Symmetry detection in early childhood as a foundation for spatial and STEM development

A deteção de simetria na infância como base para o desenvolvimento espacial e nas áreas STEM

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Abstract. Early spatial skills in children predict future mathematical and STEM performance. This study investigates 4- and 5-year-old children's (N=41) detection of symmetry transformations using simplified coloured magic squares. We examined recognition of invariance through rotations and reflections with visual tasks and tissue paper activities. ANOVA results indicate that rotational symmetry was stable across ages, except for 270° rotations, which showed improvement at age five. In contrast, reflectional symmetry showed significant gains, particularly along vertical and horizontal axes. These findings suggest that reflection tasks demand greater cognitive effort than rotations. Early educational experiences should therefore gradually introduce rotational activities before reflections to lay the groundwork for developing spatial abilities and symmetry reasoning. This study also supplements recent discussions on the pedagogical significance of spatial, hands-on learning in early STEM education.
Keywords: symmetry, STEM, spatial skills, coloured magic squares, early childhood education, dihedral group D_4.

Resumo. As competências espaciais desenvolvidas precocemente nas crianças predizem o desempenho futuro em Matemática e em áreas STEM. Este estudo investiga a capacidade de crianças de 4 e 5 anos (N = 41) para detetarem transformações de simetria utilizando quadrados mágicos coloridos e simplificados. Analisámos o reconhecimento da invariância em rotações e reflexões através de tarefas visuais e atividades com papel vegetal. Os resultados da ANOVA indicam que a simetria rotacional se manteve estável entre idades, exceto nas rotações de 270°, que evidenciaram melhorias aos cinco anos. Em contraste, a simetria reflexiva revelou ganhos significativos, sobretudo nos eixos vertical e horizontal. Estes resultados sugerem que as tarefas de reflexão exigem um esforço cognitivo superior ao das rotações. Assim, as experiências educativas precoces devem introduzir gradualmente atividades de rotação antes das de reflexão, de modo a consolidar as bases para o desenvolvimento das competências espaciais e do raciocínio sobre simetria. Este estudo

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contribui ainda para as discussões recentes sobre a relevância pedagógica de aprendizagens espaciais e manipulativas na educação STEM em idades precoces.
Palavras-chave: simetria, STEM, competências espaciais, quadrados mágicos coloridos, educação de infância, grupo diedral D4.

Background

This study is a continuation of our previous work on the pedagogical use of magic squares as a tool for introducing primary school students to the concept of symmetry, specifically invariance, or “sameness in change" (Weyl, 1952). Our previous research developed a pedagogical trajectory linking symmetry with mathematical structure, facilitating the emergence of early symmetry reasoning (Brasili & Piergallini, 2021).

Building on the theory and pedagogy developed in the "*Mathematics beyond numbers, symmetry and the search for invariants*" project, launched in 2018 at the Montegranaro School, this study extends the findings of the previous investigation to younger children. Specifically, these experiments were carried out at the I.C. Fracassetti Capodarco school in Fermo to explore symmetry-based visual and spatial tasks in 4- and 5-year-old children using 2x2 coloured squares adapted to their perceptual and cognitive abilities.

Such 2x2 squares simplify the exploration of coloured 3x3 squares, a method used in previous experiments in which coloured patterns were used to identify the eight equivalent squares according to the solutions of 3x3 magic squares (Brasili & Piergallini, 2022), using both visual and manipulative methods.

Objectives/Research Question

This study builds on prior research into symmetry-based mathematics and geometry learning. It aims to investigate how very young children recognise invariance through rotations and reflections using simplified coloured 2×2 squares.

To understand both the age-appropriateness of the task and the nature of early symmetry processing, the investigation addresses two research questions:

RQ1. Is the symmetry questionnaire suitable for 4- and 5-year-old children, as determined by its psychometric properties?

RQ2. Do children aged four and five differ in their ability to recognize rotations and reflections, and how do these differences manifest across transformations?

Overall, addressing these questions will provide evidence to identify which symmetry operations are accessible in the early childhood, and how these operations potentially support the development of spatial abilities and STEM-related skills.

Theoretical framework

Spatial skills encompass a range of human cognitive abilities that enable the generation, retention, manipulation, and reasoning about spatial relationships among objects and in space (Uttal et al., 2013).

Under the umbrella of spatial thinking, they include mental rotation, spatial visualization, perspective-taking, and spatial working memory. Spatial thinking underpins human perception and interaction with the environment and constitutes a foundational component of learning in STEM domains (Maresch & Sorby, 2022).

