifying the Gestalt and the exercise of rethinking the pedagogical practices adopted, as recommended by Batista et al. (2017) and Burow and Scherpp (1985).

In addition to the geometric construction of the pieces, the making of the stained-glass window depended on correctly predicting the fittings (the assembly itself). Although tiling with the same piece (using its front and back) has the property of covering the plane infinitely, the aperiodic nature makes covering a predefined area with the assembly an exercise of considerable difficulty. In this case, the students sought a reference containing a sufficient quantity to reproduce the assembly. However, they did so in an orderly manner, preserving the proportion of the window to ensure coverage once the parts were effectively cut and assembled.

For this purpose, the tool for defining the part from two given points was useful, so the exercise was to evaluate where the initial points for the placement of the part should be, according to the reference model[12]. In addition, to achieve better harmony in the final presentation, it was suggested to the students that the assembly of the pieces be done in accordance with the Four Color Theorem[13], a fact already foreseen in the digital model. This aspect reinforces the connection among the arts, geometry, and technology in the reported experience, integrating principles of visual design with mathematical rigor and anticipating the synthesis presented in the final reflection. The images of the digital representation and the final window can be seen in Figure 7.



(a) (b) (c)

Figure 7. Representation of the mosaic made in GeoGebra and in the window

This final stage exemplifies how the project fostered autonomy in planning, combining mathematical rigor, aesthetic sensitivity, and technological competencies. The articulation of physical manipulation and digital construction, which permeated the entire process, was decisive in enhancing students' active and reflective learning (Borba et al., 2020) by fostering dialogue among geometric reasoning, visual perception, and the use of technologies in the maker environment.

The application of the Four Color Theorem in the final assembly of the stained-glass window demonstrates how the students mobilized principles of geometric design and visual perception to achieve a harmonious, mathematically structured aesthetic, integrating the arts, geometry, and technology into the final product. This path, characterized by the exploration of different strategies and solutions for the construction of the stained glass window, concretizes in practice the concept of the open task (Sullivan, 2018) that supported the proposal, by allowing multiple paths of resolution and promoting the development of the STEAM practice, in addition to exemplifying how the Gestalt principles operated transversely throughout the experience, to the extent that the students visually organized the structures to generate meaning and coherence to the final product.

The critical seam between Gestalt and the study of tiling lies in the exploration of how structural forms and relationships are perceived in artistic and mathematical creation. The combined use of physical and digital resources (such as GeoGebra and laser cutters) enhances the construction of meaning while requiring the student to attend to the visual and structural properties of the mathematical object in question. This approach reinforces active learning, pattern discovery, and structure reorganization, which are core elements in both Gestalt and artistic tiling practice. This link between visual perception, geometric construction, and technological design proved essential in developing the competencies implemented during the task, consolidating the integration of the arts, mathematics, and technology in an articulated way.

The decision to explore an irregular aperiodic tiling pattern for the creation of the stained-glass window connects directly to the principles of Gestalt in geometry teaching. The stained-glass window, when designed based on a 13-sided polygon that fits in tessellation without overlaps, materializes the perceptual principles of visual organization, such as unification, for example, by identifying the sides of the same length for fitting of pieces, segregation, and simplification (Batista et al., 2017; Lefrançois, 2008). Students' construction of the pattern involved the mental reorganization of shapes to identify spatial relationships and create a visually harmonic arrangement, a process in which insight, a central concept of Gestalt theory, plays an essential role. This closeness between geometry and the arts, mediated by the principles of perception, not only enriches creative work but also enhances mathematical learning by favoring the relational understanding of geometric structures in the plane. This process shows how the proposed formative practice transcended simple technical execution, becoming a co-creation experience, characteristic of STEAM practices integrated into the maker culture.