Spatial thinking shows high neuroplasticity during childhood, making its practice a particularly effective and productive approach (Bansil & Yabut, 2025; Gilligan-Lee et al., 2022). It has been shown that early spatial skills are good predictors of later success in mathematics and STEM-related pathways, even when controlling for verbal and general cognitive abilities (Mix & Cheng, 2012; Tosto et al., 2014). As a result, the development of spatial thinking in early childhood has been identified as an important area for fostering STEM readiness (Yang et al., 2020).

Within the spectrum of spatial abilities, symmetry plays a central role as a class of transformations that support spatial reasoning through structured variation and invariance.

Symmetry-related processing involves recognising geometric transformations that preserve relational structure, linking perceptual organisation with higher-order cognitive operations (Seah & Horne, 2019). Empirical studies suggest that symmetry influences visual working memory and contributes to cognitive efficiency beyond low-level visual processing (Sztuka & Kühn, 2025). Engagement with symmetric structures has been shown to reduce working memory load by enabling compact visual representations and by supporting orientation-independent strategies rather than effortful mental rotation (He et al., 2022). In educational contexts, early experiences with shape recognition and geometric transformations have been shown to support broader mathematical development (Sarama & Clements, 2009).

In this context, the primary task in symmetry learning is to identify invariant properties that allow different transformations to be recognised as instances of the same underlying structure (Glattfelder, 2019). Developmental evidence suggests that this capacity emerges initially at a global level. Hu and Zhang (2019) demonstrate that preschool children first construct a general concept of symmetry, which is subsequently refined through differentiation into specific symmetry types. As argued in earlier theoretical work (Brasili & Piergallini, 2021), this trajectory suggests that symmetry is initially perceived as a holistic sense of balance and unity, before becoming an explicit conceptual and analytical tool. In this sense, symmetry functions not merely as a perceptual attribute but as an epistemic principle for detecting invariants across transformations and over time (Shaw et al., 1974; Yang, 1996).

Developmental and perceptual studies further suggest that early symmetry sensitivity is not evenly distributed across symmetry types. Research on infants indicates a marked sensitivity to vertical reflection symmetry, especially in upright human faces, which has been attributed to anatomical, motor, and experiential factors (Bornstein et al., 2023). Similar asymmetries are observed in later childhood and educational contexts, where bilateral symmetry is often favoured over rotational or diagonal transformations in both spontaneous productions and instructional designs (Crisci et al., 2024; Lipták, 2023; Zaballa et al., 2023).

Despite growing empirical interest in early symmetry processing, few studies have systematically examined how young children distinguish between rotational and reflective transformations in controlled task environments. This gap is especially evident in preschool populations, where symmetry is often treated as a single concept rather than as a set of transformations with distinct cognitive demands.

Building on this theoretical and empirical background, the present study investigates how 4- and 5-year-old children recognise invariance across rotations (90°, 180°, 270°) and reflections (vertical, horizontal, and diagonal) using coloured 2×2 square configurations within a controlled D_4 symmetry framework.

By examining children's performance across transformation types, the study aims to provide a more nuanced understanding of early symmetry reasoning and to inform the design of developmentally appropriate approaches to early STEM education.

Methodology

This section is divided into subsections outlining the study's methods: participants, tasks, materials, and data collection and analysis. The approach was designed to ensure the activities were appropriate for preschool children and sensitive to differences in symmetry recognition across ages.

Participants

The number of participants is 41 children (27 boys and 14 girls) divided in two age groups: 20 4-year-olds (14 boys, 6 girls) and 21 5-year-olds (13 boys, 8 girls). Initially, the sample was of 51 participants. The reduction to 41 participants was due to early departures from the school, limited Italian fluency, or inability/unwillingness to complete the task.

All children attended one of three kindergartens within the I.C. Fracassetti Capodarco institute in the Fermo area: San Giuliano (S.G.), San Salvatore (S.S.), or San Marco (S.M.). Five teachers carried out the activities. Initially, the teachers considered symmetry concepts potentially advanced for pre-schoolers and were concerned that tasks might cause stress for both children. This concern proved unfounded. The structured nature of the symmetry activities and well-designed materials encouraged children's engagement and productive responses. Teachers also reported a strong sense of ease and confidence while implementing the exercises. Table 1 provides a detailed breakdown of the children's sample composition.

Table 1. Sample distribution by kindergarten, age, and gender.

Kindergarten	**4-years-old**	**5-years-old**
San Giuliano (S.G.)	9 boys, 3 girls (12 total)	6 boys, 4 girls (10 total)
San Salvatore (S.S.)	5 boys, 3 girls (8 total)	7 boys, 5 girls (12 total)
San Marco (S.M.)	0 boys, 0 girls (0 total)	0 boys, 0 girls (0 total)
Total	**14 boys, 6 girls (20)**	**13 boys, 8 girls (21)**

Task and procedure

Each child received a 2×2 coloured square printed on paper and a matching blank square on tissue paper. The reference square displayed a unique arrangement of four different colours, one in each quadrant.

Children first coloured the tissue paper to replicate the reference square. They were then presented with eight additional squares, shown in Figure 1, each representing one of the eight transformations in the dihedral symmetry group D_4: identity (same orientation), three rotations (90°, 180°, 270°), and four reflections (vertical, diagonal-sinister from upper right to lower left, horizontal, and diagonal-dexter from upper left to lower right).

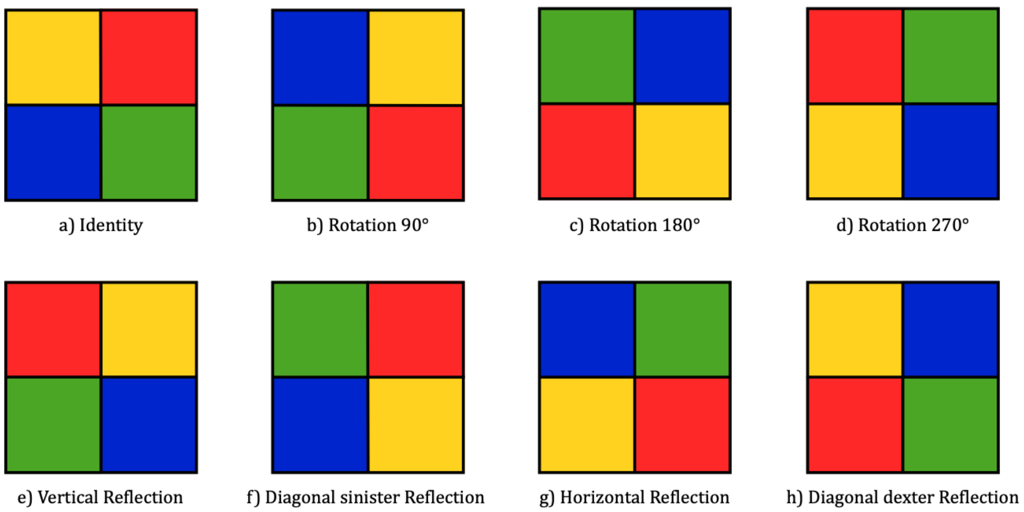

Figure 1. Children's test on all eight D_4 symmetry transforms of a 2×2 coloured square

Using the tissue paper overlay, children physically manipulated their coloured square by rotating or flipping it to match each target transformation. For each match, they described how they completed the task. Teachers recorded whether the child correctly recognised the transformation and matched the reference square to the target configuration.

This hands-on method ensured that every child actively engaged with all eight symmetry operations in an enjoyable and tangible way, initiating their emerging sense of symmetry reasoning in terms of invariance under rotations and reflections. Prior to the assessment, the children had not received any special training or practice in symmetry recognition, nor had they participated in any preparatory activities to help them succeed in the test.

To control for potential variability in teaching approaches, all teachers received the same training and standardised instructions for implementing the task.

Statistical analyses

To address the two research questions, we conducted analyses to evaluate the developmental appropriateness of the symmetry questionnaire (RQ1), and potential age-related differences in children's performance (RQ2).

For RQ1, we assessed the questionnaire's psychometric quality using Item Consistency Trend (ICT) and Classical Test Theory (CTT). ICT examined the logical progression and difficulty of each item to ensure the tasks were appropriate for 4- and 5-year-old children. CTT provided standard indices of item difficulty, discrimination, and reliability, confirming that the questionnaire consistently measured symmetry reasoning across the sample.

To account for potential classroom-level effects, preliminary analyses using ANOVA and intraclass correlation coefficients (ICCs) indicated no significant differences in mean accuracy across the three kindergartens, suggesting minimal influence from instructional variability. Analyses used aggregated scores per child rather than per teacher or classroom to ensure that observed differences reflected children's cognitive performance rather than teacher interaction.