In the reported experience, the use of digital technologies (such as GeoGebra and laser-cutting software) and non-digital technologies (such as a ruler, a pen, and a laser-cutting machine) enabled students to transition among different forms of representation and construction of geometric knowledge. This integration reinforces the perspective of distributed cognition (Noss & Hoyles, 1996), in which subject, tool, and knowledge constitute an inseparable system in the learning process. The interaction between digital resources and physical materials expanded the possibilities for exploring geometric patterns and promoted visual, dynamic, and interactive mathematical reasoning (Borba et al., 2020; Healy & Sinclair, 2007). By acting in this environment, the students developed interdisciplinary competencies of an integral and contextualized education, and the experience reaffirms the transformative potential of proposals that integrate arts, mathematics, and technology in teacher education, creating bridges between theory and practice, favoring the development of a critical and creative look at teaching and learning.

Final considerations

Data analysis was conducted using a qualitative approach, articulating the geometric construction records developed in GeoGebra by the students through the lens of Gestalt principles. The four construction cycles developed throughout the experiment were interpreted in the light of the concepts of symmetry, simplification (*Prägnanz*), unification, segregation, and insight, as discussed in the theoretical foundation. This analysis helped us shed light on the role of visual perception and cognitive reorganization in the development of geometric constructions mediated by technology.

Symmetry emerged as a central element in the most advanced construction cycles (third and fourth), although it appeared procedurally in cycle 2 as point symmetry. In the third cycle, the students began using successive reflections of an equilateral triangle across their own sides, resulting in a more stable and regular structure. This procedure evolved in the fourth cycle for an even more refined strategy, using the third parts of the original triangle as basic units of symmetry. This reorganization not only yielded greater savings in steps but also a clearer perception of the geometric regularities of the piece.

The evolution from the second to the fourth cycle also reveals an increasing *simplification* of constructions. While in the first attempts the focus was on operational issues of the GeoGebra tool, subsequent cycles show a refinement of the constructive logic, with fewer steps and greater clarity in the definition of the geometric objects involved. The perception that the perimeter of the piece was based on only three distinct measures is an example of the application of the *Prägnanz* principle, which states that the perceptual system tends to organize stimuli as simply and coherently as possible. This *simplification* is not only visual but also cognitive: by dealing with fewer variables and greater regularity, the students were able to deepen their understanding of the structure of the piece, resulting in greater control over the shape during free-point manipulation. The results obtained through different processes of understanding (and construction) are consistent with those proposed by Kus and Newcombe (2025).

The construction of the final stained-glass window required the students to organize the pieces cohesively, respecting proportions and ensuring adequate window coverage. The choice to respect the Four Color Theorem, in addition to improving aesthetic harmony, also reinforced the principle of *unification*: the tendency to group visual elements that belong to the same set. The construction was understood at this stage not only as a sum of parts but also as a cohesive global structure. Therefore, *unification* favored a less fragmented learning experience, allowing students to understand the stained glass as a system rather than as a juxtaposition of isolated parts.

The second cycle revealed difficulties related to the deformation of the figures when the free points were manipulated. From the third cycle, the use of triangular meshes, bisectors, and distinct colors enabled students to better discriminate the different segments and elements of the construction, favoring perceptual segregation; that is, the ability to distinguish relevant parts within a whole. Such clarity in the delimitation of the elements is fundamental to geometric understanding and reiterates the need for the human gaze in search of balance and visual harmony, as pointed out by Gomes Filho (2009).

The transition between the second and third cycles can be understood as a moment of *insight*, as Lefrançois (2008) highlights. Through physical manipulation of the pieces, the students mentally reorganized the geometric relationships, identifying symmetries and previously unperceived patterns. This reorganization not only made the constructions more consistent but also revealed a qualitative change in how the problem was understood.

As a final perspective of this work, the pieces created have been reproduced in acrylic and in different colors in the laser cutter and shared at events and fairs of mathematical dissemination, where they have usually received good acceptance and involvement from children, adolescents, and adults, as illustrated in Figure 8.

Figure 8. Sharing of “Einstein” pieces at fairs and events

When exposed, the pieces are usually accompanied by a brief orientation to “invite” those who interact with them to cover different paper sizes (the largest being A4), so that they seek to meet the challenge, which is an open task since it presents different ways to solve it. The free exploration of the material also allows participants to perceive new assembly patterns, sometimes recognizing familiar shapes and achieving a certain harmony.

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Notes

[1] In the sense of promoting situations or problems that allow more than one (re)solution and encourage students to do more than just recall known facts (Sullivan, 2018).
[2] This concept will be presented later in this text. Technically, if the symmetry group of a mosaic has at least two non-parallel translations, then the mosaic is called periodic. A mosaic that is not periodic is said to be aperiodic.
[3] Note that, in this case, there is an emphasis on the M of mathematics.
[4] We use the capital letters T, A, and M to reinforce that the activities we develop focus on Technology, Arts, and Mathematics. For the general use of the STEAM concept, we keep all letters capitalized.
[5] As a curiosity, according to Stewart (2012, p.10), symmetry did not become a dominant idea along the expected path: geometry. The beautiful and indispensable concept of symmetry that mathematicians and physicists use today arrived through algebra.
[6] From the German, a single stone.
[7] From the English, hat, for its resemblance to the shape.
[8] First author of this article.
[9] Some copies were made available to them by the professor-researcher who supervised them. He had been given these pieces at a maker fair he attended.
[10] A step-by-step guide to this construction, developed by the students, can be found at https://www.geogebra.org/m/pb7vyspc.
[11] Note that the orange triangle highlighted in Figure 6 (a) is not exactly the same as the one with which construction was started (Figure 3 (a)) in the 2nd cycle. However, from the triangle identified in Figure 3 (a), in the 2nd construction cycle, and the line parallel to its left side, the triangle referred to herein is obtained.
[12] It is worth noting that, in fact, two models of the pieces were made, since the same physical piece can fit with its front or with its back.
[13] The Four Color Theorem ensures that four colors are sufficient to color the regions of any map, so that two adjacent regions do not have the same color. By adjacent, two regions with a common border (not just a vertex) are considered. In addition, for the validity of the theorem, an important condition is that it is only about maps with directly connected regions.

References

Amaral, C. A. N., Mariano, J. C. F., & Wrobel, J. S. (2014). Oficina de pavimentação com triângulos equiláteros. *Anais do Encontro Internacional de Ensino de Matemática, 2*(1), 1-12.

André, M. E. D. A. (1995). *Etnografia da prática escolar*. Papirus.

Arnal-Palacián, M. (2021). Mathematical flexibility of degree of primary education students in solving an area problem: Pick's theorem. *Mathematics Teaching Research Journal, 12*(4), 21–35.

Barros, F. C. (2016). P*avimentações do Plano: Propostas lúdicas de aula. Trabalhando com Ângulos Internos, Simetrias, Isometrias, Obras de Artes e Mediatrize*s [Dissertação de Mestrado]. Instituto de Matemática Pura e Aplicada, Rio de Janeiro. Retrieved from https://impa.br/wp-content/uploads/2016/12/TCC_Felipe_de_Carvalho_Barros.pdf.

Batista, L. S, Souza, M. D. O, Nobrega, M. R. A., & Silva, M. F. (2017). Atividades matemáticas à luz da teoria da Gestalt. In *Anais IV CONEDU* (pp. 1-9). CEMEP/UEPB.

Berry, R. Q., & Larson, M. R. (2019). The need to catalyze change in high school mathematics. *Phi Delta Kappan, 100*(6), 39–44. https://doi.org/10.1177/0031721719834027

Blanco, T. F., González-Roel, V., Diego-Mantecón, J. M., & Ortiz-Laso, Z. (2021). Análisis de la conexión arte-matemáticas en los libros de texto de Educación Primaria. *Educación matemática, 33*(3), 67-93. https://doi.org/10.24844/EM3303.03

Bogdan, R. C., & Biklen, S. K. (1994). *Investigação qualitativa em educação*: uma introdução à teoria e aos métodos. Portugal: Porto Editora.