For RQ2, we analysed children's responses to each transformation type. One-sample t-tests determined whether performance differed from chance within each age group, and a global ANOVA evaluated differences in symmetry recognition across age groups and transformation types. The sample size, determined by school availability, provided sufficient statistical power for within- and between-group analyses.

Item consistency trend (ICT) analysis

The ICT analysis evaluated the logic and developmental appropriateness of item difficulty. Mean scores and standard deviations were used to assess the consistency of difficulty across symmetry types. Consistent patterns for the logical difficulty of each symmetry item are shown in Table 2. The sequence of items assessed reflects a progression of difficulty: Identity, Rotations, Reflections, and Diagonals.

Floor and ceiling effects were not a concern, as the easiest item (Identity) was not answered correctly by all participants, and even the hardest item (Diagonal-sinister Reflection) had a success rate exceeding 40%. Good variance across the items suggests that the assessment can effectively distinguish between levels of performance.

Table 2. Summary of Item Difficulty (Mean Scores)

Item	Mean Accuracy	SD	Interpretation
Identity	0.976	0.109	Very easy
Rotation 90°	0.866	0.274	Easy
Rotation 180°	0.793	0.316	Moderate
Rotation 270°	0.756	0.298	Moderate
V. Reflection	0.622	0.458	Moderately difficult
H. Reflection	0.683	0.429	Moderate
D. s. Reflection	0.415	0.314	Difficult
D. d. Reflection	0.463	0.324	Difficult

Classical test theory (CTT) and item discrimination

All items were scored dichotomously (correct = 1, incorrect = 0). Item difficulty (p) was calculated as the proportion of correct responses.

Discrimination indices (D) were computed by comparing performance between the top and bottom 27% of total scorers, specifically the 11 highest and 11 lowest scores. Except for the identity item, all items showed adequate variability, supporting individual differentiation.

Logit-transformed difficulty values were also computed for interpretability in latent trait models. The logit is a scale-invariant measure of item difficulty (p), defined by the formula: $Logit(p) = ln(p/(1-p))$. These values confirm the progression from easily solved items (Identity: 3.74 logits) to more challenging ones (Diagonal-sinister Reflection: -0.34 logits).

Table 3. Summary of Item Difficulty (Mean Scores).

Item	Difficulty (p)	Logit	Discrimination (D)	Interpretation
Identity	0.976	3.74	0.09	Very easy; low discrimination
Rotation 90°	0.866	1.88	0.27	Easy; moderate discrimination
Rotation 180°	0.793	1.35	0.36	Moderate; good discrimination
Rotation 270°	0.756	1.13	0.27	Moderate; moderate discrimination
V. Reflection	0.622	0.50	0.46	Moderate; high discrimination
H. Reflection	0.683	0.77	0.55	Moderate; excellent discrimination
D. s. Reflection	0.415	-0.34	0.37	Difficult; good discrimination
D. d. Reflection	0.463	-0.15	0.28	Difficult; moderate discrimination

Test reliability

Internal consistency of the symmetry questionnaire was assessed using two standard indices for dichotomously scored items: Cronbach's alpha (α) and the Kuder–Richardson formula 20 (KR-20). Cronbach's alpha measures the extent to which the items of a test are related and indicates whether they consistently reflect a single underlying construct. KR-20 is a related reliability index specifically developed for binary (true/false) responses and accounts for both item difficulty and total score variance.

The questionnaire demonstrated strong internal consistency ($\alpha = 0.81$), indicating that most items reflect a single underlying cognitive domain: symmetry reasoning.

The KR-20 score was low at 0.14, probably due to two main factors: first, the presence of very simple items, such as the identity transformation, which almost all children answered correctly; and second, the test's short length, which limits the variability required for high KR-20 scores.

A low KR-20 score does not imply poor task quality; rather, it reflects the questionnaire's diagnostic nature, which primarily evaluates different types of symmetry detection rather than producing a single composite score. Overall, the strong Cronbach's alpha and item-level analyses confirm that the questionnaire is a reliable tool providing informative variance for identifying developmental patterns in symmetry detection.

Ethical considerations

The study was conducted in accordance with standard ethical guidelines for educational research involving minors. Authorization to conduct the research was obtained from the school administration and the teachers involved. Families were informed in advance about the aims, procedures, and materials used in the research, and written informed consent was obtained from parents or legal guardians prior to children's participation. Children's participation was voluntary, and withdrawal was possible at any stage without penalty.