Borba, M. C., Silva, R. R. S., & Gadanidis, G. (2020). *Fases das tecnologias digitais em educação matemática: Sala de aula e internet em movimento*. Autêntica Editora.

Borba, M. C., & Villarreal, M. E. (2005). *Humans-with-Media and the Reorganization of Mathematical Thinking: Information and Communication Technologies, Modeling, Experimentation and Visualization*. Springer.

Burow, O-A., & Scherpp, K. (1985). *GestaltPedagogia: Um Caminho para a Escola e a Educação.* Summus.

Carniel, M., Lieban, D. E., & Deconti, G. S. (2023). Um vitral como perspectiva STEAM combinando arte, matemática e tecnologia. In *MTC- Mostra Técnico -Científica 2023- Campus Bento Gonçalves* (p. 1)*.* IFRS*.*

Chen, S. Y., & Lin, S. W. A. (2019). Cross-cultural study of mathematical achievement: From the perspectives of one's motivation and problem-solving style. *International Journal of Science and Mathematics Education,* 18, 1149–1167. https://doi.org/10.1007/s10763-019-10011-6

Diego-Mantecón, J. M., Blanco, T. F., Búa, J. B., & Sequeiros, P. G. (2019). Is the relationship between art and mathematics addressed thoroughly in Spanish secondary school textbooks? *Journal of Mathematics and the Arts, 13*(1–2), 25–47. https://doi.org/10.1080/17513472.2018.1552068

Diego-Mantecón, J. M., Ortiz-Laso, Z., & Blanco, T. F. (2022). Implementing STEM projects through the EDP to learn mathematics: the importance of teachers' specialization. In P. R. Richard, M. P. Vélez, & S. Van Vaerenbergh (Eds.), *Mathematics education in the age of artificial intelligence* (pp. 399-415). Springer. https://doi.org/10.1007/978-3-030-86909-0_17

Doorman, M., Bos, R., de Haan, D., Jonker, V., Mol, A., & Wijers, M. (2019). Making and implementing a mathematics day challenge as a makerspace for teams of students. *International Journal of Science and Mathematics Education, 17*, 149–165. https://doi.org/10.1007/s10763-019-09995-y

English, L. D. (2020). Facilitating STEM Integration Through Design. In J. Anderson & Y. Li (Eds.), *Integrated Approaches to STEM Education: An International Perspective* (pp. 45-66). Springer. https://doi.org/10.1007/978-3-030-52229-2_4

Gomes Filho, J. (2009). *Gestalt do objeto: sistema de leitura visual da forma*. Escrituras.

Gómez-Gálvez, P., Vicente-Munuera, P., Tagua, A., Forja, C., Castro, A. M., Letrán, M., ... & Escudero, L. M. (2018). Scutoids are a geometrical solution to three dimensional packing of epithelia*. Nature Communications*, *9*(1), 2960. https://doi.org/10.1038/s41467-018-05376-1

Harron, J. R., Jin, Y., Hillen, A., Mason, L., & Siegel, L. (2022). Maker math: Exploring mathematics through digitally fabricated tools with K–12 in-service teachers. *Mathematics*, *10*(17), 3069. https://doi.org/10.3390/math10173069

Healy, L., & Sinclair, N. (2007). If this is our mathematics, what are our stories?. *International Journal of Computers for Mathematical Learning*, *12*(1), 3-21. https://doi.org/10.1007/s10758-006-9109-4

Hsu, P. S., Lee, E. M., Ginting, S., Smith, T. J., & Kraft, C. (2019). A case study exploring non-dominant youths' attitudes toward science through making and scientific argumentation. *International Journal of Science and Mathematics Education, 17*, 185–207. https://doi.org/10.1007/s10763-019-09997-w

Kurti, R. S., Kurti, D. L., & Fleming, L. (2014). The philosophy of educational makerspaces part 1 of making an educational makerspace. *Teacher Librarian*, *41*(5), 8–11. https://newblankets.org/worth_a_look/philosophy_of_makerspace.pdf

Kus, M., & Newcombe, N. S. (2025). Facilitation of students' disembedding in an online visual arts and mathematics education program. *International Journal of STEM Education, 12*(8), 1-29. https://doi.org/10.1186/s40594-024-00524-0

Lefrançois, G. R. (2008). *Teorias da aprendizagem*. Cengage Learning.