All data were collected anonymously and handled in accordance with established privacy and confidentiality standards. The activities were non-invasive and were carried out within regular classroom practices.

Results

This section presents children's accuracy on rotation and reflection tasks, comparing 4- and 5-year-olds using descriptive and inferential analyses.

Overview of performance across symmetry types

To address RQ2, we analysed children's accuracy on different types of transformation. Transformations were grouped into two categories: rotation and reflection tasks, for each age group. Results are presented as mean accuracy values with standard error of the mean (SEM) to illustrate variability and facilitate comparison.

Figures 2 and 3 show mean accuracy for rotation and reflection tasks, respectively, by age group. ANOVA tests were conducted on the rotation and reflection tasks separately to examine group-level performance. Descriptive statistics, including means and variances, are summarized in the tables below.

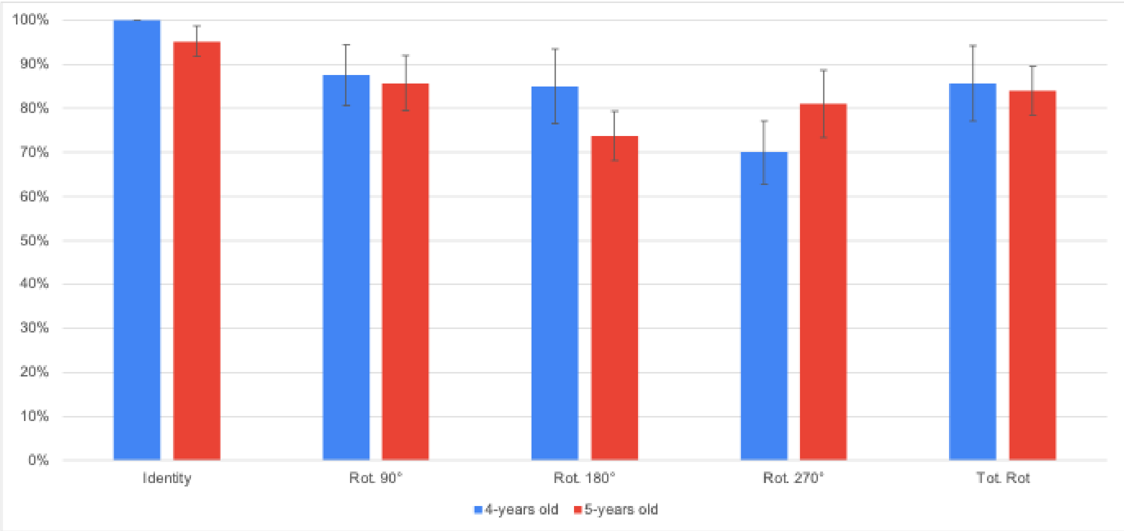

Figure 2. Mean accuracy on rotation tasks by age group, with error bars representing SEM

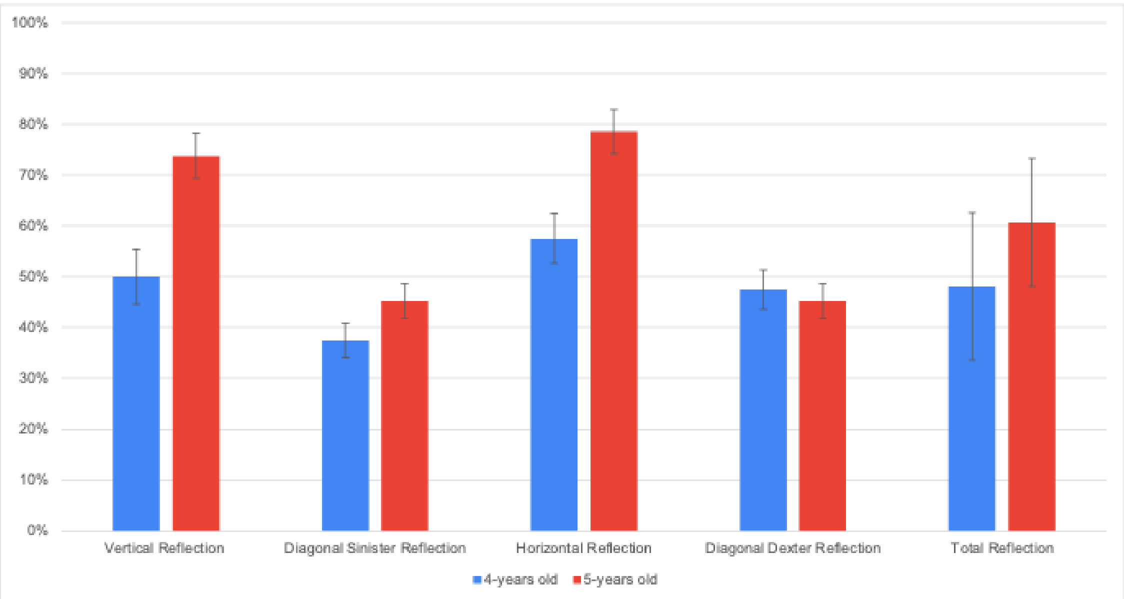

Figure 3. Mean accuracy on reflection tasks by age group, with error bars representing SEM

Rotation tasks by age group

Children showed consistently high accuracy on rotation tasks across both age groups, with only minor decreases on more complex rotations. 5-year-olds performed consistently across all three rotation types, while 4-year-olds showed a slight decrease in accuracy on the 270° transformation. Descriptive statistics and ANOVA results for each rotation task are presented in Table 4, highlighting age-dependent performance trends.

Table 4. Descriptive statistics and ANOVA for rotation tasks by age group

Item	N (4 yrs)	Mean (4 yrs)	Var. (4 yrs)	N (5 yrs)	Mean (5 yrs)	Var. (5 yrs)
Identity	20	0.975	0.012	21	0.976	0.012
Rotation 90°	20	0.875	0.076	21	0.857	0.079
Rotation 180°	20	0.850	0.082	21	0.738	0.115
Rotation 270°	20	0.700	0.116	21	0.810	0.062
Tot. Rotation	20	0.808	0.089	21	0.801	0.085
	ANOVA 4 years: $F(2,57) = 1.97$, p=0.149			**ANOVA 5 years:** $F(2,60) = 0.88$, p=0.419		

To complement these findings, Figure 4 displays boxplots of rotation accuracy, illustrating score distributions and variability by age group and transformation type.

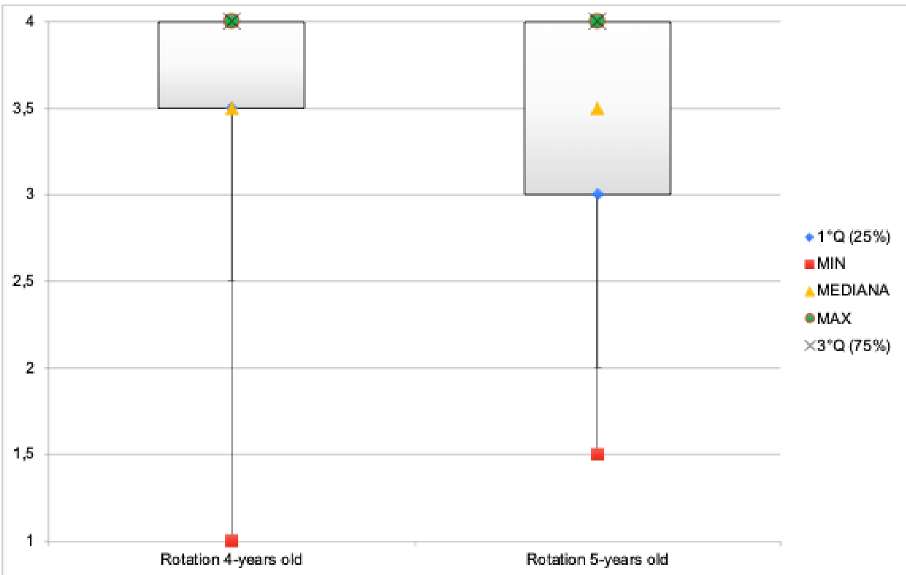

Figure 4. Boxplots of rotation task scores by age group

Although 4-year-olds showed slightly lower accuracy on the 270° rotation, ANOVA results indicated no statistically significant differences across the three rotation tasks in either age group.

This suggests that while rotational complexity may introduce minor performance variations, rotational symmetry is largely well understood by age four and does not show marked improvement between the two age groups.