Lévy, P. (1997). *As tecnologias da inteligência*. Editora 34.

Namukasa, I. K., Gecu-Parmaksiz, Z., Hughes, J., & Scucuglia, R. (2023). Technology maker practices in mathematics learning in STEM contexts: A case in Brazil and two cases in Canada. *ZDM – Mathematics Education, 55*(7), 1331–1350. https://doi.org/10.1007/s11858-023-01534-y

Neto, A., Loss, C., Silva, M., Maria, S., & Dal Col, A. (2023). Tesselações: Ladrilhos de Penrose e sua construção com o GeoGebra. *Professor de Matemática Online (PMO)*, *11*(6), 81–96. https://doi.org/10.21711/2319023x2023/pmo1106

Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings: Learning cultures and computers*. Kluwer Academic Publishers.

Ortiz-Laso, Z. (2023). *El enfoque integrado STEAM y su impacto en la enseñanza de las matemáticas en educación secundaria: un estudio experimental* [Tese de doutoramento, Universidad de Cantabria]. Repositorio Abierto de la Universidad de Cantabria. https://hdl.handle.net/10902/30200

Ortiz-Laso, Z., Diego-Mantecón, J. M., Lavicza, Z., & Blanco, T. F. (2023). Teacher growth in exploiting mathematics competencies through STEAM projects. *ZDM – Mathematics Education, 55*(7), 1283-1297. https://doi.org/10.1007/s11858-023-01528-w

Paganelli, A., Cribbs, J. D., Huang, X. S., Pereira, N., Huss, J., & Chandler, W. (2016). The makerspace experience and teacher professional development. *Professional Development in Education, 43*(2), 232-235. https://doi.org/10.1080/19415257.2016.1166448

Santos Neto, A., Barbosa, S. M., & Dal Col, A. (2023). Fundamentos para a Construção no GeoGebra de Tesselações Aperiódicas usando um Único Polígono. *arXiv, 1*, 1-13. https://doi.org/10.48550/arXiv.2311.14753

Smith, D., Myers, J. S., Kaplan, C. S., & Goodman-Strauss, C. (2024). An aperiodic monotile. *Combinatorial Theory, 4*(1), 1-91. https://doi.org/10.5070/C64163843

Stake, R. E. (2010). *Qualitative research: Studying how things work*. The Guilford Press.

Stewart, I. (2012). *Uma história da Simetria na Matemática*. Zahar.

Sua, C., Gutiérrez, A., & Jaime, A. (2024). Analogies: a way to promote the learning of proof in 3d geometry using dynamic geometry environments. In T. Lowrie, A. Gutiérrez & F. Emprin (Eds.), *Proceedings of the 26th ICMI Study Conference* (pp. 409-416). INSPÉ. Université de Reims Champagne-Ardenne.

Stohlmann, M. (2018). A vision for future work to focus on the "M" in integrated STEM. *School Science and Mathematics, 118*(7), 310-319. https://doi.org/10.1111/ssm.12301

Sullivan, P. (2018). *Challenging mathematical tasks: Unlocking the potential of all students*. Oxford University Press.

Yin, Y., Hadad, R., Tang, X., & Lin, Q. (2020). Improving and assessing computational thinking in maker activities: The integration with physics and engineering learning. *Journal of Science Education and Technology, 29*, 189–214. https://doi.org/10.1007/s10763-019-09995-y

Zulatto, R. B. A. (2002). Professores de Matemática que Utilizam Softwares de Geometria Dinâmica: suas características e perspectivas. [Dissertação de Mestrado, Universidade Estadual Paulista]. Repositório da UNESP. http://hdl.handle.net/11449/91012