This interpretation is further supported by the boxplot in Figure 4 below, which displays the distribution of total rotation scores for both age groups. The medians are almost identical, and the interquartile ranges overlap considerably. Both groups also have the same maximum score, indicating a similar ceiling effect; however, the 4-year-old group has a slightly lower minimum score, suggesting that overall variability is similar. Thus, the data visually confirm that performance on the rotation task does not differ substantially between 4- and 5-year-old children.

Reflection tasks by age group

In contrast to the rotation tasks, the children's performance on the reflection tasks showed greater variability and lower overall accuracy, particularly among the 4-year-olds.

As shown in Table 5, 5-year-olds generally outperformed 4-year-olds on all types of reflections (vertical, horizontal, diagonal-sinister, and diagonal-dexter), although the degree of improvement varied by axis.

Table 5. Descriptive statistics and ANOVA for reflection tasks by age group

Item	N (4 yrs)	Mean (4 yrs)	Var. (4 yrs)	N (5 yrs)	Mean (5 yrs)	Var. (5 yrs)
V. Reflection	20	0.500	0.237	21	0.738	0.165
H. Reflection	20	0.375	0.102	21	0.452	0.098
D. s. Reflection	20	0.575	0.191	21	0.786	0.164
D. d. Reflection	20	0.475	0.118	21	0.452	0.098
Tot. Reflection	20	0.482	0.210	21	0.622	0.185
	ANOVA 4 years: $F(3,76) = 0.84$, $p=0.475$			**ANOVA 5 years:** $F(3,80) = 5.17$, $p=0.003$		

Vertical and horizontal reflections were recognized more accurately, while diagonal reflections, especially diagonal-sinister, were more challenging for both age groups. ANOVA results indicated stable performance across reflection types for 4-year-olds but significant differences for 5-year-olds, suggesting age-related specialization in reflection skills.

This pattern is visually illustrated in Figure 5, which presents the boxplots of performance on the reflection task by age group. The median scores and interquartile ranges for the 5-year-olds are consistently higher and show less dispersion, indicating both improved accuracy and more consistent performance. In contrast, the medians were lower and more variable for 4-year-olds, particularly for the more complex diagonal reflections.

These outcomes confirm that reflections, especially along diagonal axes, are more cognitively demanding, and these abilities continue to develop after the age of four.

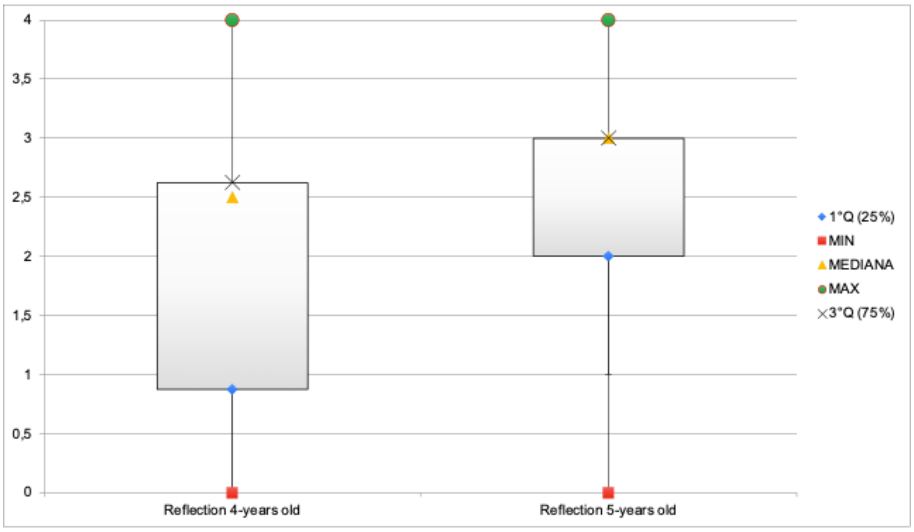

Figure 5. Boxplots of reflection task scores by age group

Discussion

This study assessed the suitability of the symmetry test for preschool children and examined age-related differences. The results reveal a clear developmental dissociation between rotational and reflection symmetry.

Rotational symmetry appears largely mastered by age four, with only minor refinement at age five, whereas reflection symmetry, especially along diagonal axes, showed significant improvements in the older group. Near-ceiling performance on rotation tasks suggests that even younger children already deploy effective spatial strategies for rotational transformations.

These findings align with previous research on early spatial cognition. Pedrett et al. (2022) reported that mental representations of rotational motion are present by approximately 3.5 years of age. However, their ability with more complex or irregular shapes continues to develop until around 5.5 years of age.

Similarly, Frick et al. (2013) observed a marked increase in mental rotation accuracy between ages 3 and 5, with performance approaching ceiling levels by age 5.

The present results support this pattern, suggesting that basic rotational skills develop early in the preschool years.

Further support for the precedence of rotation over reflection comes from Krüger (2018), who showed that although 3-year-olds perform above chance on rotation tasks, they do not exhibit the typical linear increases in reaction time associated with analogue mental rotation. This pattern suggests reliance on task-specific heuristics rather than fully continuous spatial transformations. Comparable dissociations between performance and explicit reasoning have been observed in spatial perspective-taking tasks, in which children can perform geometric transformations before verbalizing them (Ives & Rakow, 1978).

In contrast, reflection tasks impose greater cognitive demands, particularly when diagonal axes are involved, leading to greater variability and slower developmental progression.

From a cognitive perspective, reflection requires an additional operation of inversion across an axis, rather than the continuous transformation involved in planar rotation. This inversion introduces an extra spatial dimension, shifting the task from a purely two-dimensional transformation to a plane–space (2D–3D) correspondence. This added complexity increases working memory and attentional demands.

Piaget's (1952) notion of horizontal décalage provides a useful interpretive framework, suggesting that related spatial concepts may emerge at different times within the same conceptual domain. Neurocognitive accounts further support this distinction: reflection processing appears to rely more heavily on integrative parietal networks, including the intraparietal sulcus, which mature later than those supporting simpler rotational transformations (Dehaene, 2005; Vallortigara, 2006).

From a didactic perspective, these findings have direct implications for the design of early geometry and spatial education tasks. Early instructional activities should begin by focusing on children's strong sensitivity to rotation before progressing to reflection. Activities involving turning or reorienting objects, such as rotating tiles, blocks, or simple geometric patterns, can build on children's existing abilities and help them develop more precise spatial reasoning. In contrast, the observed difficulty with reflections, particularly diagonal reflections, underlines the need for more carefully scaffolded instructional designs.

Didactic tasks that emphasise axis identification, mirror correspondence, folding activities, or embodied gestures may help children gradually internalise reflective transformations. Instructional sequences should therefore distinguish between symmetry types, progressing from rotations to vertical and horizontal reflections before introducing diagonal axes.

These findings support the design of didactic environments that progress from perceptually salient and cognitively economical transformations to those requiring higher cognitive load and explicit geometric reasoning. At later stages, different types of symmetry can be integrated into a unified conceptual framework.

Several limitations should be considered when interpreting these results. Although task instructions were standardised, variability in teaching styles and classroom contexts across schools may have influenced the children's engagement and performance. Social and contextual factors, such as prior informal exposure to spatial play, were not controlled and may have contributed to individual differences. Future studies could address these limitations by explicitly modelling classroom-level effects or by comparing outcomes across a wider range of educational contexts.

In summary, rotation and reflection are distinct cognitive symmetry operations with different developmental trajectories. Rotation appears early, possibly as early as age 3, and can be executed before it becomes verbally accessible, whereas reflection, even along horizontal axes, requires greater cognitive and neural resources.

This asymmetry has direct implications for early geometry instruction, supporting a developmental sequencing of spatial transformations that mirrors children's emerging capacity to detect invariance under increasingly complex transformations.

Conclusion

Our results indicate that rotational and reflectional symmetry do not develop along the same timeline, following distinct developmental trajectories.

Rotational symmetry is typically mastered by age four, often before it becomes verbally accessible, as it is a more complex skill. In contrast, reflectional symmetry, particularly diagonal reflectional symmetry, continues to develop beyond age four, reflecting the higher cognitive demands associated with mental inversion.

These findings support a sequential approach to symmetry instruction: rotational symmetry should be introduced in early childhood. In contrast, reflectional symmetry can be addressed more explicitly around age five, consistent with the progressive development of children's spatial reasoning and invariance detection.

Furthermore, the questionnaire proved to be an effective instrument for both research and educational purposes, demonstrating robust psychometric properties, particularly in terms of internal consistency and item informativeness. These results provide a framework for designing developmentally appropriate symmetry tasks, moving from simpler rotational activities to more cognitively demanding reflection-based exercises.

Future research would benefit from longitudinal studies and neuroimaging approaches to further elucidate the cognitive mechanisms underlying symmetry perception.

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